In this chapter we consider the problem of economic dispatch. During operation of the plant, a generator may be in one of the following states:

i) Base supply without regulation: the output is a constant.

ii) Base supply with regulation: output power is regulated based on system load.

iii) Automatic non-economic regulation: output level changes around a base setting as area control error changes.

iv) Automatic economic regulation: output level is adjusted, with the area load and area control error, while tracking an economic setting. Regardless of the units operating state, it has a contribution to the economic operation, even though its output is changed for different reasons.

The factors influencing the cost of generation are the generator efficiency, fuel cost and transmission losses. The most efficient generator may not give minimum cost, since it may be located in a place where fuel cost is high. Further, if the plant is located far from the load centers, transmission losses may be high and running the plant may become uneconomical. The economic dispatch problem basically determines the generation of different plants to minimize total operating cost.

Modern generating plants like nuclear plants, geo-thermal plants etc, may require capital investment of millions of rupees. The economic dispatch is however determined in terms of fuel cost per unit power generated and does not include capital investment, maintenance, depreciation, start-up and shut down costs etc.

Performance Curves Input-Output Curve

This is the fundamental curve for a thermal plant and is a plot of the input in British

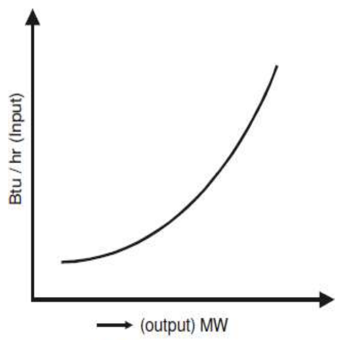

Fig 1: Input – output curve

Thermal units (Btu) per hour versus the power output of the plant in MW as shown in Fig1

Incremental Fuel Rate Curve

The incremental fuel rate is equal to a small change in input divided by the corresponding change in output.

$$\text{Incremental fuel rate} = \frac{\Delta Input}{\Delta Output}$$

The unit is again Btu / KWh. A plot of incremental fuel rate versus the output is shown in Fig 3

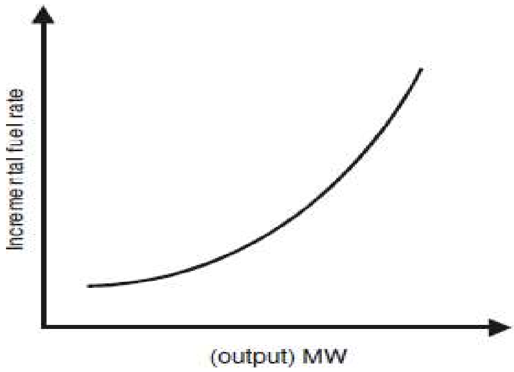

Fig 3: Incremental fuel rate curve

Incremental cost curve

The incremental cost is the product of incremental fuel rate and fuel cost (Rs / Btu) the curve is shown in Fig. 4. The unit of the incremental fuel cost is Rs / MWhr.

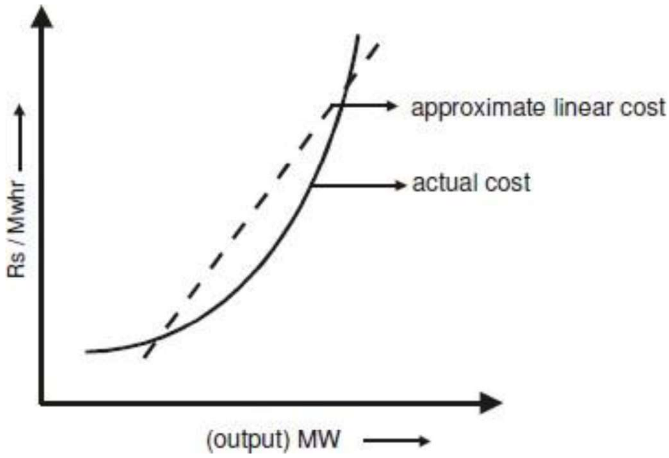

Fig 4: Incremental cost curve

In general, the fuel cost Fi for a plant, is approximated as a quadratic function of the generated output PGi.

