Zeeman Effect

100 Years 1920-2020

University of Lucknow | Centenary Year

लखनऊ विश्वविद्यालय | शताब्दी वर्ष

Zeeman Effect

Splitting up of an energy levels into a number of symmetrically, equally spaced levels when a weak magnetic field is applied.

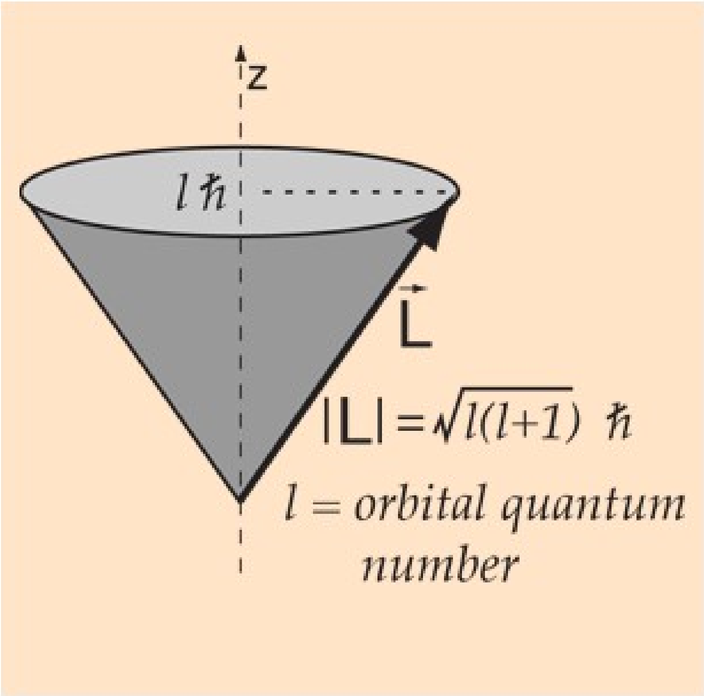
Transition between two sets of these levels, subject to certain selection and intensity rules, gives rise to observed spectral frequencies.

Vector Model

The orbital angular momentum for an atomic electron is visualized in terms of a vector.

Angular momentum vector is seen as precessing about a direction in space. Angular momentum vector has the magnitude

$$|L| = \sqrt{l(l+1)}\ \hbar$$

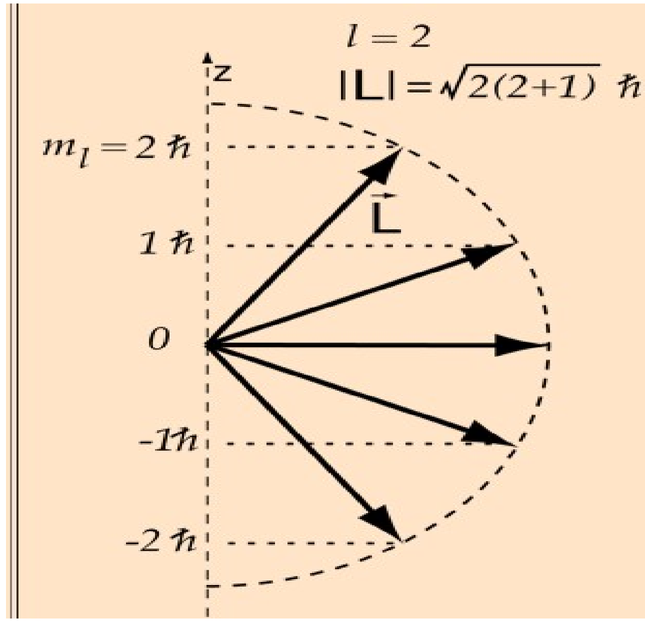


http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/vecmod.html

Only a maximum of l units can be measured along a given direction, where l is the orbital quantum number.

Since there is a magnetic moment associated with the orbital angular momentum, the precession can be compared to the precession of a classical magnet caused by the torque exerted due a magnetic field.

This precession is called Larmor precession and has a characteristic frequency called the Larmor frequency.

