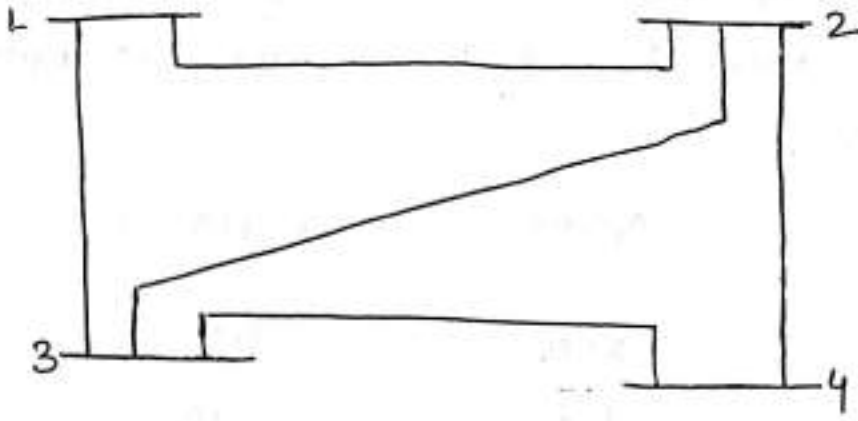


Qus



Line Bus to Bus	R (pu)	X (pu)
1-2	0.05	0.5
1-3	0.10	0.30
2-3	0.15	0.45
2-4	0.10	0.30
3-4	0.05	0.15

From the sample system shown in figure, the generators are connected at all the four buses, while loads are at buses 2 and 3, values of real and reactive power are listed in table. All buses other than the slack are PQ type. Assuming a flat voltage start, find the voltages and bus angles at the three buses at the end of the first GS ^{method} iteration.

Input data

Bus	P_i (pu)	Q_i (pu)	V_i (pu)	Remarks
1.	—	—	1.04 $\angle 0^\circ$	Slack Bus
2.	0.5	-0.2	—	PQ Bus
3.	-1.0	-0.5	—	PQ Bus
4.	0.3	-0.1	—	PQ Bus.

551. Calculation of Y_{bus} .

Change the value of impedances to each buses in admittance.

Line Bus to Bus	G (pu)	B (pu)
1-2	2.0	-6.0
1-3	1.0	-3.0
2-3	0.666	-2.0
2-4	1.0	-3.0
3-4	2.0	-6.0

$$Y_{BUS} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} y_{12} + y_{13} & -y_{12} & -y_{13} & 0 \\ -y_{21} & y_{21} + y_{23} + y_{24} & -y_{23} & y_{24} \\ -y_{31} & -y_{32} & y_{31} + y_{32} + y_{34} & -y_{34} \\ 0 & -y_{42} & -y_{43} & y_{42} + y_{43} \end{bmatrix}$$

$$= \begin{bmatrix} 3-j9 & -2+6j & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Bus voltages at the end of the first iteration are calculated using eqn. (13)

$$V_2' = \frac{1}{Y_{22}} \left\{ \frac{P_2 - jQ_2}{(1-j0)} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0 \right\}$$

$$= \frac{1}{Y_{22}} \left\{ \frac{0.5 + j0.2}{1-j0} - 1.04(-2+j6) - (-0.666+j2)(1+j0) - (-1+j3) \right\}$$

$$= \frac{1}{(3.666 - j11)} \left\{ \frac{0.5 + j0.2}{1} - 1.04(-2 + j6) + 0.666 - j2 + 1 - j3 \right\}$$

$$= \frac{1}{(3.666 - j11)} \left\{ 0.5 + j0.2 + 2.08 - j6.24 + 0.666 - j5 \right\}$$

$$= \frac{1}{(3.666 - j11)} (4.246 - j11.04)$$

$$= 1.019 + j0.046 \text{ pu.}$$

$$V_3' = \frac{1}{Y_{33}} \left\{ \frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^0 - Y_{32} V_2' - Y_{34} V_4^0 \right\}$$

↳ Always same due to slack Bus

$$= \frac{1}{Y_{33}} \left\{ \frac{-1 - j0.5}{1 - j0} - 1.04(-1 + j3) - (-0.666 + j2)(1.019 + j0.046) - (-2 + j6)(1 + j0) \right\}$$

$$= \frac{2.01 - j11.627}{3.666 - j11} = (1.028 - j0.087) \text{ pu.}$$

$$V_4' = \frac{1}{Y_{44}} \left\{ \frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41} V_1^0 - Y_{42} V_2' - Y_{43} V_3' \right\}$$

$$= \frac{1}{Y_{44}} \left\{ \frac{0.3 + j0.1}{1 - j0} - (-1 + j3)(1.019 + j0.046) - (-2 + j6)(1.028 - j0.087) \right\}$$

$$= \frac{2.991 - j9.253}{3 - j9} = 1.025 - j0.0053 \text{ pu.}$$

Ques- In above example, let Bus 2 be PV bus now with $N_2 = 1.04 \text{ pu}$. Once again assuming a flat voltage start, find Q_2, δ_2, V_3, V_4 at the end of the first GS iteration. (Given: $0.2 \leq Q_2 \leq 1$)