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad Rs / h$$

The incremental fuel cost is given by

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \quad Rs / MWh$$

The incremental fuel cost is a measure of how costly it will be produce an increment of power. The incremental production cost, is made up of incremental fuel cost plus the incremental cost of labor, water, maintenance etc. which can be taken to be some percentage of the incremental fuel cost, instead of resorting to a rigorous mathematical model. The cost curve can be approximated by a linear curve. While there is negligible operating cost for a hydel plant, there is a limitation on the power output possible. In any plant, all units normally operate between PGmin, the minimum loading limit, below which it is technically infeasible to operate a unit and PGmax, which is the maximum output limit. **Section I: Economic Operation of Power System**

- **Economic Distribution of Loads between the Units of a Plant**
- **Generating Limits**
- **Economic Sharing of Loads between Different Plants**

In an early attempt at economic operation it was decided to supply power from the most efficient plant at light load conditions. As the load increased, the power was supplied by this most efficient plant till the point of maximum efficiency of this plant was reached. With further increase in load, the next most efficient plant would supply power till its maximum efficiency is reached. In this way the power would be supplied by the most efficient to the least efficient plant to reach the peak demand. Unfortunately however, this method failed to minimize the total cost of electricity generation. We must therefore search for alternative method which takes into account the total cost generation of all the units of a plant that is supplying a load.

Economic Distribution of Loads between the Units of a Plant

To determine the economic distribution of a load amongst the different units of a plant, the variable operating costs of each unit must be expressed in terms of its power output. The fuel cost is the main cost in a thermal or nuclear unit. Then the fuel cost must be expressed in terms of the power output. Other costs, such as the operation and maintenance costs, can also be expressed in terms of the power output. Fixed costs, such as the capital cost, depreciation etc., are not included in the fuel cost.

The fuel requirement of each generator is given in terms of the Rupees/hour. Let us define the input cost of an unit- i, f_i in Rs/h and the power output of the unit as P_i . Then the input cost can be expressed in terms of the power output as

$$f_i = \frac{a_i}{2} P_i^2 + b_i P_i + c_i \quad \text{Rs/h} \qquad (1.1)$$

The operating cost given by the above quadratic equation is obtained by approximating the power in MW versus the cost in Rupees curve. The incremental operating cost of each unit is then computed as

$$\lambda_i = \frac{df_i}{dP_i} = a_i P_i + b_i \quad \text{Rs/MWhr} \qquad (1.2)$$

Let us now assume that only two units having different incremental costs supply a load. There will be a reduction in cost if some amount of load is transferred from the unit with higher incremental cost to the unit with lower incremental cost. In this fashion, the load is transferred from the less efficient unit to the more efficient unit thereby reducing the total operation cost. The load transfer will continue till the incremental costs of both the units are same. This will be optimum point of operation for both the units. The above principle can be extended to plants with a total of *N* number of units. The total fuel cost will then be the summation of the individual
fuel cost f_i , $i = 1, \dots, N$ of each unit, i.e.,

$$f_T = f_1 + f_2 + \cdots + f_N = \sum_{k=1}^{N} f_k \tag{1.3}$$

Let us denote that the total power that the plant is required to supply by P_T , such that

$$P_T = P_1 + P_2 + \cdots + P_N = \sum_{k=1}^{N} P_k \tag{1.4}$$

Where $P_1 , \dots , P_N$ are the power supplied by the *N* different units.
The objective is minimizing f_T for a given P_T. This can be achieved when the total difference df_T becomes zero, i.e.

$$df_T = \frac{\partial f_T}{\partial P_1} dP_1 + \frac{\partial f_T}{\partial P_2} dP_2 + \cdots + \frac{\partial f_T}{\partial P_N} dP_N = 0 \tag{1.5}$$

