14

Time Series

14.1 INTRODUCTION

Time series are a kind of observations measured at time or space intervals arranged in chronological order. For instance, population, sales, production, prices etc.

Spiegel defined time series as follows:

A time series is a set of observations taken at specified times usually at equal intervals.

14.2 UTILITY OF TIME SERIES

(1) It is very much useful to know the **past history** of the time series data.

(2) In order **to predict** the future demand, production, prices, weather conditions etc.

(3) It also helps in **planning** of the future operations.

(4) Two or more time series can be **compared** belonging to the same reference period.

14.3 COMPONENTS OF TIME SERIES

There are four types of component of a time series.

(*i*) Secular trend or simply trend

(*ii*) Seasonal fluctuations

(*iii*) Cyclical fluctuations and

(*iv*) Irregular components

(1) Secular trend or Simply trend:

If the data of a time series are observed for a long time, then smooth, regular movements are known as trends. Sometimes series may provide upward or downward trend but some may take reverse direction after a period of growth and enter a period of decline.

(2) Seasonal fluctuations

A periodic movement in a time series where the period is not longer than one year is called seasonal fluctuations. This movement repeats at regular intervals of time or periods. For example, sales of departmental stores during the 12 months of a year, regular classes attended by students during 5 days of a week, issue of library books during the seven days of a week and so on.

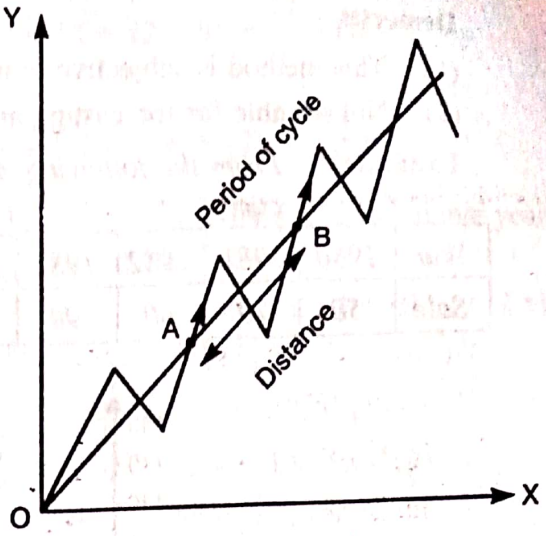
(3) Cyclical fluctuations

The oscillatory movement in a time series is called the cyclical fluctuations. The period of oscillation may be taken more than a year and one complete period makes a cycle. The cyclical fluctuations are not necessarily periodic. One complete period which normally lasts from 7 to 9 years in termed as a "cycle". Most of the economic and business series e.g. those relating to price, income, investment, wage, production etc. reveal this tendency.

Measurement of cyclical variation

The cyclical period can be found by plotting the time series. Then we can fit straight line by

st fit method. This line will cut the original series at veral points. The upper portion is positive and lower rtion is negative. When the series changes sign from gative to positive is called upcross. The various tances are noted between two adjoining upcross. The ean distance of these distances is called mean period the cycle.



If moving averages are calculated with this mean riod, then the cyclical component C can be removed.

In the case of multiplicative model of a time ies, the cyclic component is obtained in combined rm, by dividing the original series by trend and asonal component.

In the case of additive model of a time series, the cyclical components are obtained by btracting trend and seasonal components from the data.

Cyclical variations are found by removing the random variation form the residual variation, hich were obtained by eliminating trend and seasonal variation from the given table. This is done averaging these residuals directly or through moving average.

(4) Irregular Components

The fluctuations which are either totally unaccountable or happened by such unforeseen events as wars, floods, strikes etc. Such variations do not exhibit any definite pattern and there is no regular period or time of their occurrence, hence they are named irregular variations.