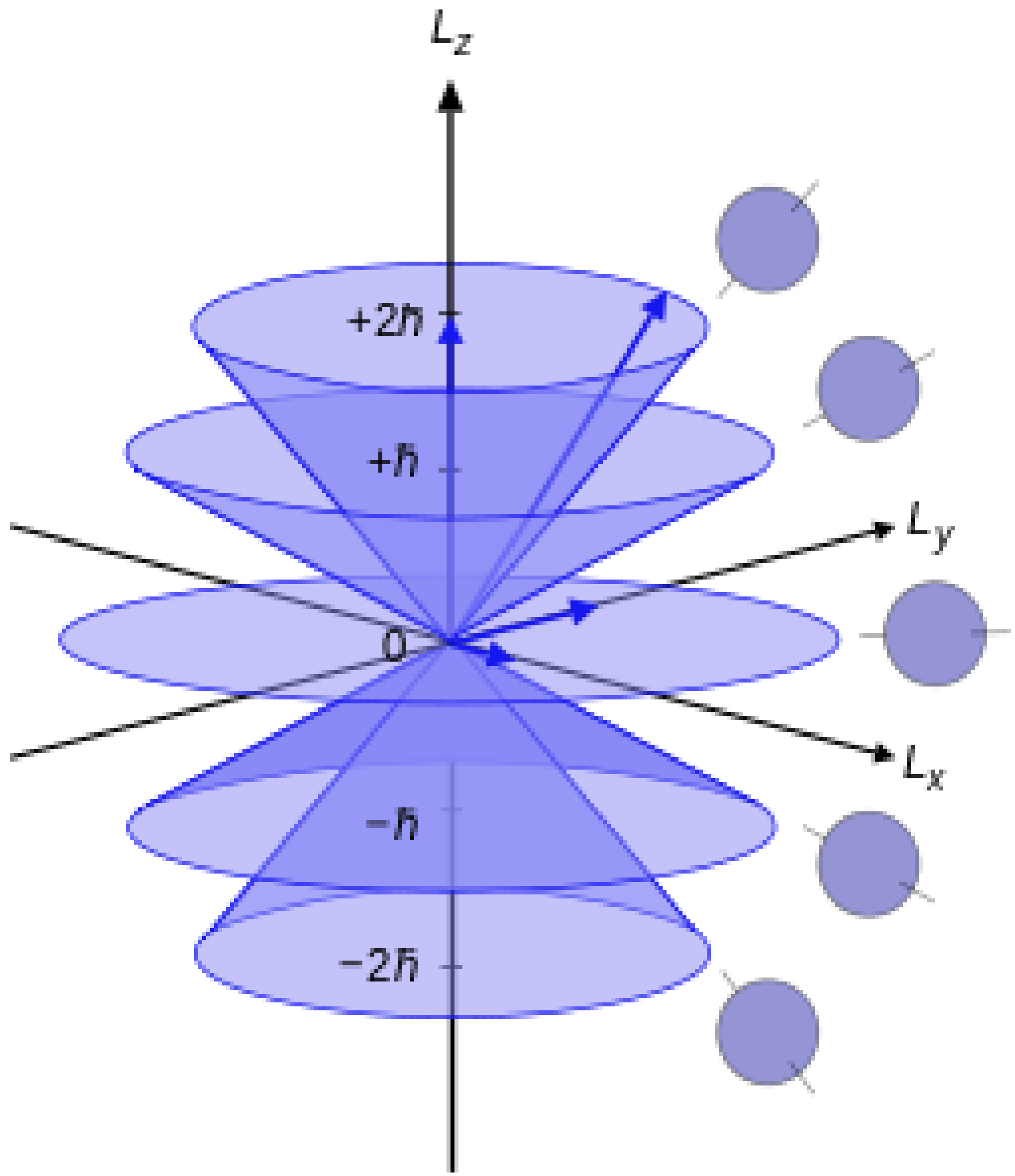


http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/vecmod.html

The projection of L along a direction in space is quantized. The diagram shows the possible values for the magnetic quantum number ml for l=2 can take the values