Now since the power supplied is assumed to be constant we have

$$dP_T = dP_1 + dP_2 + \cdots + dP_N = 0 \tag{1.6}$$

Multiplying (1.6) by λ and subtracting from (1.5) we get

$$\left(\frac{\partial f_T}{\partial P_1} - \lambda\right) dP_1 + \left(\frac{\partial f_T}{\partial P_2} - \lambda\right) dP_2 + \cdots + \left(\frac{\partial f_T}{\partial P_N} - \lambda\right) dP_N = 0 \tag{1.7}$$

The equality in (5.7) is satisfied when each individual term given in brackets is zero. This gives us

$$\frac{\partial f_T}{\partial P_i} - \lambda = 0, \quad i = 1, \dots, N \tag{1.8}$$

Also the partial derivative becomes a full derivative since only the term f_i of f_T varies with P_i, $i = 1 \ldots N$. We then have

$$\lambda = \frac{df_1}{dP_1} = \frac{df_2}{dP_2} = \cdots = \frac{df_N}{dP_N} \qquad (1.9)$$

Generating Limits

It is not always necessary that all the units of a plant are available to share a load. Some of the units may be taken off due to scheduled maintenance. Also it is not necessary that the less efficient units are switched off during off peak hours. There is a certain amount of shut down and start up costs associated with shutting down a unit during the off peak hours and servicing it back on-line during the peak hours. To complicate the problem further, it may take about eight hours or more to restore the boiler of a unit and synchronizing the unit with the bus. To meet the sudden change in the power demand, it may therefore be necessary to keep more units than it necessary to meet the load demand during that time. This safety margin in generation is called spinning reserve.

The optimal load dispatch problem must then incorporate this startup and shut down cost for without endangering the system security.

The power generation limit of each unit is then given by the inequality constraints

$$P_{\min,i} \le P_i \le P_{\max,i}, \quad i = 1, \ldots, N \qquad (1.10)$$

The maximum limit P_{Gmax} is the upper limit of power generation capacity of each unit. On the other hand, the lower limit P_{Gmin} pertains to the thermal consideration of operating a boiler in a thermal or nuclear generating station. An operational unit must produce a minimum amount of power such that the boiler thermal components are stabilized at the minimum design operating temperature.

Example 1

Consider two units of a plant that have fuel costs of

$f_1 = \frac{0.8}{2} P_1^2 + 10 P_1 + 25$ Rs/h and $f_2 = \frac{0.7}{2} P_2^2 + 6 P_2 + 20$ Rs/h

Then the incremental costs will be

$\lambda_1 = \frac{df_1}{dP_1} = 0.8 P_1 + 10$ Rs/MWhr and $\lambda_2 = \frac{df_2}{dP_2} = 0.7 P_2 + 6$ Rs/MWhr

If these two units together supply a total of 220 MW, then $P_1 = 100$ MW and $P_2 = 120$ MW will result in an incremental cost of

$\lambda_1 = 80 + 10 = 90$ Rs/MWhr and $\lambda_2 = 84 + 6 = 90$ Rs/MWhr

This implies that the incremental costs of both the units will be same, i.e., the cost of one extra MW of generation will be Rs. 90/MWhr. Then we have

$f_1 = \frac{0.8}{2}100^2 + 10 \times 100 + 25 = 5025$ Rs/h and $f_2 = \frac{0.7}{2}120^2 + 6 \times 120 + 20 = 5780$ Rs/h

And total cost of generation is p

$f_T = f_1 + f_2 = 10{,}805$ Rs/h

Now assume that we operate instead with P 1 = 90 MW and P 2 = 130 MW. Then the individual cost of each unit will be

$f_1 = \frac{0.8}{2}90^2 + 10 \times 90 + 25 = 4{,}165$ Rs/h and $f_2 = \frac{0.7}{2}130^2 + 6 \times 130 + 20 = 6{,}175$ Rs/h

And total cost of generation is

$f_T = f_1 + f_2 = 10{,}880$ Rs./h

This implies that an additional cost of Rs. 75 is incurred for each hour of operation with this non-optimal setting. Similarly it can be shown that the load is shared equally by the two units, i.e. $P_1 = P_2 = 110$ MW, then the total cost is again 10,880 Rs/h.

