Numerical solutions of GVF problems -

Simple Numerical Methods
- Direct - Step Method
- Standard - step method

Advanced Numerical Methods
- Standard Fourth order Runge - Kutta methods
- Kutta - Merson method
- Trapezoidal Method

Simple Numerical Methods →

1→ Direct - Step Method →

→ Simplest method

→ Suitable for use in prismatic channels.

The differential energy equation of GVF is

$$\frac{dE}{dx} = S_0 - S_f$$

writing in the finite-difference form

$$\frac{\Delta E}{\Delta x} = S_0 - \bar{S}_f$$

$\bar{S}_f$ = Average friction slope in the reach Δx

$$\Delta x = \frac{\Delta E}{S_0 - \bar{S}_f}$$

and between two sections 1 and 2

$$(x_2 - x_1) = \Delta x = \frac{(E_2 - E_1)}{S_0 - \frac{1}{2}(S_{f_1} + S_{f_2})}$$

Procedure →

→ Let it be required to find the water-surface profile between two sections 1 and (N+1) where the depths are y_1 and y_{N+1} respectively.

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→ The channel reach is now divided into N parts of known depths. i.e. values of y_i $i = 1, N$ are known.

→ It is required to find the distance Δx_i between y_i and y_{i+1}.

→ Between the two sections i and $i+1$

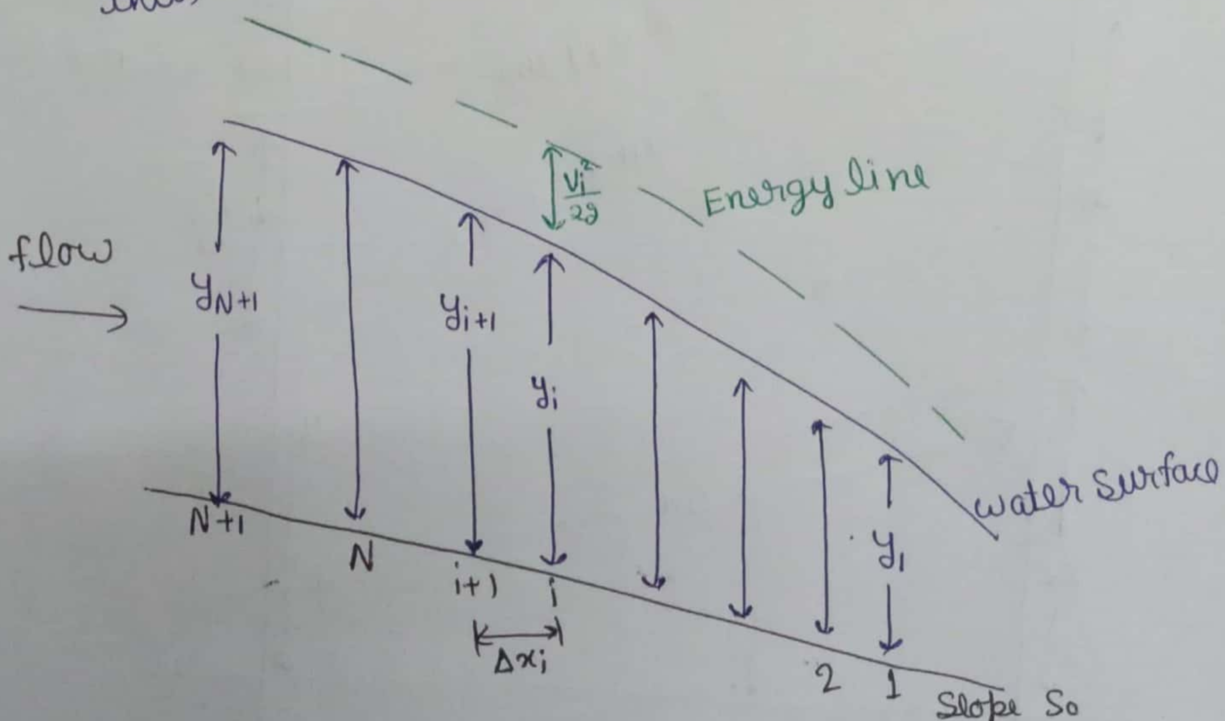
→ $$\Delta E = \Delta\left(y + \frac{V^2}{2g}\right) = \Delta\left(y + \frac{Q^2}{2gA^2}\right)$$