4. METHODS OF MEASURING TREND

(1) Free hand or graphic method
(2) Semi-average method
(3) Moving average method
(4) Least square method

(1) Free Hand or Graphic Method

This is the simplest and most flexible method of estimating the secular trend and consists of first obtaining a histogram by plotting the time series value on a graph paper and then drawing it free hand smooth curve through these points so that it accurately reflects the long term tendency of the data.

Important Points for drawing a curve

(1) It should be smooth.
(2) Number of points above and below should be nearly equal.
(3) The sum of the vertical deviations of the given points about the trend line should be approximately equal to sum of the vertical deviations of the points below the trend line.
(4) Sum of the squares of the vertical deviations of the given points from the trend line should be minimum possible.

Merits:

This method is
(1) Simple and time saving method, does not require any mathematical calculation.
(2) Flexible method for all types of trends.

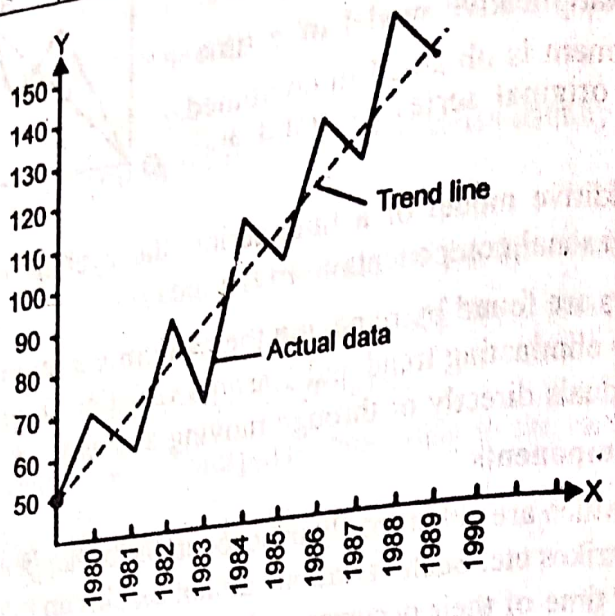
Demerits

(1) This method is subjective in nature.

(2) Not suitable for forecasting and for making decisions.

Example 1. *From the following data of the sales determine the trend line by free hand curve.*

Year	1980	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Sale	50	70	60	90	70	110	100	130	120	150	140



(2) Method of Semi-Averages

Under this method the data for which trend values are to be computed are divided into two equal parts and averages are computed for both the parts. If there is odd number of years, the value of the middle year is omitted.

The average value of these two halves should be calculated. These average values would be plotted against the mid value of each half. By joining these two points we get the trend line. This line can be extended on both ways.

Merits

(1) It is a simple method.

(2) The trend figures are objective.

(3) Future and past estimates can be obtained by extending the lines.

Demerits

(1) This method is based on linear relationship but fails in non-linear relationship.

(2) This method ignores the extreme values.

(3) The prediction is not reliable.

Example 2. *Calculate trend values from the following data by the method of semi-averages:*