Example 2

Let us consider a generating station that contains a total number of three generating units. The fuel costs of these units are given by

$$f_1 = \frac{0.8}{2}P_1^2 + 10P_1 + 25 \text{ Rs/h}$$

$$f_2 = \frac{0.7}{2}P_2^2 + 5P_2 + 20 \text{ Rs/h}$$

$$f_3 = \frac{0.95}{2}P_3^2 + 15P_3 + 35 \text{ Rs/h}$$

The generation limits of the units are

$$30 \text{ MW} \le P_1 \le 500 \text{ MW}$$

$$30 \text{ MW} \le P_2 \le 500 \text{ MW}$$

$$30 \text{ MW} \le P_3 \le 250 \text{ MW}$$

The total load that these units supply varies between 90 MW and 1250 MW. Assuming that all the three units are operational all the time, we have to compute the economic operating settings as the load changes.

The incremental costs of these units are

$$\frac{df_1}{dP_1} = 0.8P_1 + 10$$ Rs/MWhr

$$\frac{df_2}{dP_2} = 0.7P_2 + 5$$ Rs/MWhr

$$\frac{df_3}{dP_3} = 0.95P_3 + 15$$ Rs/MWhr

At the minimum load the incremental cost of the units are

$$\frac{df_1}{dP_1} = \frac{0.8}{2}30^2 + 10 = 34$$ Rs/MWhr

$$\frac{df_2}{dP_2} = \frac{0.7}{2}30^2 + 5 = 26$$ Rs/MWhr

$$\frac{df_3}{dP_3} = \frac{0.95}{2}30^2 + 15 = 43.5$$ Rs/MWhr

Since units 1 and 3 have higher incremental cost, they must therefore operate at 30 MW each. The incremental cost during this time will be due to unit-2 and will be equal to 26 Rs/MWhr. With the generation of units 1 and 3 remaining constant, the generation of unit-2 is increased till its incremental cost is equal to that of unit-1, i.e., 34 Rs/MWhr. This is achieved when P_2 is equal to 41.4286 MW, at a total power of 101.4286 MW.

An increase in the total load beyond 101.4286 MW is shared between units 1 and 2, till their incremental costs are equal to that of unit-3, i.e., 43.5 Rs/MWhr. This point is reached when P_1 = 41.875 MW and P_2 = 55 MW. The total load that can be supplied at that point is equal to 126.875. From this point onwards the load is shared between the three units in such a way that the incremental costs of all the units are same. For example for a total load of 200 MW, from (5.4) and (5.9) we have

$$P_1 + P_2 + P_3 = 200$$

$$0.8P_1 + 10 = 0.7P_2 + 5$$

$$0.7P_2 + 5 = 0.95P_3 + 15$$

Solving the above three equations we get P_1 = 66.37 MW, P_2 = 80 MW and P_3 = 50.63 MW and an incremental cost (λ) of 63.1 Rs./MWhr. In a similar way the economic dispatch for various other load settings are computed. The load distribution and the incremental costs are listed in Table 5.1 for various total power conditions.

At a total load of 906.6964, unit-3 reaches its maximum load of 250 MW. From this point onwards then, the generation of this unit is kept fixed and the economic dispatch problem involves the other two units. For example for a total load of 1000 MW, we get the following two equations from (1.4) and (1.9)

$$P_1 + P_2 = 1000 - 250$$

$$0.8P_1 + 10 = 0.7P_2 + 5$$

Solving which we get P_1 = 346.67 MW and P_2 = 403.33 MW and an incremental cost of 287.33 Rs/MWhr. Furthermore, unit-2 reaches its peak output at a total load of 1181.25. Therefore any further increase in the total load must be supplied by unit-1 and the incremental cost will only be borne by this unit. The power distribution curve is shown in Fig. 5.