$$m_{l=} -2, -1, 0, 1, 2$$

$$m_{l=} -l, -l+1, \dots, l-1, l$$



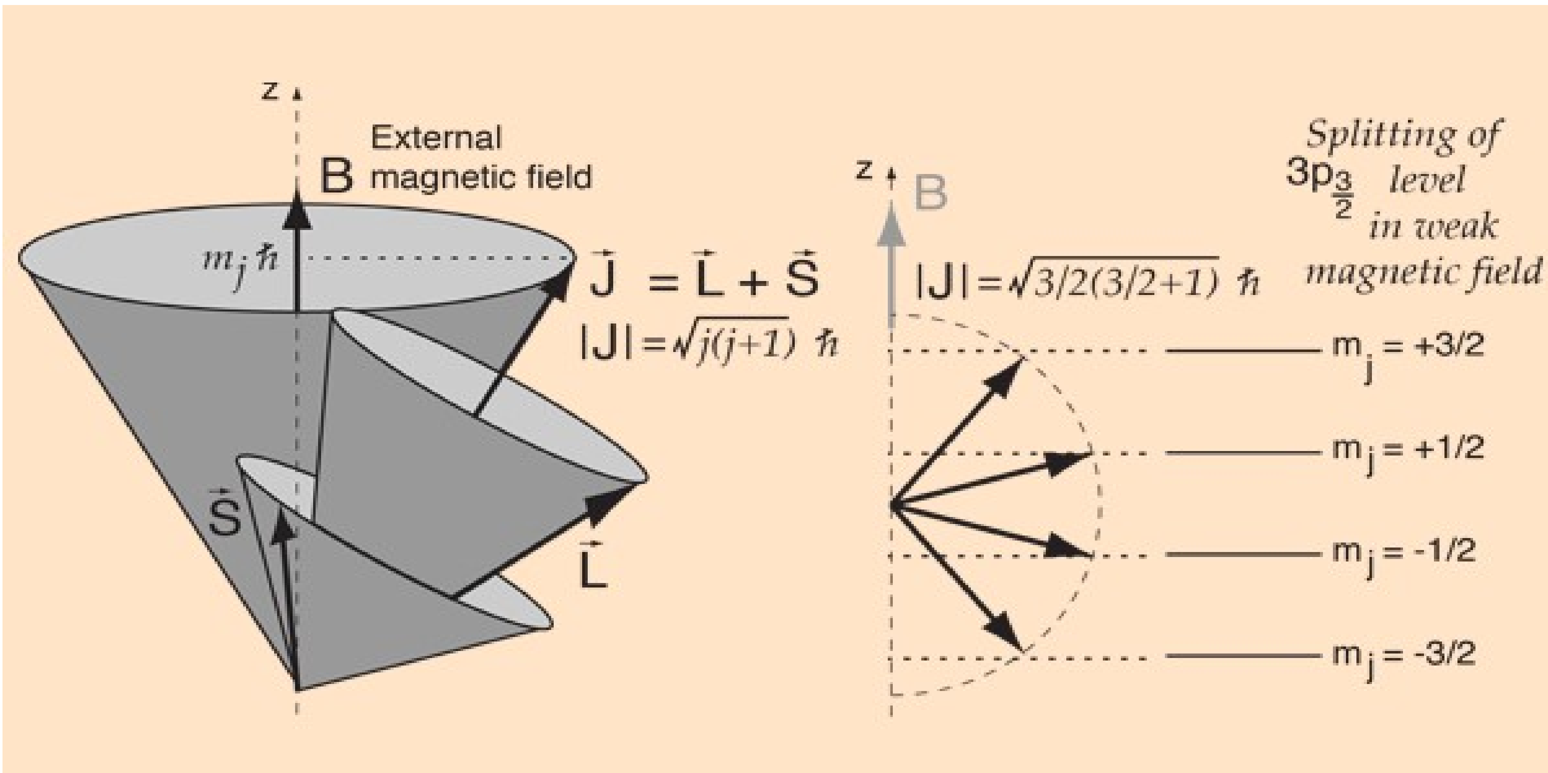
https://commons.wikimedia.org/wiki/File:Vector_model_of_orbital_angular_momentum.svg

Once we have combined orbital and spin angular momenta according to the vector model, the resulting total angular momentum can be visuallized as precessing about any externally applied magnetic field.

