

Assignment 5

Q.1)

Solve completely the equation

$$d^2y / dt^2 = c^2 (d^2y / dx^2)$$

Representing the vibration of a string of length l fixed at both the sides

Given,

$$y(0,t)=0$$

$$y(l,t)=0$$

$$y(x,0)=f(x) \text{ and,}$$

$$(\partial y / \partial t)_{t=0}=0$$

Q.2)

The vibrations of an elastic strings are governed by the

Partial Differential Equation

$$\partial^2 u / \partial t^2 = \partial^2 u / \partial x^2$$

The length of the string is π and the ends are fixed, the initial velocity is 0 and the initial deflection is

$$u(x,0)=2(\sin x + \sin 3x)$$

Find the deflection $u(x,t)$ of the vibrating string for $t > 0$.

Q.3)

Using the method of separation of variables,

Solve:

$$\partial u / \partial x = (2\partial u / \partial t) + u$$

$$\text{Where } u(x,0) = 6e^{-3x}$$

Q.4)

A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. It is set vibrating by given each point a velocity

$$\lambda x(l-x)$$

Find the displacement of string $y(x,t)$

Q.5)

Solve

$$\partial u / \partial t = a^2 (\partial^2 u / \partial x^2)$$

Given that,

- $u=0$ when $x=0$
- $u=0$ when $x=l$
- $u = 3\sin(\pi x/l)$ when $t=0$

Q.6)

Solve the equation

$$U_{xx} + U_{yy} = 0 \quad \text{when,}$$

- $u(0,y)=0$
- $u(\pi,y)=0$
- $u(x,0)=K$
- $\lim_{y \rightarrow \infty} u(x,y)=0$

Q.7)

Find the solution of Laplace's equation

$$\nabla^2 \Psi = 0$$

in cartesian coordinates in the region $0 \leq x \leq a$, $0 \leq y \leq b$

To satisfy the condition

$$y=0 \text{ on } x=0, y=b \text{ and } \Psi=a(a-x), 0 \leq x \leq a$$