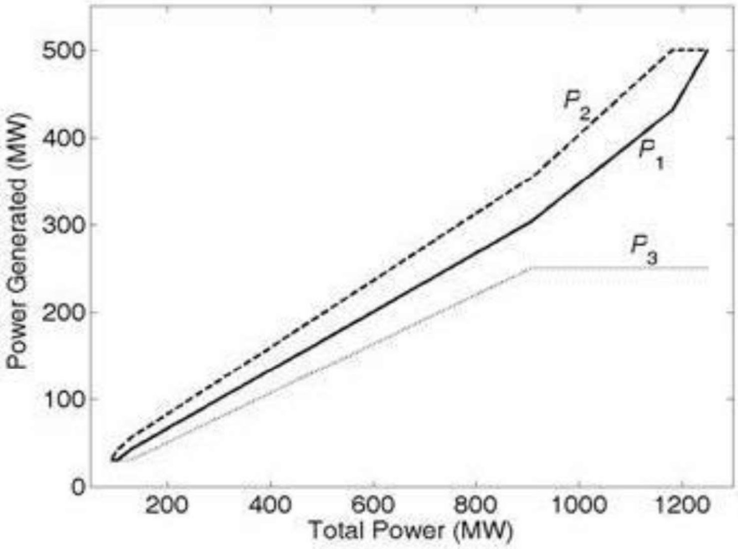

Fig5. Power distribution between the units of Example 2

Example 3

Consider two generating plant with same fuel cost and generation limits. These are given by

$$f_i = \frac{0.8}{2}P_i^2 + 10P_i + 25 \text{ Rs./h} \quad i = 1,2$$

$$100 \text{ MW} \le P_i \le 500 \text{ MW}, \quad i = 1,2$$

For a particular time of a year, the total load in a day varies as shown in Fig. 5.2. Also an additional cost of Rs. 5,000 is incurred by switching of a unit during the off peak hours and switching it back on during the during the peak hours. We have to determine whether it is economical to have both units operational all the time.

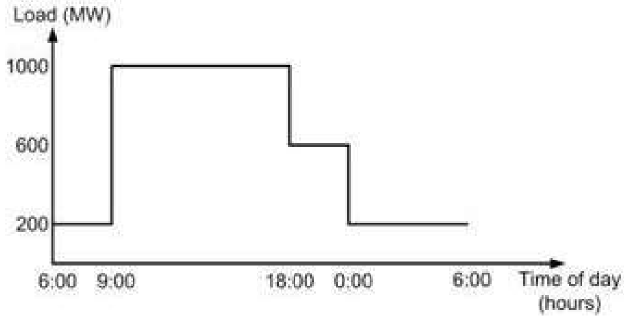

Fig.6. Hourly distribution of a load for the units of Example 2

Since both the units have identical fuel costs, we can switch of any one of the two units during the off peak hour. Therefore the cost of running one unit from midnight to 9 in the morning while delivering 200 MW is

$$\left(\frac{0.8}{2}200^2+10\times 200+25\right)\times 9=162{,}225 \text{ Rs.}$$

Adding the cost of Rs. 5,000 for decommissioning and commissioning the other unit after nine hours, the total cost becomes Rs. 167,225. 0

On the other hand, if both the units operate all through the off peak hours sharing power equally, then we get a total cost of

$$\left(\frac{0.8}{2}100^2+10\times 100+25\right)\times 9\times 2=90{,}450 \text{ Rs.}$$

Which is significantly less that the cost of running one unit alone?