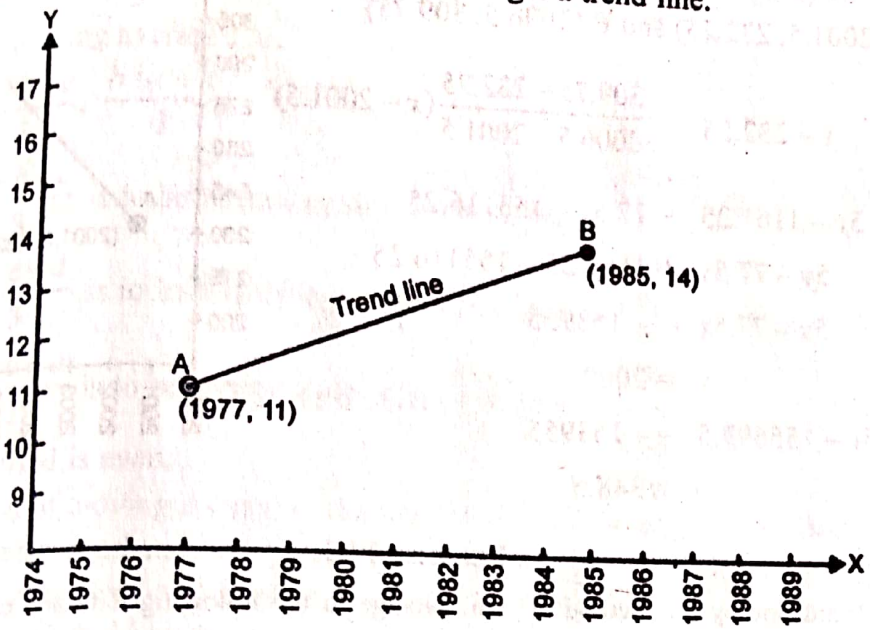
Year :	1974	1975	1976	1977	1978	1979	1980	1981
Sales :	10	11	13	8	14	12	9	14
Year :	1982	1983	1984	1985	1986	1987	1988	
Sales :	13	10	12	16	14	16	17	

Solution. We have odd number of years, therefore we neglect the value of middle year 1981

Average for 1974 to 1980 $= \frac{1}{7}(10 + 11 + 13 + 8 + 14 + 12 + 9) = \frac{77}{7} = 11$

Average for 1982 to 1988 $= \frac{1}{7}(13 + 10 + 12 + 16 + 14 + 16 + 17) = \frac{98}{7} = 14$

11 is plotted the mid year 1977 of the first half and 14 should be plotted against the mid year of the other half. These two points are joined to get a trend line.



The equation of the trend line

The trend line passes through the point A (1977, 11) and the point B (1985, 14).

$$\left[y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

$$y - 11 = \frac{14 - 11}{1985 - 1977}(x - 1977)$$

$$\Rightarrow \quad y - 11 = \frac{3}{8}(x - 1977)$$

$$\Rightarrow \quad y = 11 + 0.375\,(x - 1977)$$ **Ans.**

Example 3. *From the following series of annual data, find the trend line of semi-averages. Also estimate the value for 2009.*

Years	2000	2001	2002	2003	2004	2005	2006	2007	2008
Actual Value	170	231	261	267	278	302	299	298	340

(U.P. III Semester, 2010-11)

Solution: We have odd number of years, therefore we neglect the value of middle year 2004.

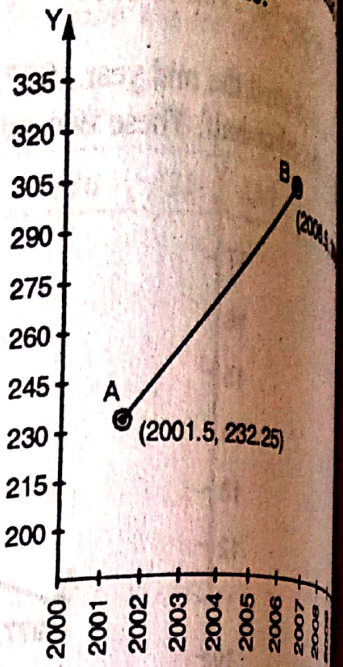
Average for 2000 to 2003 $= \frac{1}{4}(170 + 231 + 261 + 267) = \frac{929}{4} = 232.25$

Mid year $= \frac{1}{4}(2000 + 2001 + 2002 + 2003) = 2001.5$

Average for 2005 to 2008 $= \frac{1}{4}(302 + 299 + 298 + 340) = \frac{1239}{4} = 309.75$

Mid year $= \frac{1}{4}(2005 + 2006 + 2007 + 2008) = 2006.5$

232.25 is plotted the mid year 2001.5 of the first half and 309.75 is plotted the
2006.5 of the other half. These two points are joined to get a trend line.



