Laser Lecture 2 Line Shape Function

University of Lucknow | Centenary Year
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Relation between Einstein Coefficient A and spontaneous life time of level 2

Assume that an atom in level 2 can make transition (spontaneous) to level 1 only

Rate of transition per unit volume

$$\frac{dN_2}{dt} \propto -\mathrm{N}_2$$

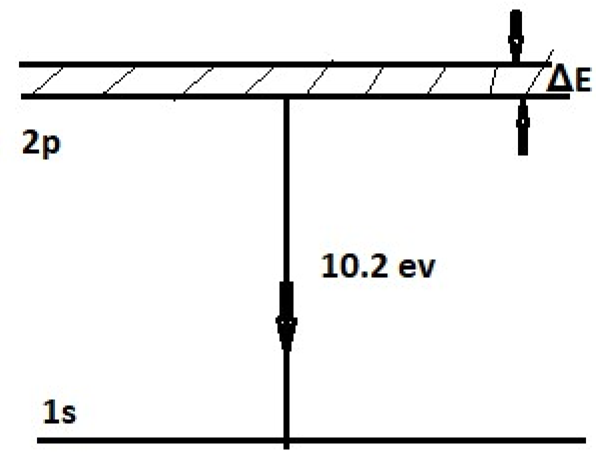
$$\Rightarrow \quad \frac{dN_2}{dt} = -\mathrm{A}_{21}\,\mathrm{N}_2$$

$$\Rightarrow \quad N_2(t) = N_2(0)e^{-A_{21}t}$$

Population of level 2 reduces to 1/e of its initial value in time t_{sp}= 1/A_{21}

This time span is known as spontaneous life time associated with the transition 2 → 1

Example: Hydrogen atom transition 2p→1s



The lifetime of 2p state is t_{sp}=1.6 x 10^{-9} sec

∴ $A_{21} = 1/t_{sp} = 6 \times 10^{8}\ sec^{-1}$

$E_2 - E_1 = 10.2$ ev $=\hbar\omega$, $\omega = 1.55 \times 10^{16}\ sec^{-1}$

$$B_{21} = \frac{\pi^2 c^3}{\hbar\omega^3 n_o^3} A_{21} \approx 4.1x10^{20} m^3/Js^2$$

Here no=1 has been assumed

Line Shape Function

Each energy state has a finite life time. Due to this each energy state has a certain width ΔE (=$\hbar\omega$).

This implies that the atom can absorb or emit radiation over a range of frequencies $\Delta\omega$. This range is usually much less than ω.

When a radiation is incident the strength of interaction of this energy state with radiation is a function of frequency.

This function is called the line shape function. The line shape function is represented by $g(\omega)$.

This function is usually normalized according to the relation

$$\int g(\omega)d\omega = 1$$

Thus out of the total N_2' and N_1' atoms per unit volume, only $N_2' \; g(\omega)\, d\omega$ and $N_1'\, g(\omega) d\omega$ atoms per unit volume are capable of interacting with radiation in the frequency range ω and ω+dω

Thus the number of stimulated emissions per unit volume per unit time will be

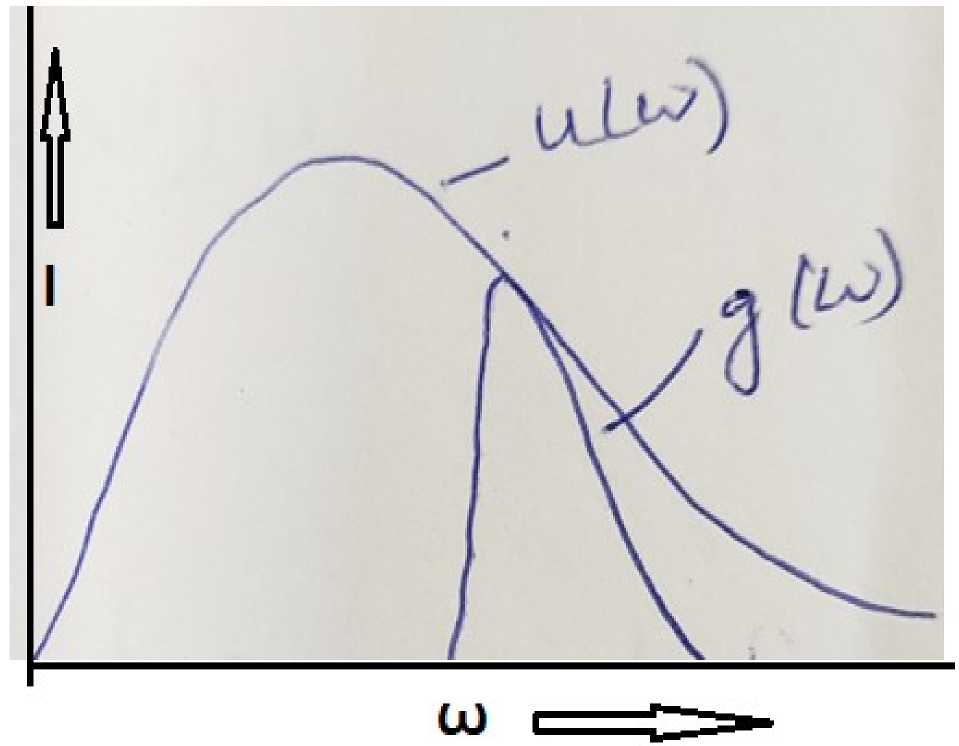
$$\Gamma_{21} = \int B_{21} u(\omega) \mathrm{N}_2 \, \mathrm{g}(\omega)\, \mathrm{d}\omega = \mathrm{N}_2 \frac{\pi^2 c^3}{\hbar\, n_o^3} \frac{1}{t_{sp}} \int \frac{u(\omega) g(\omega)}{\omega^3} \mathrm{d}\omega$$