This is a useful model for dealing with interactions such as the Zeeman effect in sodium.

The magnetic energy contribution is proportional to the component of total angular momentum along the direction of the magnetic field, which is usually defined as the z-direction.

The z-component of angular momentum is quantized in values one unit apart, so for the upper level of the sodium doublet with j=3/2, the vector model gives the splitting shown.



http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/vecmod.html

l^* and s^* precess with uniform speed around j^*. When atom is placed in a weak magnetic field, μ_j associated with $p_j = j^*(h/2\pi)$ causes the atom to precess like a top around the field direction H.

Quantum conditions imposed on this motion :

Projections of $j^*(h/2\pi)$ on H direction can have values **m**$(h/2\pi)$; $m = -j$ to $+j$
i.e. $2j+1$ values

The discrete orientation of atoms in space, and the small change in energy due to precession, give rise to various discrete Zeeman levels.

The number of these levels is determined by the mechanical moment $j^*(h/2\pi)$.

Magnitude of separation is determined by field strength H and magnetic moment μ.

In the field free space energy level is determined by n, l, j

In the presence of the field energy level is determined by n, l, j, **m**.

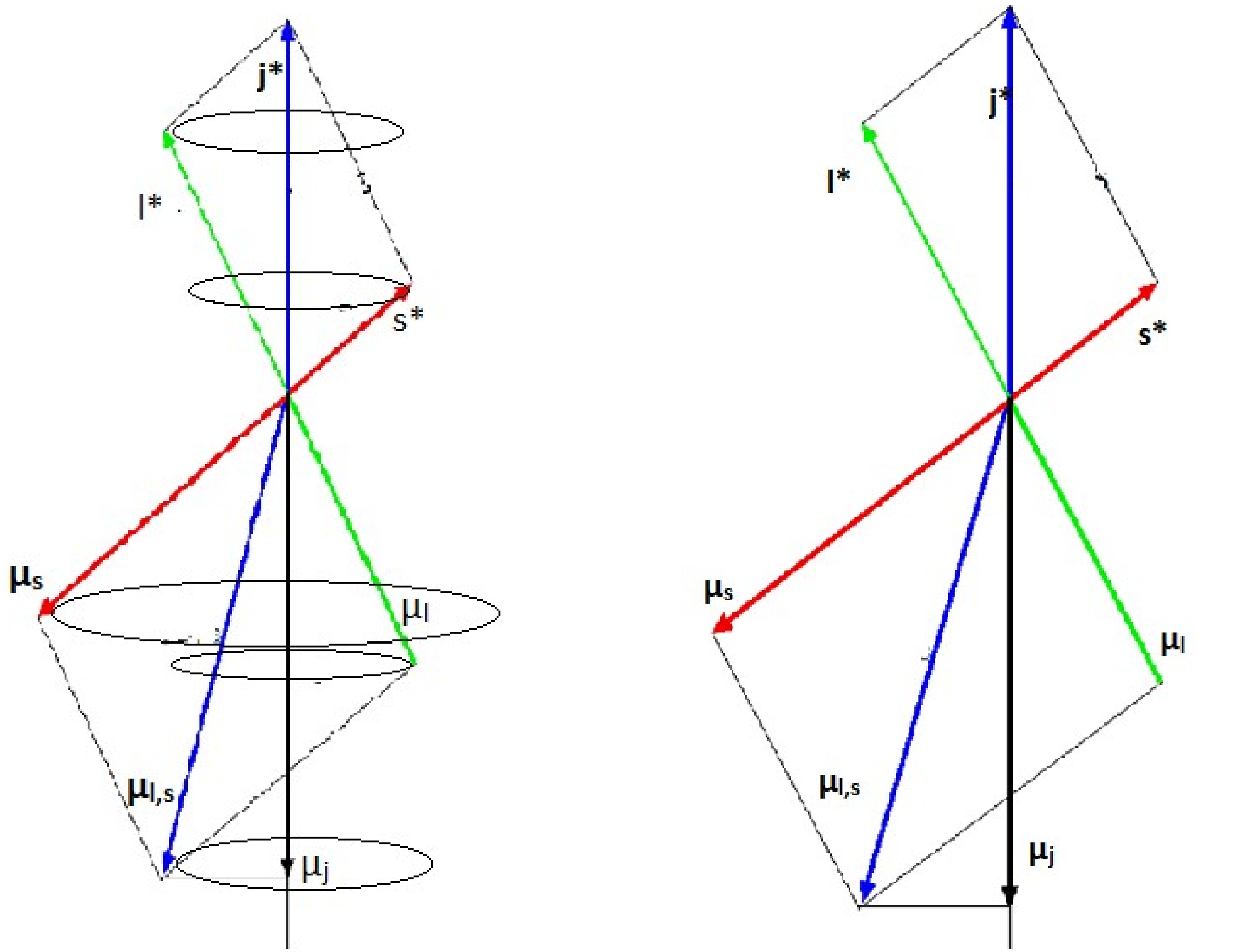
The relationship between the magnetic moment and mechanical angular momentum is given by