The equation of the trend line

The trend line possess through the points
A (2001.5, 232.25) and B (2006.5, 309.75)

$$y - 232.25 = \frac{309.75 - 232.25}{2006.5 - 2001.5}(x - 2001.5)$$

$$5y - 1161.25 = 77.5x - 155116.25$$

$$5y - 77.5x = 1161.25 - 155116.25$$

$$5y - 77.5x = -153955$$

If $x = 2009$

$\therefore 5y - 155697.5 = -153955$

$\Rightarrow y = 348.5$

Exercise 14.1

1. Fit a trend line by the method of semi average to the following data and estimate the sale
1987:

Year	1979	1980	1981	1982	1983	1984	1985	1986
Sales (in '000) :	412	438	444	454	470	482	490	500

Hint: The trend line will pass through the points (1980.5, 437) and (1984.5, 480.5)

2. Fit a trend line.....

Year :	1980	1981	1982	1983	1984	1985	1986
Production :	12	10	14	11	13	15	17

Hint: The trend line will pass through the points (1981, 12) and (1985, 15)

Ans. $y = 12 + 0.75(x - 198$

3. Fit a trend line....

Years	1971	1972	1973	1974	1975
Profit (in ₹ lakhs)	28.0	29.4	30.2	27.0	32.5

Ans. $y = 28.7 + 0.35(x - 197$

4. The sales of a commodity in tonnes varied from 1985 to 1997 as follows :

280 300 280 270 240 230 230 220 200 210 200

Fit a trend line by the method of semi-averages.

Ans. $y = 275 + 8.57(x - 199$

14.5 METHOD OF MOVING AVERAGES

Moving averages consists of a series of arithmetic means calculated from overlapping gro
of successive values of a time series.

Case I. When period is odd:

Let the period of moving averages be 3 years and the items *a, b, c, d, e,*

Average of first three $= \frac{a+b+c}{3}$

Dropping the first year value and adding the value of the next years the average becomes

$$= \frac{b+c+d}{3}$$

Next average $= \frac{c+d+e}{3}$

In this way moving averages are

$$\frac{a+b+c}{3}, \frac{b+c+d}{3}, \frac{c+d+e}{3}$$

$\frac{a+b+c}{3}$ is to be written against the mid year b.

$\frac{b+c+d}{3}$ is to be written against the mid year c.

$\frac{c+d+e}{3}$ is to be written against the mid year d.

ase II. When period is even.

Let the period of moving averages be 4-years then the average of first four figures will be placed between second and third year like-wise the average of second group of four years will be placed between third and fourth year. These two moving averages will then be averaged and this average would be written against the third year. This process is called centering of the averages.

1	2	3	4	5	6	7	8
a	b	c	d	e	f	g	h

$\frac{a+b+c+d}{4}$ against to be placed between 2 and 3.

$\frac{b+c+d+e}{4}$ against to be placed between third and fourth and so on.

These two moving averages will then be average and this new average would be written against the third year. This process is called centering of the averages.

In case of even period of moving averages the trend values are obtained after centering the averages.

Note: Centering of the moving averages is required only in the even period and not in odd period.

Merits: This method is

(1) Simple
(2) Flexible to add more figures
(3) Objective
(4) To eliminate the fluctuation
(5) Used for determining seasonal cyclic and irregular variations.

Demerits

(1) There is no trend value for some years in the beginning and at the end. It means there will be no trend values for the first three years and the last three years.
(2) There is no fluctuation relationship between the values and time.
(3) This method is not helpful in forecasting and predicting the value.
(4) Selection of period of moving average is a difficult task.
(5) In case of non-linear trend the values obtained by this method are biased.

Example 4. *Computation of moving average taking a 3 years period and find a trend.*

Years	*1985*	*1986*	*1987*	*1988*	*1989*	*1990*	*1991*	*1992*
Profits (in ₹)	*15420*	*14470*	*15520*	*21020*	*26120*	*31950*	*35370*	*35670*

Solution. Procedure:

(1) Put the original data of years and profits in column (i) & (ii) as shown in Table :

(2) Obtain the 3 years moving totals starting from first year and put against the middle of the years.