$$\Delta E = E_{i+1} - E_i = \left[y_{i+1} + \frac{Q^2}{2gA_{i+1}^2}\right] - \left[y_i + \frac{Q^2}{2gA_i^2}\right]$$

→ and $$\bar{S_f} = \frac{(S_{f_{i+1}} + S_{f_i})}{2} = \frac{n^2Q^2}{2}\left[\frac{1}{A_{i+1}^2 R_{i+1}^{4/3}} + \frac{1}{A_i^2 R_i^{4/3}}\right]$$

→ $$\Delta x_i = \frac{\Delta E}{S_0 - \bar{S_f}} = \frac{E_{i+1} - E_i}{S_0 - \bar{S_f}}$$

→ The sequential evaluation of Δx_i starting from $i = 1$ to N, will give the distances between the N sections and thus the GVF profile.



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(2) Standard Step Method →

→ Can be applied to natural channel.

→ Given the cross-sectional information at two adjacent sections and the discharge and stage at one section, it is required to determine the stage at the other section.

→ $$E_2 = E_1$$

$$z_2 + y_2 + \alpha_2 \frac{V_2^2}{2g} = z_1 + y_1 + \frac{\alpha_1 V_1^2}{2g} + h_f + h_e$$

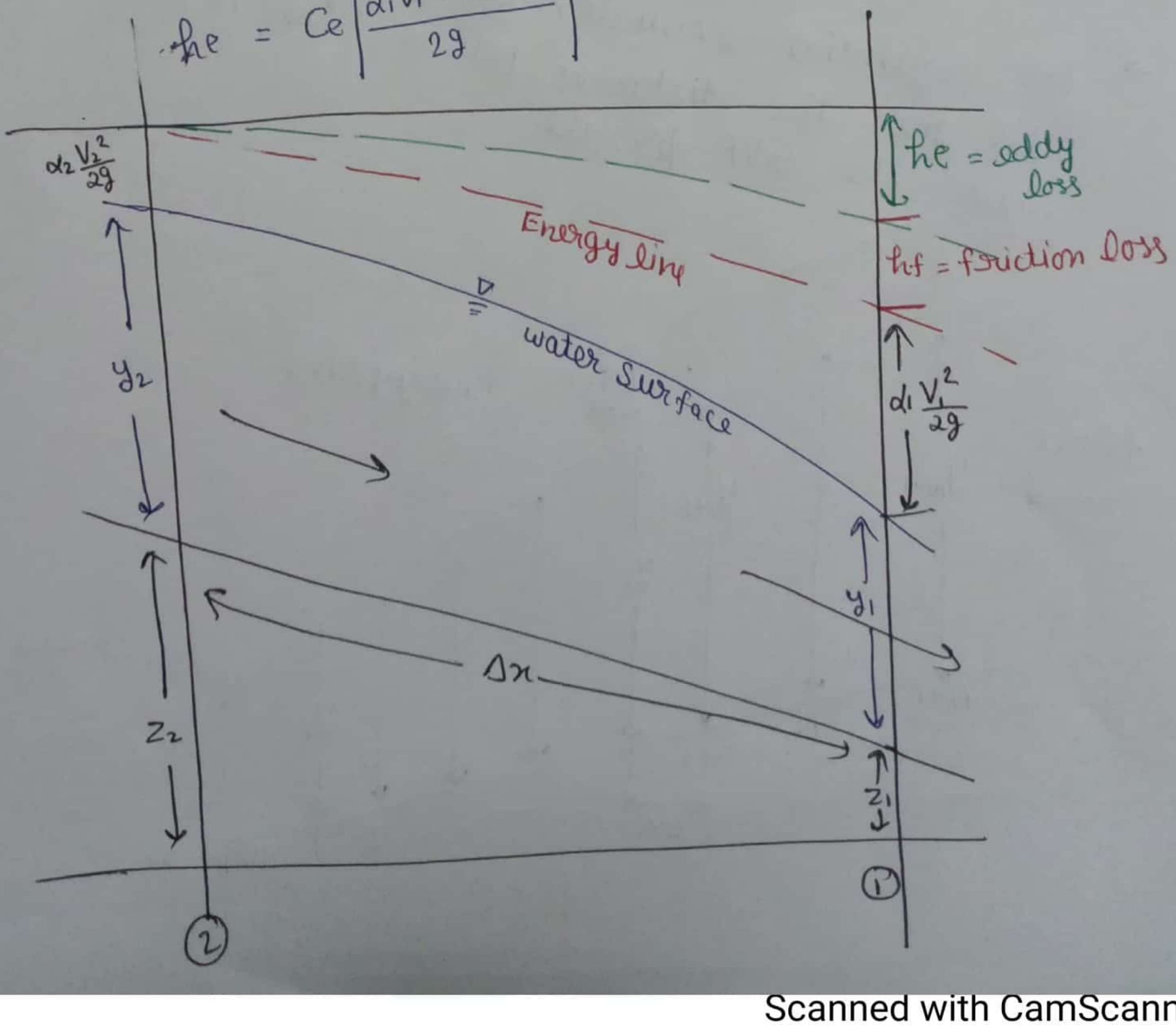
where h_f = friction loss

$$h_f = \bar{S}_f \cdot \Delta x = \frac{(S_{f_1} + S_{f_2})}{2} \cdot \Delta x$$

where $S_f = \dfrac{n^2 v^2}{R^{4/3}} = \dfrac{n^2 Q^2}{A^2 R^{4/3}}$

h_e = eddy loss

$$h_e = C_e \left| \frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2g} \right|$$



Denoting the stage $= Z + y = h$

$$H = h + \frac{\alpha V^2}{2g} \quad \text{——— (1)}$$

$$H_2 = H_1 + h_f + h_e \quad \text{——— (2)}$$

Procedure →

