Differential eq of SVF with decreasing discharge -

Assumptions →

1. The pressure distribution is hydrostatic.
2. The one-dimensional method of analysis is used.
3. The friction losses are adequately represented by Manning's formula.
4. Withdrawal of water does not affect the energy content per unit mass of water in the channel.
5. The flow is steady.
6. The channel is prismatic and is of small slope.

H = Total Energy head.

Z = Datum head

y = Pressure head or depth of water at a perticular Section.

α = Kinetic energy correction factor.

V = velocity of water at a perticular section.

Consider the total energy at a section x,

$$H = Z + y + \frac{\alpha V^2}{2g}$$

differentiating with respect to x

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{\alpha V^2}{2g}\right) \quad \text{——— (1)}$$

$$\frac{dH}{dx} = -S_f \rightarrow \text{Energy slope} \quad \text{——— (2)}$$

$$\frac{dZ}{dx} = -S_o \rightarrow \text{Bed Slope} \quad \text{——— (3)}$$

$$\because V = \frac{Q}{A}$$

$$\frac{d}{dx}\left(\frac{\alpha V^2}{2g}\right) = \frac{d}{dx}\left(\frac{\alpha Q^2}{2g A^2}\right)$$

$$= \frac{\alpha}{2g}\left[\frac{d}{dx}\frac{Q^2}{A^2}\right]$$

$$= \frac{\alpha}{2g}\left[\frac{d}{dQ}\frac{Q^2}{A^2}\frac{dQ}{dx} + \frac{d}{dA}\frac{Q^2}{A^2}\cdot\frac{dA}{dx}\right]$$

$$= \frac{\alpha}{2g}\left[\frac{2Q}{A^2}\frac{dQ}{dx} + \frac{(-2)Q^2}{A^3}\frac{dA}{dx}\right] \qquad \because \frac{dQ}{dx} = q_*$$

$$= \frac{2\alpha}{2g}\left[\frac{Q}{A^2}q_* - \frac{Q^2}{A^3}\frac{dA}{dy}\frac{dy}{dx}\right] \qquad \because \frac{dA}{dy} = T$$

$$\frac{d}{dx}\left(\frac{\alpha V^2}{2g}\right) = \frac{\alpha}{g}\left[\frac{Q}{A^2}q_* - \frac{Q^2 T}{A^3}\frac{dy}{dx}\right] \quad \text{———} (4)$$

eq (1) →

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{\alpha V^2}{2g}\right)$$

By putting the values of eq (2), (3) & (4)

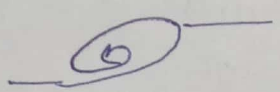
$$-S_f = -S_o + \frac{dy}{dx} + \frac{\alpha Q q_*}{g A^2} - \frac{\alpha Q^2 T}{g A^3}\frac{dy}{dx}$$

$$\frac{dy}{dx} - \frac{\alpha Q^2 T}{g A^3}\frac{dy}{dx} = S_o - S_f - \frac{\alpha Q q_*}{g A^2}$$

$$\frac{dy}{dx}\left[1 - \frac{\alpha Q^2 T}{g A^3}\right] = S_o - S_f - \frac{\alpha Q q_*}{g A^2}$$

$$\boxed{\frac{dy}{dx} = \frac{S_o - S_f - \dfrac{\alpha Q q_*}{g A^2}}{1 - \dfrac{\alpha Q^2 T}{g A^3}}}$$

→ When the flow occures from Supercritical to subcritical jump happens



Control Section→ A control section is defined as a section in which a fixed relationship exists between the discharge and depth of flow.

→ Supercritical flows have control sections existing at the upstream end of the channel section.

→ Subcritical flows have control sections in down stream end.

Transitional depth→ It is defined as the depth at which the normal discharge Q_n is equal to the critical discharge Q_c and slope of the Gradually varied flow is horizontal.

$$Q_n = Q_c$$

$$\frac{dy}{dx} = S_0 \left[\frac{1 - \left(\frac{Q}{Q_n}\right)^2}{1 - \left(\frac{Q}{Q_c}\right)^2} \right] = S_0$$

→ Transitional depth depends only on the channel geometry, roughness and slope and is independent of the actual discharge.

$$Q_n = Q_c$$

$$\frac{A}{n} R^{2/3} S_0^{1/2} = \frac{A\sqrt{Ag}}{\sqrt{T}}$$

$$\frac{A^{2/3} S_0^{1/2}}{n \cdot P^{2/3}} = \sqrt{\frac{Ag}{T}}$$

$$\frac{A^{4/3} S_0}{n^2 P^{4/3}} = \frac{Ag}{T}$$

$$\boxed{\frac{S_0}{n^2 g} = \frac{P^{4/3}}{T \cdot A^{1/3}}}$$

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