Table 1.1 Load distribution and incremental cost for the units of Example 1

P_T (MW)	P_1 (MW)	P_2 (MW)	P_3 (MW)	λ (Rs./MWh)
90	30	30	30	26
101.4286	30	41.4286	30	34
120	38.67	51.33	30	40.93
126.875	41.875	55	30	43.5
150	49.62	63.85	36.53	49.7
200	66.37	83	50.63	63.1
300	99.87	121.28	78.85	89.9
400	133.38	159.57	107.05	116.7
500	166.88	197.86	135.26	143.5

600	200.38	236.15	163.47	170.3
700	233.88	274.43	191.69	197.1
800	267.38	312.72	219.9	223.9
906.6964	303.125	353.5714	250	252.5
1000	346.67	403.33	250	287.33
1100	393.33	456.67	250	324.67
1181.25	431.25	500	250	355
1200	450	500	250	370
1250	500	500	250	410

DERIVATION OF TRANSMISSION LOSS FORMULA:

An accurate method of obtaining general loss coefficients has been presented by Kroc. The method is elaborate and a simpler approach is possible by making the following assumptions:

(i) All load currents have same phase angle with respect to a common reference

(ii) The ratio X / R is the same for all the network branches

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig a

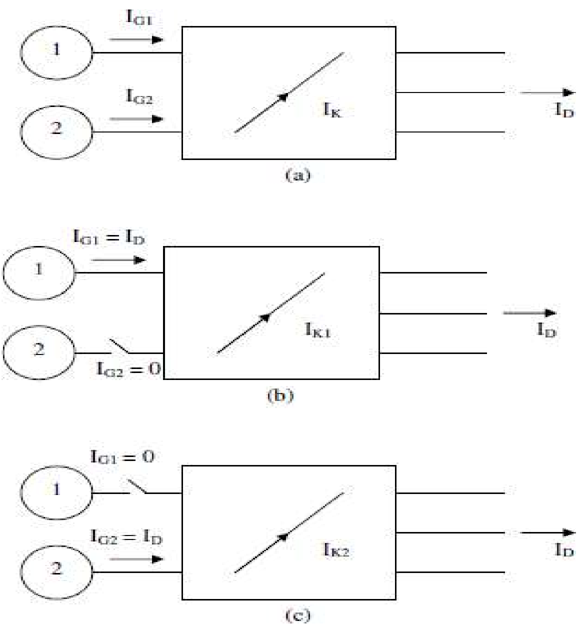

Fig.2.1 Two plants connected to a number of loads through a transmission network

Let"s assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be IK1. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that IG1 = ID in this case. Similarly with only plant 2 supplying the load
Current ID, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

NK1 and NK2 are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of ID. When both generators are supplying the load, then by principle of superposition IK = NK1 IG1 + NK2 IG2
Where IG1, IG2 are the currents supplied by plants 1 and 2 respectively, to meet the demand ID. Because of the assumptions made, IK1 and ID have same phase angle, as do IK2 and ID. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}|\angle\sigma_1 \text{ and } I_{G2} = |I_{G2}|\angle\sigma_2.$$

Where σ_1 and σ_2 are phase angles of IG1 and IG2 with respect to a common reference. We can write

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2\right)^2$$

$$= \begin{array}{l} N_{K1}{}^2|I_{G1}|^2\left[\cos^2\sigma_1 + \sin^2\sigma_1\right] + N_{K2}{}^2|I_{G2}|^2\left[\cos^2\sigma_2 + \sin^2\sigma_2\right] \\ + 2\left[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2\right] \end{array}$$

$$= N_{K1}{}^2|I_{G1}|^2 + N_{K2}{}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

$$\text{Now } |I_{G1}| = \frac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1} \text{ and} |I_{G2}| = \frac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$$

Where PG1, PG2 are three phase real power outputs of plant1 and plant 2; V1, V2 are the line to line bus voltages of the plants and Φ_1 and Φ_2 are the power factor angles.
The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

Where the summation is taken over all branches of the network and RK is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K + \frac{P_{G2}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^2 R_K$$

$$P_L = P_{G1}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K$$

$$B_{22} = \frac{1}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}^2 + \dots\dots + \frac{P_{Gn}^2}{|V_n|^2(\cos\phi_n)^2}\sum_K N_{Kn}^2 R_K$$
$$+2\sum_{\substack{p,q=1\\p\neq q}}^{n} \frac{P_{GP}P_{Gq}\cos(\sigma_p-\sigma_q)}{|V_p||V_q|\cos\phi_p\cos\phi_q}\sum_k N_{KP}N_{Kq}R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n}\sum_{q=1}^{n} P_{Gp}B_{Pq}P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p-\sigma_q)}{|V_p||V_q|\cos\phi_p\cos\phi_q}\sum_K N_{KP}N_{Kq}R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