→ Select a trial value of h_2 and calculate H_2, h_f and h_e and check whether eq (2) is satisfied. If there is a difference, improve the assumed value of h_2 and repeat calculations till the two sides of eq (2) match to an acceptable degree of tolerance.

Advanced Numerical Method →

→ The basic differential eq of GVF is

$$\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2 T}{g A^3}} = F(y)$$ for a given S_0, n, Q and channel geometry.

→ we have to found y at $(x + \Delta x)$ given y at x.

→ $y_i =$ depth at $x_i = y(x_i)$

and $x_{i+1} = x_i + \Delta x$

and $y_{i+1} = y(x_{i+1})$

(1) Standard Fourth order Runge-Kutta methods (SRK) →

$$y_{i+1} = y_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

where $K_1 = \Delta x \, F(y_1)$

$K_2 = \Delta x \cdot F\left(y_i + \frac{K_1}{2}\right)$

$K_3 = \Delta x \cdot F\left(y_i + \frac{K_2}{2}\right)$

$K_4 = \Delta x \cdot F(y_i + K_3)$

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(b) Kutta - Merson Method (KM) →

$$y_{i+1} = y_i + \frac{1}{2}(K_1 + 4K_4 + K_5)$$

where,
$$K_1 = \frac{1}{3}\Delta x \, F(y_i)$$

$$K_2 = \frac{\Delta x}{3} F(y_i + K_1)$$

$$K_3 = \frac{\Delta x}{3} F\left(y_i + \frac{K_1}{2} + \frac{K_2}{2}\right)$$

$$K_4 = \frac{\Delta x}{3} F\left(y_i + \frac{3}{8}K_1 + \frac{9}{8}K_3\right)$$

$$K_5 = \frac{\Delta x}{3} F\left(y_i + \frac{3}{2}K_1 - \frac{9}{2}K_3 + 6K_4\right)$$

