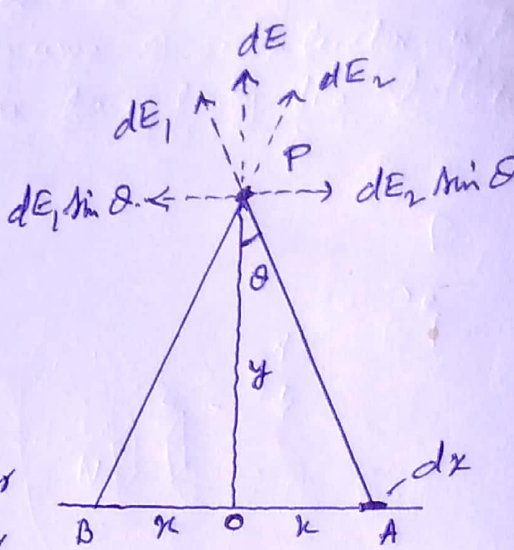
Intensity of Electric field at a point due to.

(1) an infinitely long uniform charged rod

(2) a uniformly charged ring at a point on its axis

Intensity of Electric field at a point due to an infinitely long uniform charged rod :- Let us consider there be a uniformly charged infinitely long rod situated along x-axis on which charge per unit length i.e. linear charge density is λ coulomb/metre.



To calculate the intensity of electric at a point P situated at distance y on the perpendicular bisector of the rod. Let us consider a small element of length dx at a point A which is at a distance x apart from the centre O.

The charge on it will be $dq = \lambda dx$

The intensity of electric field at the point P due to charge dq situated at point A is

$$\vec{dE_1} = \frac{1}{4\pi\epsilon_0}\frac{dq}{(x^2+y^2)} \quad \text{in direction } \vec{AP}$$

Since $AP = \sqrt{OA^2 + OP^2} = \sqrt{x^2+y^2}$

Similarly, the intensity of electric field at the point P due to charge dq situated at point B at distance x from centre on other side, is

$$\vec{dE_2} = \frac{1}{4\pi\epsilon_0}\frac{dq}{(x^2+y^2)} \quad \text{in direction } \vec{BP}.$$

The magnitude of $\vec{dE_1}$ and $\vec{dE_2}$ are same.

The magnitude of electric field at point $\vec{P}$ is calculate, by The resultant intensity of electric field at point P is calculate, by resolving $\vec{dE_1}$ and $\vec{dE_2}$ in components along OP and perpendicular to OP, we find that their horizontal components perpendicular to OP cancel each other as they are equal and in opposite directions, but their components along OP get added together being in one direction (i.e. in direction $\vec{OP}$).

Hence the resultant intensity of electric field at point P is

$$\vec{dE} = \vec{dE_1}\cos\theta + \vec{dE_2}\cos\theta \quad \text{in direction } \vec{OP}$$

$$= 2\cdot\frac{1}{4\pi\epsilon_0}\frac{dq}{(x^2+y^2)}\cos\theta$$

$$= 2\cdot\frac{1}{4\pi\epsilon_0}\frac{dq}{(x^2+y^2)}\cdot\frac{y}{\sqrt{x^2+y^2}}$$

Since in right angled triangle POA $\cos\theta = \frac{OP}{\ldots} = \frac{y}{\ldots}$

or $\quad \vec{dE} = \frac{1}{4\pi\epsilon_0} \frac{2y\,\lambda\,dx}{(x^2+y^2)^{3/2}}$ ——— (1)

Hence the intensity of electric field at the point P due to an infinitely long rod can be calculated by integrating the above expression between the limits of x from 0 to ∞,

$$\vec{E} = \int_0^\infty \frac{1}{4\pi\epsilon_0} \times \frac{2y\,\lambda\,dx}{(x^2+y^2)^{3/2}} \text{ in direction } \vec{OP}$$