$$\frac{\mu_l}{p_l} = \frac{e}{2mc}$$

$$\mu_l = l * \frac{h}{2\pi}\frac{e}{2mc}\frac{erg}{gauss}$$

$$\frac{\mu_s}{p_s} = \frac{e}{2mc}$$

$$\mu_s = 2s * \frac{h}{2\pi}\frac{e}{2mc}\frac{erg}{gauss}$$


j*
l*
s*
μs
μl
μl,s
μj
j*
l*
s*
μs
μl
μl,s
μj

Component of μ_l along j*

$$l^* \frac{h}{2\pi}\frac{e}{2mc}\cos(l^* j^*)$$

Component of μ_s along j*

$$2s^* \frac{h}{2\pi}\frac{e}{2mc} cos\ s^* j^*$$

Total Component along j*

$$[l^* \cos(l^* j^*) + 2s^* \cos(s^* j^*)](\frac{h}{2\pi}\frac{e}{2mc})$$

$$\frac{h}{2\pi}\frac{e}{2mc} = 1\ Bohr\ Magneton$$

$$j^* g = l^* \cos(l^* j^* + 2s^* \cos(s^* j^*)$$

$$s^{*^2} = l^{*^2} + j^{*^2} - 2l^* j^* cos(l^* j^*)$$

$$l^* \cos(l^* j^*) = \frac{j^{*2} + l^{*2} - s^{*2}}{2j^*} \qquad s^* \cos(s^* j^*) = \frac{j^{*2} - l^{*2} + s^{*2}}{2j^*}$$

$$g = 1 + \frac{j^{*2} + s^{*2} - l^{*2}}{2j^{*2}} = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}$$

Magnetic Interaction Energy

$$\frac{\mu_j}{p_j} = g\frac{e}{2mc} \qquad p_j = j^* \frac{h}{2\pi}$$

The precession of j* around H is the result of a torque acting on both l* and s*. If the field is not too strong the coupling between l* and s* is remains intact so that j* remains constant and j* is able to precess with a compromise angular velocity

$$\omega_L = gH\frac{e}{2mc}$$

Energy of precession is given by product of ω_L with component of j*(h/2π) along H.

$$\Delta\zeta = \omega_L j^* \frac{h}{2\pi} \cos(j^* H) = gH \frac{e}{2mc} j^* \frac{h}{2\pi} \cos(j^* H)$$