Consider two cases

⇨ The atoms are interacting with radiation whose spectrum is very broad compared to that of g(ω).

Over the region of interaction where g(ω) is appreciable $u(\omega)/\omega^3$ may be treated to be essentially constant and thus may be taken out of the integral

$$\Gamma_{21} = N_2 \frac{\pi^2 c^3}{\hbar\, n_o^3} \frac{1}{t_{sp}} \frac{u(\omega)}{\omega^3}$$



Here

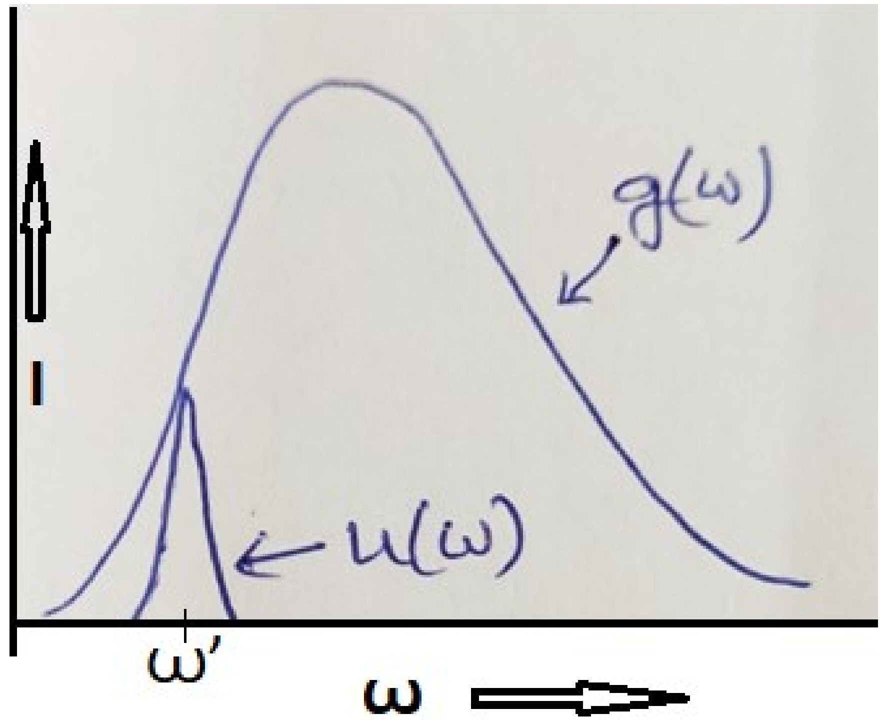
$$\int g(\omega) d\omega = 1$$

Here ω represents transition frequency

The atoms are interacting with near monochromatic radiation

Let the frequency of the incident radiation is ω'

u(ω) curve will be sharply peaked at ω=ω'



$g(\omega)/\omega^3$ may be taken out of the integral

$$\Gamma_{21} = N_2 \frac{\pi^2 c^3}{\hbar\, n_0^3 \omega'^3} \frac{1}{t_{sp}} g(\omega') \int u(\omega) d\omega = N_2 \frac{\pi^2 c^3}{\hbar\, n_0^3 \omega'^3} \frac{1}{t_{sp}} g(\omega') u$$

u is the energy density of the incident near monochromatic radiation. u has dimension of energy per unit volume. u(ω) has dimension of energy per unit volume per unit spectral width.

Thus when the atoms described by the line shape function g(ω) interact with near monochromatic ration at frequency ω' the rate of stimulated emission per unit volume is

$$\Gamma_{21} = N_2 \frac{\pi^2 c^3}{\hbar\, n_o^3 \omega'^3} \frac{1}{t_{sp}} g(\omega') u$$

Similarly, rate of stimulated absorptions per unit volume

$$\Gamma_{12} = N_1 \frac{\pi^2 c^3}{\hbar\, n_o^3 \omega'^3} \frac{1}{t_{sp}} g(\omega') u$$

YouTube Channel Link:
https://www.youtube.com/channel/UC3rdRYA605bdDdsJdEf0oJw/featured

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