(3) Now, leaving first year value, add a successive year value in the group. Find the total of this group and place it against the middle of the years.

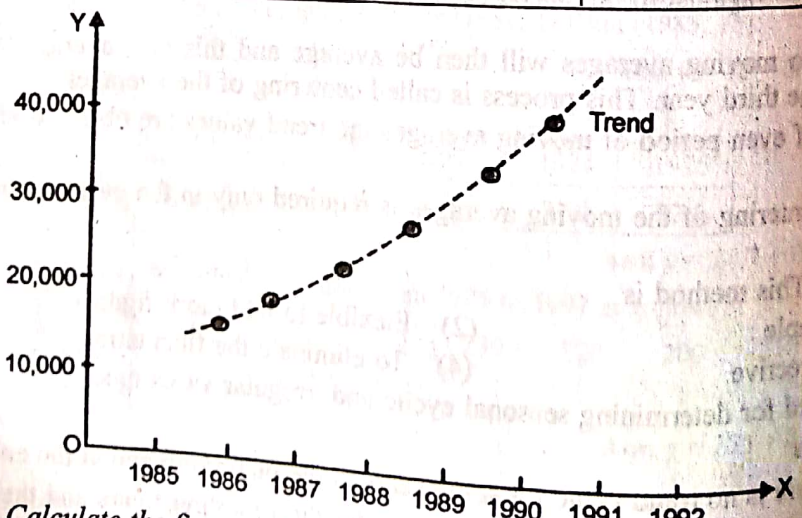
(4) Keep on continuing unless all values are utilized.

(5) Now, divide each 3 years moving total by 3 to get the moving average as per the years and shown in (*iv*) column.

(6) We can plot the moving averages on a graph paper by taking years along x-axis and moving averages along y-axis by choosing suitable scales. The resulting graph provides the trend.

Table

Year (*i*)	Profits (in ₹) (*ii*)	3- Year moving totals (*iii*)	3- year moving average (*iv*)
1985	15420	—	—
1986	14470	45410	15136.7
1987	15520	51010	17003.3
1988	21020	62660	20886.7
1989	26120	79090	26363.3
1990	31950	93440	31146.7
1991	35370	102990	34330.0
1992	35670		



Example 5. *Calculate the five yearly moving average of acres under tea in India from the following data :*

Year	*1925*	*1926*	*1927*	*1928*	*1929*	*1930*	*1931*	*1932*	*1933*	*1934*
Area (in 1000 acres)	*672*	*679*	*690*	*702*	*712*	*802*	*807*	*809*	*816*	*821*

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Statistical Quality Control Methods
(Control Charts)

15.1 INTRODUCTION

Quality control is very important in case of product manufactured in factory. The sale/profit of the products depends upon the quality of a product. So it is necessary to control the quality of a product.

15.2 WAYS TO CONTROL OF QUALITY OF A PRODUCT

There are two ways to control the quality of product

1. Physical inspection

2. Statistical quality control.

(1) Physical Inspection.

This method is suitable in those cases where the product produced are costly and of big size such as engines, boilers - plants etc. They are manufactured generally on make **to order.**

Here the number of product is small.

(2) Statistical Quality Control (S.Q.C.)

When the number of the products is on large scale or mass production basis such as biscuits, screws, medicines, etc.

Statistical quality control refers to statistical techniques which are employed for the control and maintenance of the uniform quality of the product manufactured in a factory through continuous flow of production.

(S.Q.C.) is defined as an economic and effective system of maintaining and improving the quality of the products throughout the whole operating process of specification. SQC depends upon quality of raw material, man power, machines and management. The objective of SQC is

(1) to fulfill the specification

(2) to control the variation

(3) to find causes of variation

15.3 CAUSES OF VARIATION

(*i*) Chance variations or Random Variations

(*ii*) Assignable variations

(i) Chance variation or Random variation

These are natural inherent and small variations.