$$j^* \frac{h}{2\pi} \cos(j^* H) = m \frac{h}{2\pi}$$

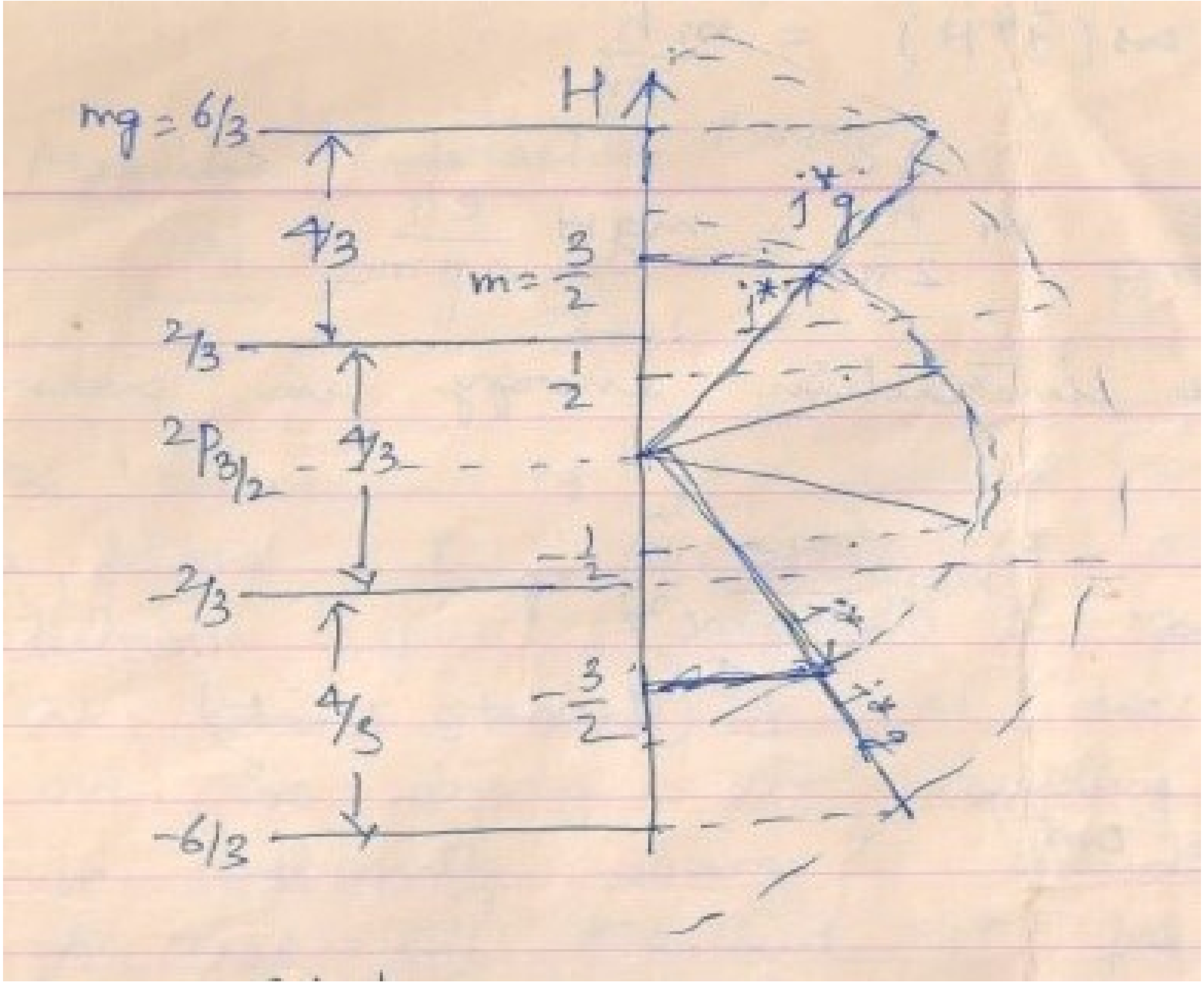
$$\Delta\zeta = gH \frac{e}{2mc} m \frac{h}{2\pi} = mgH \frac{eh}{4\pi mc}$$