Since $\quad x = y\tan\theta$

$\therefore \quad dx = y\sec^2\theta\,d\theta$

$x^2+y^2 = (y\tan\theta)^2 + y^2 = y^2(1+\tan^2\theta) = y^2\sec^2\theta$

$$\therefore \quad \vec{E} = \int_0^{\pi/2} \frac{1}{4\pi\epsilon_0} \frac{2y\,\lambda\, y\sec^2\theta\,d\theta}{(y^2\sec^2\theta)^{3/2}} = \int_0^{\pi/2} \frac{1}{4\pi\epsilon_0} \frac{\lambda \cdot 2\cos\theta}{y}\,d\theta$$

$$= \frac{2\lambda}{(4\pi\epsilon_0)y} (\sin\theta)_0^{\pi/2} = \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{y} \text{ N/C in direction } \vec{OP}$$

Thus the intensity of electric field due to a uniformly positively charged rod of infinite length is radially outwards and its magnitude is inversely proportional to the distance of the point from the rod.

* If the length of rod 'l' are not infinite, then from Eq^n (1), the intensity of electric field at point P is

$$\vec{E} = \int_0^{l/2} \frac{1}{4\pi\epsilon_0} \cdot \frac{2y\lambda\,dx}{(x^2+y^2)^{3/2}} \quad \text{in direction } \vec{OP}$$

or $$\vec{E} = \int_0^{\alpha} \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{y} \cos\theta\,d\theta.$$

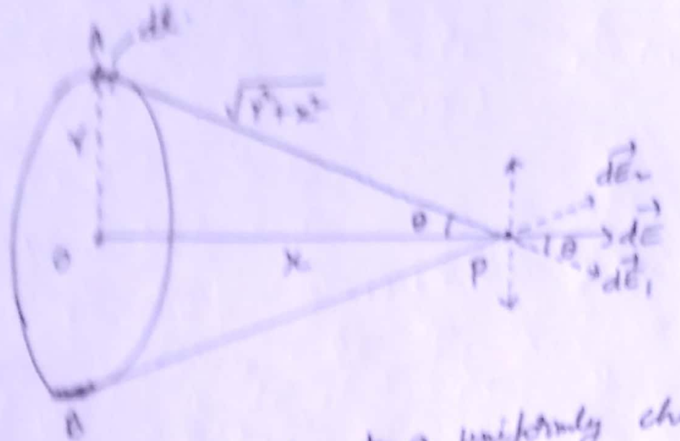
$$= \frac{1}{4\pi\epsilon_0} \frac{2\lambda}{y} \sin\alpha$$

where α is the angle subtended by one end of the rod at the point P.

Then $\quad \sin\alpha = \frac{l/2}{\sqrt{(l/2)^2+y^2}}$

Hence $$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0 y} \cdot \frac{2\lambda\,l}{\sqrt{l^2+4y^2}}} \text{ in direction } \vec{OP}$$

Intensity of Electric Field due to a Uniformly charged Ring at a Point on its Axis :- Let a charge q coulomb be uniformly distributed on a uniform ring of radius 'r'.



Electric field due to a uniformly charged ring

To calculate the intensity of electric field at point P situated at distance x from centre O on its axis.
charge on unit length of the circumference of ring i.e. linear charge density

$$\lambda = \frac{q}{2\pi r} \quad \text{coulomb / metre}$$

Consider a small element of length dl at a point A on the circumference of the ring on which the charge is $dq = \lambda\, dl$.
The intensity of electric field at point P due to this charge element

$$\vec{dE_1} = \frac{1}{4\pi\epsilon_0} \frac{\lambda\, dl}{(r^2+x^2)} \quad \text{in direction } \vec{AP}$$

Similarly, the intensity of electric field at point P due to an element of length dl at point B diametrically opposite to the point A is

$$\vec{dE_2} = \frac{1}{4\pi\epsilon_0} \frac{\lambda\, dl}{(r^2+x^2)} \quad \text{in direction } \vec{BP}$$

We see that $\vec{dE_1}$ and $\vec{dE_2}$ are equal in magnitude.
The resultant intensity of electric field at the point P is calculated by using $\vec{dE_1}$ and $\vec{dE_2}$ are resolved into horizontal and vertical components, we find that their vertical components being equal and in opposite directions cancel each other while the horizontal components get added together.