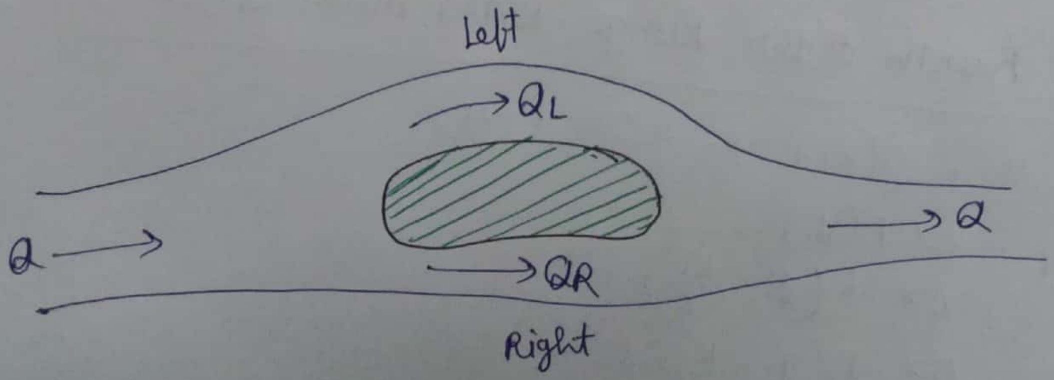
(c) Trapezoidal Method (TRAP) →

$$y_{i+1} = y_i + \frac{\Delta x}{2}\left\{F(y_i) + F(y_{i+1})\right\}$$

Flow profiles in divided channels-

Divided channels also known as island-type flow.