Dividing by hc, the interaction energy in wave numbers becomes

$$\frac{\Delta\zeta}{hc} = mg\left(\frac{He}{4\pi mc^2}\right) cm^{-1}$$

$$\frac{He}{4\pi mc^2} \quad Lorentz\ Unit$$

$$\Gamma - mgL\ cm^{-1}$$

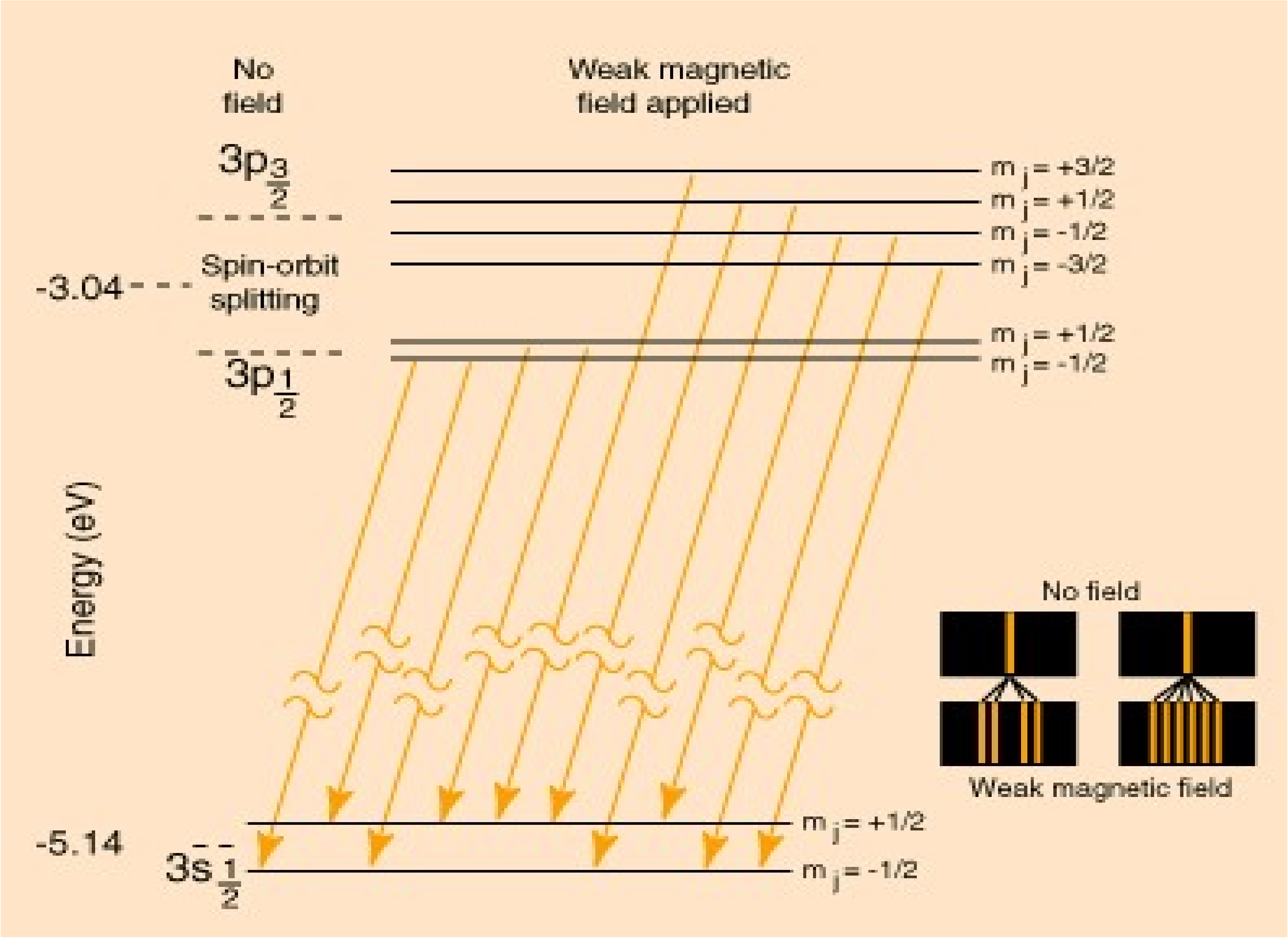

H
mg = 6/3
4/3
m = 3/2
2/3
1/2
$^2P_{3/2}$
4/3
−1/2
−2/3
−3/2
4/3
−6/3

Selection rule: As an example of calculation of Zeeman patterns, consider the simple case of a principal series doublet like sodium yellow D lines.