Hence, resultant intensity of electric field at the point P is

$$\vec{dE} = d\vec{E}_1 \cos\theta + d\vec{E}_2 \cos\theta \quad \text{in direction } \vec{OP}$$

$$= 2 \times \frac{1}{4\pi\epsilon_0} \frac{\lambda\, dl}{(r^2+x^2)} \cdot \frac{x}{\sqrt{r^2+x^2}}$$

Since $\cos\theta = \frac{OP}{AP} = \frac{x}{\sqrt{r^2+x^2}}$

or $$d\vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda\, dl\, x}{(r^2+x^2)^{3/2}}$$

Now the intensity of electric field at the point P due to the entire ring will be equal to the sum of intensities of electric field due to all these small elements of the ring. i.e.

$$\vec{E} = \sum \frac{1}{2\pi\epsilon_0} \frac{\lambda\, dl\, x}{(r^2+x^2)^{3/2}} \quad \text{in direction } \vec{OP}$$

$$= \frac{1}{2\pi\epsilon_0} \frac{\lambda x}{(r^2+x^2)^{3/2}} \sum dl$$

Since for half circumference of the ring $\sum dl = \pi r$

$$\therefore \quad \vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda x}{(r^2+x^2)^{3/2}} \cdot \pi r$$

$$= \frac{1}{2\pi\epsilon_0} \cdot \frac{x}{(r^2+x^2)^{3/2}} \cdot \frac{q}{2\pi r} \cdot \pi r \quad \text{putting } \lambda = \frac{q}{2\pi r}$$

$$\boxed{\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{qx}{(r^2+x^2)^{3/2}} \ \text{N/C}} \quad \text{——— (1)}$$

It is clear that the intensity of electric field at any point on its axis due to a uniformly positively charged ring will be directed along the axis of the ring outwards. But if the ring is negatively charged, then its direction will be along the axis towards the ring.

<u>Case (i)</u> :- If $x \gg r$, i.e. if P lies at a very large distance from O, r can be neglected and we get

$$E = \frac{1}{4\pi\epsilon_0} \frac{qx}{x^3} = \frac{1}{4\pi\epsilon_0} \frac{q}{x^2}$$

which is the intensity of electric field due to a point charge at a distance x.

Case (ii): If $x << r$ i.e. if the point P lies very near to the centre, the expression for E reduces to

$$E = \frac{1}{4\pi\epsilon_0}\frac{qx}{r^3}$$

Case (iii): At $x=0$ i.e. at the centre of ring, $E=0$.

From Eq^n(1), it can be shown that the intensity of electric field is maximum at distance $\frac{r}{\sqrt{2}}$ from the centre of ring since the condition for E to be maximum is $\frac{\partial E}{\partial x} = 0$

or $$\frac{1}{4\pi\epsilon_0}\cdot\frac{q}{(r^2+x^2)^3}\left[(r^2+x^2)^{3/2} - x\cdot\frac{3}{2}(r^2+x^2)^{\frac{1}{2}}\cdot 2x\right] = 0$$

or $$r^2 - 2x^2 = 0$$

or $$x = \pm\frac{r}{\sqrt{2}}$$

Now at $x = \frac{r}{\sqrt{2}}$, $\frac{\partial^2 E}{\partial x^2}$ will be negative, hence the intensity of electric field will be maximum at distance $x = \frac{r}{\sqrt{2}}$ from the centre of ring

$$\boxed{E_{max} = \frac{q}{6\sqrt{3}\,\epsilon_0 r^2}}$$

We see that in case of $x << r$,

$$E = \frac{1}{4\pi\epsilon_0}\frac{qx}{r^3}$$

Hence if a negative charge $(-Q)$ is left free at a distance x from the centre of a positively charged ring, experience a force

$$F = -QE = \frac{-1}{4\pi\epsilon_0}\frac{qQx}{r^3}$$

will act towards the centre of the ring which is directly proportional to x from the centre. If negative charge is electron then ~~hence~~ the electron will execute simple harmonic oscillations. The equation of motion of the electron can be written as,

$$F = m\frac{d^2x}{dt^2} = -\frac{qe}{4\pi\epsilon_0}\frac{x}{r^3}$$

where m is the mass of the electron.

The period of oscillation of the electron will be

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} = 2\pi\sqrt{\frac{4\pi\epsilon_0 r^3 m}{qe}}$$

$\therefore$ The frequency, $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{qe/(4\pi\epsilon_0 r^3 m)}$