(ii) Assignable variations

The variations due to fault in machine, mistake of workman, defective tools, defective raw materials are called assignable variations.

15.4 ADVANTAGE OF S.Q.C.

(1) It increases the confidence of the customer rely on the quality of the product

(2) Effective Check up

(3) Consciousness of the workers

(4) Protection against losses to the producer.

(5) Avoids rejection of the large number of products.

(6) Reduces the cost of inspection

(7) Limits of the quality

(8) We can take corrective measures

(9) Guide for setting up a new plant.

(10) Good will of the product.

(11) working life of equipment.

15.5 TYPES OF QUALITY CONTROL

1. Process control

2. Product control

Process Control

Here the quality is controlled while the product being produced. A chart is also prepared to control the quality.

Product control

The quality of the product is checked before sale

15.6 CONTROL CHART

A control chart is a graphical chart used for presenting a sequence of suitable sample characteristic. According to Dr. Walter Chart serves three purposes.

(i) To define the goal or standard for the process.

(ii) It is an instrument to maintain the goal.

(iii) It serves as means of judging whether the goal is being achieved or not.

It is the most important tool of statistical quality control.

There are three horizontal lines in a control chart. They start from R.H.S. of the vertical line and parallel to the base line of the chart. The vertical line represents the quality statistic of each sample. These three horizontal lines are known as control lines.

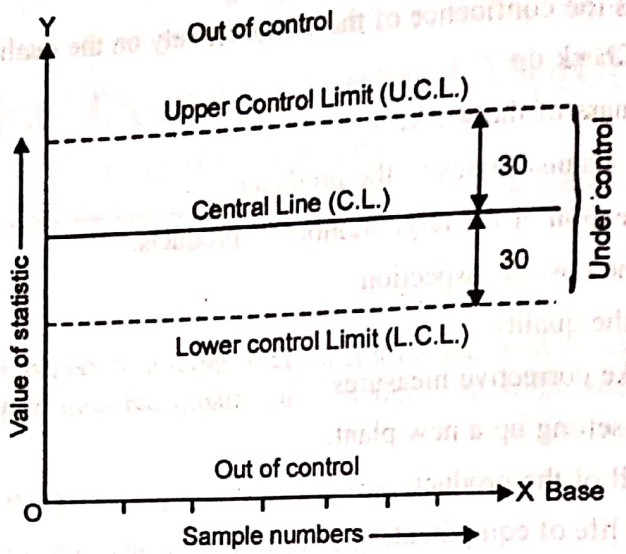
1. **Central line. (C.L.)** It passes through the middle of the chart and is parallel to the base. It represents the prescribed standard quality of the product.

2. Upper Control Limit (U.C.L.)

It is shown in the chart by a dotted line that passes through the chart above and parallel to the central line and represent the upper limit of tolerance.

3. Lower Control Limit (L.C.L.)

It is shown in the chart by a dotted line that passes through the chart below and parallel to the central line. It represents the lower limit of tolerance.



15.7 ADVANTAGES OF THE USE OF A CONTROL CHART

1. It acts as a guide for getting up of a future standard quality.
2. It is a flexible tool for controlling the quality of the products between two tolerance limits.
3. It detects any unusual variations in the production.
4. It gives warning immediately when the process goes out of control.
5. It is economical and time saving device which needs no inspector.
6. It indicates whether the process is satisfactory or not.

15.8 DETERMINATION OF CONTROL LIMIT

If a variable x is normally distributed then the probability lies between $\mu \pm 3\sigma$ (0.997), which is very high, where μ is the mean and σ is the standard deviation.

If a sample point falls out side the three sigma limits it may be assumed that it happens due to the presence of some assignable causes in the process of production that the said point indicates to some factor contributing to the quality variation in the process.