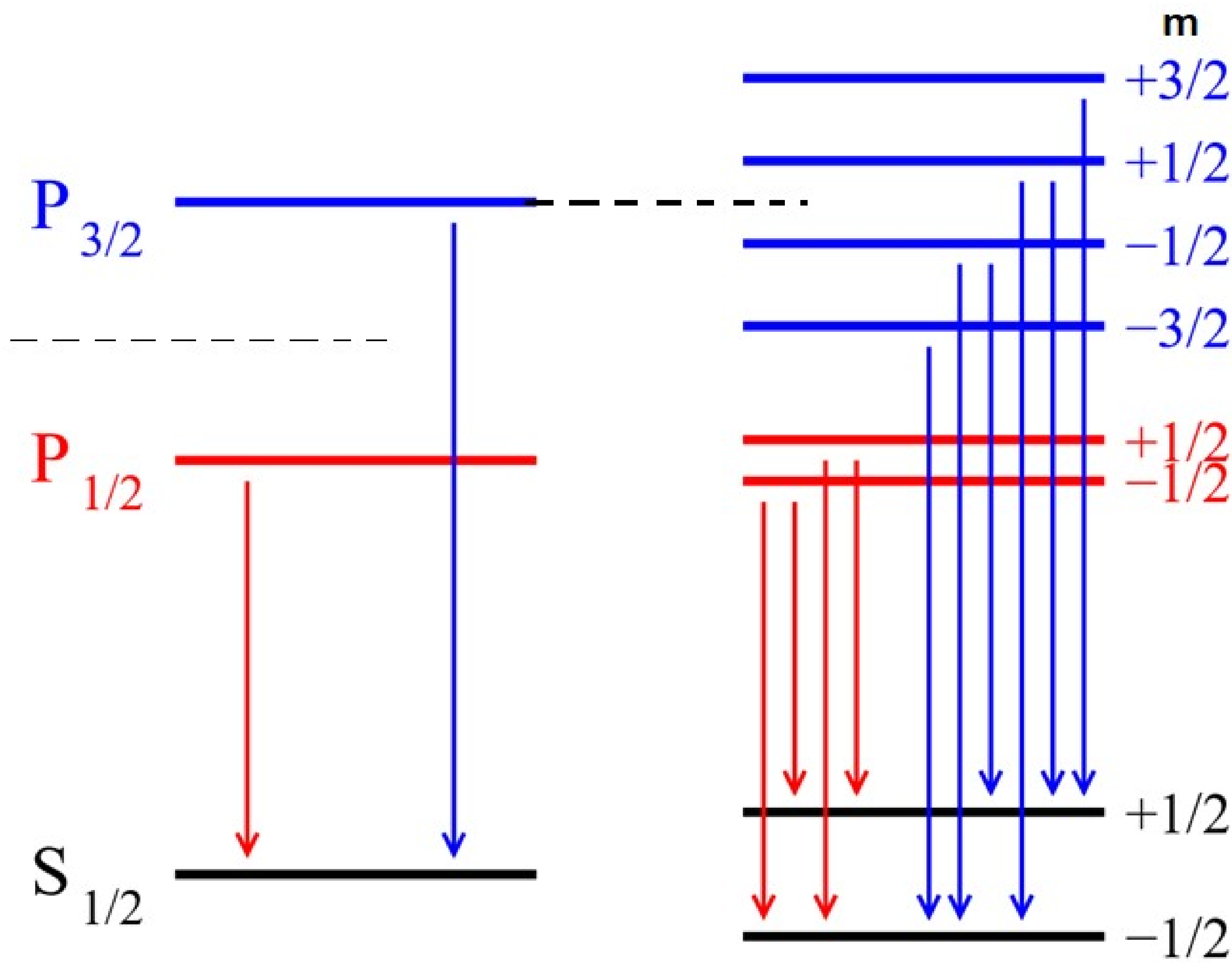
$^2P_{1/2}$ **g=2/3** $^2P_{3/2}$ **g=4/3** $^2S_{1/2}$ **g=2**

Splitting of these levels is schematically shown in figure starting with the field free level at the left. Dotted lines in each case represent the centre of gravity of associated levels.

Selection rule: $\Delta m = 0, \pm 1$



Selection rule: Δm = 0, ±1



Prof. Narendra Kumar Pandey
Department of Physics
University of Lucknow
Lucknow-226007
Email: profnarendrapandey137@gmail.com