Economic Sharing of Loads between Different Plants

So far we have considered the economic operation of a single plant in which we have discussed how a particular amount of load is shared between the different units of a plant. In this problem we did not have to consider the transmission line losses and assumed that the losses were a part of the load supplied. However if now consider how a load is distributed between the different plants that are joined by transmission lines, then the line losses have to be explicitly included in the economic dispatch problem. In this section we shall discuss this problem.

When the transmission losses are included in the economic dispatch problem

$$P_T = P_1 + P_2 + \cdots + P_N - P_{LOSS} \quad (2.1)$$

$$0 = dP_1 + dP_2 + \cdots + dP_N - dP_{LOSS} \quad (2.2)$$

Where P_{LOSS} is the total line loss. Since P_T is assumed to be constant, we have

$$dP_{LOSS} = \frac{\partial P_{LOSS}}{\partial P_1} dP_1 + \frac{\partial P_{LOSS}}{\partial P_2} dP_2 + \cdots + \frac{\partial P_{LOSS}}{\partial P_N} dP_N \quad \ldots\ldots\ldots\ldots\ldots (2.3)$$

In the above equation dP_{LOSS} includes the power loss due to every generator, i.e.,

Also minimum generation cost implies $df_T = 0$ as given in (1.5). Multiplying both (2.2) and (2.3) by λ and combining we get

$$0 = \left(\lambda \frac{\partial P_{LOSS}}{\partial P_1} - \lambda\right) dP_1 + \left(\lambda \frac{\partial P_{LOSS}}{\partial P_2} - \lambda\right) dP_2 + \cdots + \left(\lambda \frac{\partial P_{LOSS}}{\partial P_N} - \lambda\right) dP_N \quad (2.4)$$

$$0 = \sum_{i=1}^{N} \left(\frac{\partial f_T}{\partial P_i} + \lambda \frac{\partial P_{LOSS}}{\partial P_i} - \lambda \right) dP_i \quad (2.5)$$

Adding (2.4) with (1.5) we obtain

$$\frac{\partial f_T}{\partial P_i} + \lambda \frac{\partial P_{LOSS}}{\partial P_i} - \lambda = 0, \quad i = 1, \ldots, N \quad (2.6)$$

The above equation satisfies when

$$\frac{\partial f_T}{\partial P_i} = \frac{df_T}{dP_i}, \quad i = 1, \ldots, N$$

Again since

$$\lambda = \frac{df_1}{dP_1} L_1 = \frac{df_2}{dP_2} L_2 = \cdots = \frac{df_N}{dP_N} L_N \quad (2.7)$$

From (2.6) we get

$$L_i = \frac{1}{1 - \partial P_{LOSS} / \partial P_i}, \quad i = 1, \ldots, N \quad (2.8)$$

Where L_i is called the **penalty factor** of load- *i* and is given by

$$P = \begin{bmatrix} P_1 & P_2 & \cdots & P_N \end{bmatrix}^T$$

Consider an area with *N* number of units. The power generated are defined by the vector

$$P_{LOSS} = P^T BP \quad (2.9)$$

Then the transmission losses are expressed in general as

Where *B* is a symmetric matrix given by

$$B = \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1N} \\ B_{12} & B_{22} & \cdots & B_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ B_{1N} & B_{2N} & \cdots & B_{NN} \end{bmatrix}$$

The elements B_{ij} of the matrix *B* are called the **loss coefficients.** These coefficients are not constant but vary with plant loading. However for the simplified calculation of the penalty factor L_i these coefficients are often assumed to be constant.

When the incremental cost equations are linear, we can use analytical equations to find out the economic settings. However in practice, the incremental costs are given by nonlinear equations that may even contain nonlinearities. In that case iterative solutions are required to find the optimal generator settings.