① Simple Island type flow →



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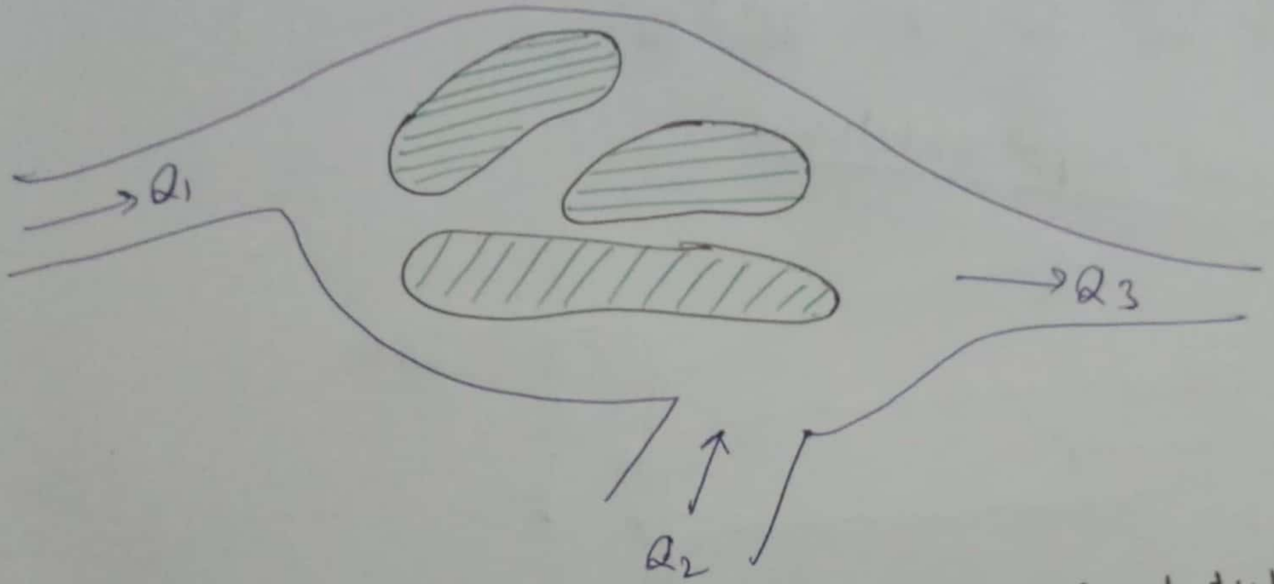
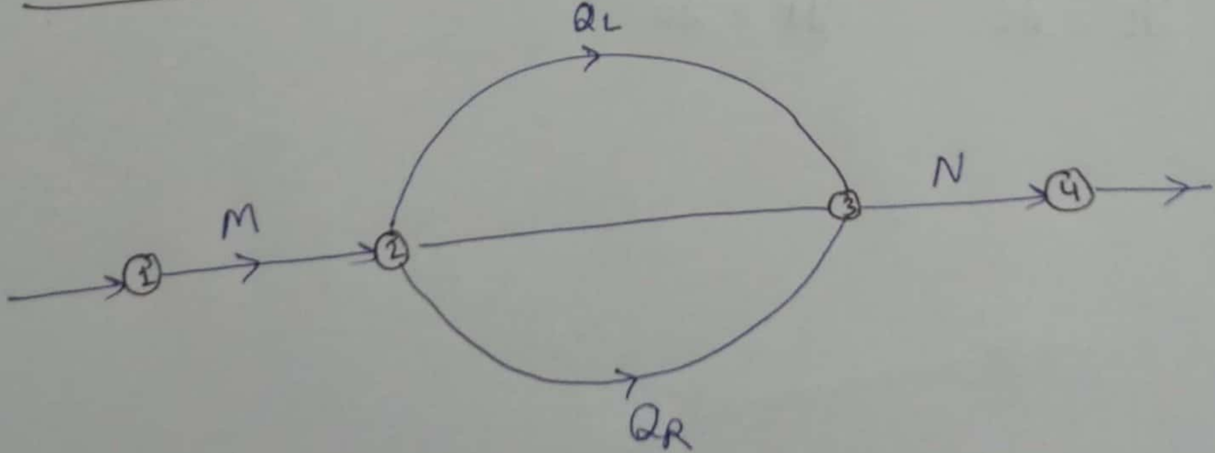
② Multi- Island type Flow →



Schematic representation of Multi Island type →



Role of End Conditions →

The kinetic energy of the stream is recovered as potential energy. The depth of flow at the outlet side depends on the depth of water in the lake.

