

Solving systems of linear equations

- Given a $n \times n$ matrix A and a $n \times 1$ vector b , it is required to solve $Ax=b$, for an unknown $n \times 1$ vector x . For $n=4$,

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 = b_1,$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 = b_2,$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 = b_3,$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 = b_4.$$

An SIMD Algorithm- $t(n) = O(n^x)$ where $2 < x < 2.5$

procedure SIMD GAUSS JORDAN (A, b, x)

Step 1: **for** $j = 1$ **to** n **do**
 for $i = 1$ **to** n **do in parallel**
 for $k = j$ **to** $n + 1$ **do in parallel**
 if ($i \neq j$)
 then $a_{ik} \leftarrow a_{ik} - (a_{ij}/a_{jj})a_{jk}$
 end if
 end for
 end for
end for.

Step 2: **for** $i = 1$ **to** n **do in parallel**
 $x_i \leftarrow a_{i,n+1}/a_{ii}$
end for. \square

$$2x_1 + x_2 = 3,$$

$$x_1 + 2x_2 = 4.$$

Example-

After 1 iteration-

$$a_{21} = a_{21} - (a_{21}/a_{11})a_{11} = 1 - \left(\frac{1}{2}\right)2 = 0,$$

$$a_{22} = a_{22} - (a_{21}/a_{11})a_{12} = 2 - \left(\frac{1}{2}\right)1 = \frac{3}{2},$$

$$a_{23} = a_{23} - (a_{21}/a_{11})a_{13} = 4 - \left(\frac{1}{2}\right)3 = \frac{5}{2}.$$

- After 2 iterations-

$$a_{12} = a_{12} - (a_{12}/a_{22})a_{22} = 1 - (1/\frac{3}{2})(\frac{3}{2}) = 0,$$

$$a_{13} = a_{13} - (a_{12}/a_{22})a_{23} = 3 - (1/\frac{3}{2})(\frac{5}{2}) = \frac{4}{3}.$$

Ans- $x_1 = 2/3$, $x_2 = 5/3$.