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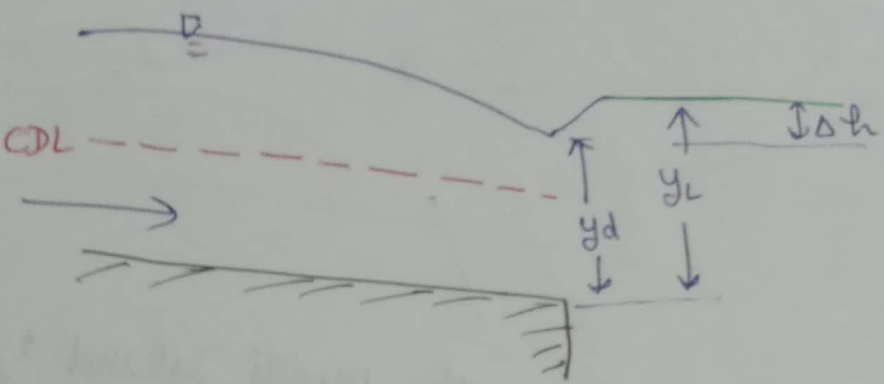
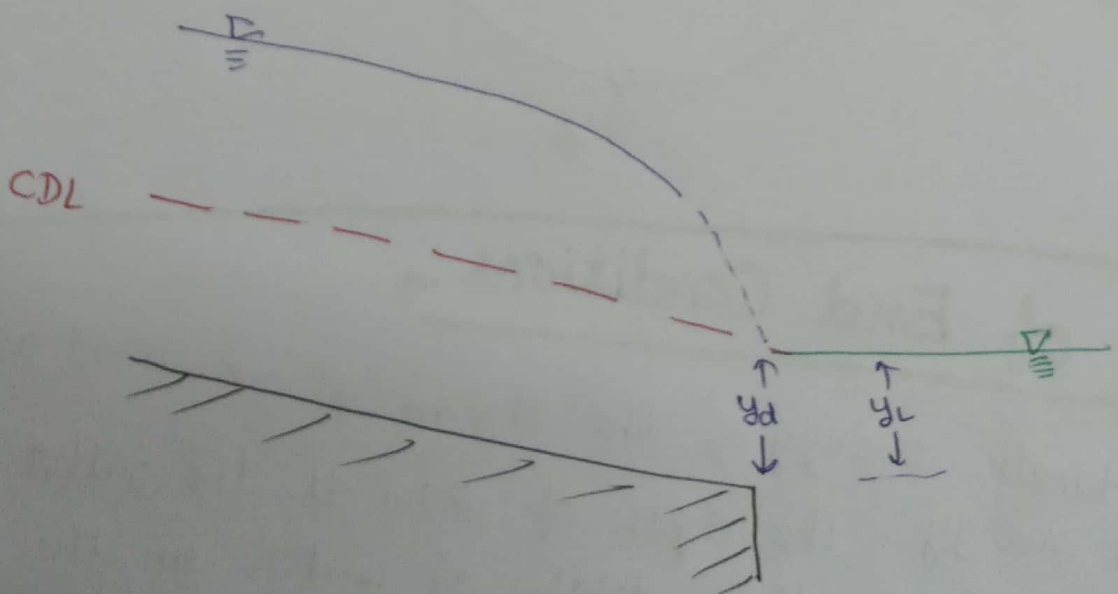
y_L = depth of water in the lake
y_d = depth of flow in the canal at the outlet
y_c = critical depth of channel.

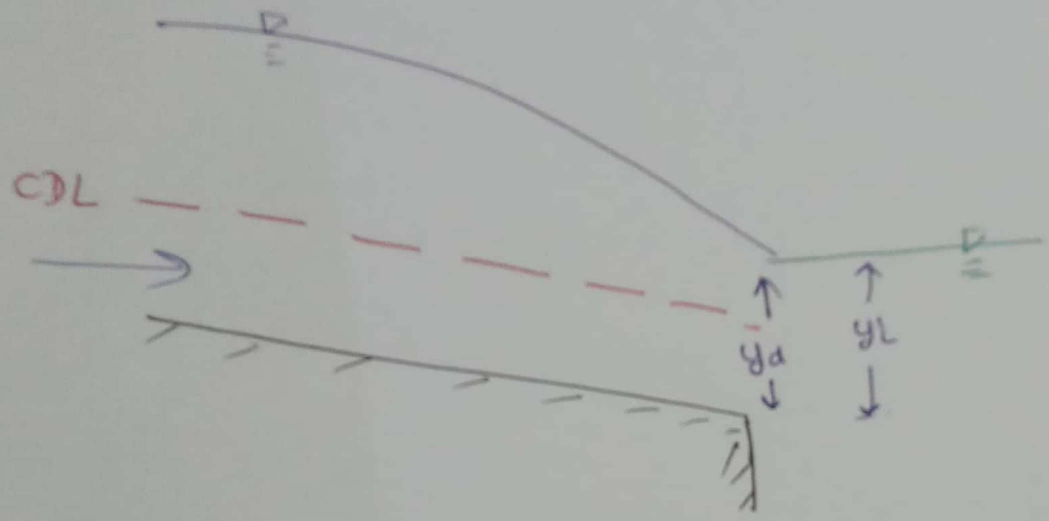
General representation →



① if $y_L = y_c$, $y_d = y_c$



(2) if $y_L > y_c$, $y_d = y_L$



(3) if $y_L < y_c$, $y_d = y_c$