Note $\delta_2^0 = 0$ i.e. $V_2^0 = 1.04 + j0$

from eqn (18)

$$Q_2^1 = -\text{Im} \left\{ (V_2^0)^* Y_{21} V_1^1 + (V_2^0)^* [Y_{22} V_2^0 + Y_{23} V_3^0 + Y_{24} V_4^0] \right\}$$

$$= -\text{Im} \left\{ 1.04(-2 + j6)1.04 + 1.04 [(3.666 - j11)1.04 + (-0.666 + j2) + (-1 + j3)] \right\}$$

$$= -\text{Im} \left\{ -0.0693 - j0.2079 \right\}$$

$Q_2^1 = 0.2079 \text{ pu}$

from eqn (19)

$$\delta_2^1 = \angle \left\{ \frac{1}{Y_{22}} \left[\frac{P_2 - jQ_2^1}{(V_2^0)^*} - Y_{21} V_1^1 - Y_{23} V_3^0 - Y_{24} V_4^0 \right] \right\}$$

$$= \angle \left\{ \frac{1}{3.666 - j11} \left[\frac{0.5 - j0.2079}{1.04 - j0} - (-2 + j6)(1.04 + j0) - (-0.666 + j2)(1 + j0) - (-1 + j3)(1 + j0) \right] \right\}$$

$$= \angle \left(\frac{4.2267 - j11.439}{3.66 - j11} \right) = \angle (1.0512 + j0.0339)$$

or $\delta_2^1 = 1.8465^\circ = 0.032 \text{ rad.}$

$$V_2^1 = 1.04 (\cos \delta_2^1 + j \sin \delta_2^1)$$

$$= 1.04 (0.99948 + j0.0322)$$

$$V_3^1 = \frac{1}{Y_{33}} \left\{ \frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31} V_1^1 - Y_{32} V_2^1 - Y_{34} V_4^0 \right\}$$

$$= \frac{1}{3.666 - j11} \left[\frac{-1 - j0.5}{(1 - j0)} - (-1 + j3)1.04 - (-0.666 + j2)(1.03946 + j0.03351) - (-2 + j6) \right]$$

$$= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937$$

$$V_4^1 = \frac{1}{Y_{44}} \left\{ \frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1^0 - Y_{42}V_2^1 - Y_{43}V_3^1 \right\}$$

$$= \frac{1}{3 - j9} \left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)(1.0394 + j0.0335) - (-2 + j6)(1.0317 - j0.08937) \right]$$

$$= \frac{2.9671 - j8.9962}{3 - j9} = 0.9985 - j0.0031$$

When the range of Q is not in sample

Now suppose the permissible limits on Q₂ (reactive power injection) are revised as follows:

$$0.25 \leq Q_2 \leq 1.0 \text{ pu}$$

It is clear, that other data remaining the same, the calculated Q₂ (= 0.2079) is now less than the Q₂ min. Hence Q₂ is set equal to Q₂ min i.e. Q₂ = 0.25 pu.

Bus 2, therefore, becomes a PQ bus from a PV Bus. Therefore, |V₂| can no longer remain fixed at 1.04 pu. The value of V₂ at the end of the first iteration is calculated as follows - (Note V₂⁰ = 1 + j0 by virtue of flat start)

$$V_2' = \frac{1}{Y_{22}} \left(\frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21}V_1' - Y_{23}V_3^0 - Y_{24}V_4^0 \right)$$

$$= \frac{1}{3.666 - j11} \left[\frac{0.5 - j0.25}{1 - j0} - (-2 + j6)1.04 - (-0.666 + j2) \right] - (-1 + j3)$$

$$= \frac{4.246 - j11.49}{3.666 - j11} = 1.0559 + j0.0341$$

$$V_3' = \frac{1}{Y_{33}} \left(\frac{P_3 - jQ_3}{(V_3^0)^*} - Y_{31}V_1' - Y_{32}V_2' - Y_{34}V_4^0 \right)$$

$$= \frac{1}{3.666 - j11} \left[\frac{-1 - j0.5}{1 - j0} - (-1 + j3)1.04 - (-0.666 + j2)(1.0559 + j0.0341) \right] - (-2 + j6)$$

$$= \frac{2.8112 - j11.709}{3.666 - j11} = 1.0347 - j0.0893 \text{ pu.}$$

$$V_4' = \frac{1}{Y_{44}} \left(\frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41}V_1' - Y_{42}V_2' - Y_{43}V_3' \right)$$

$$= \frac{1}{3 - j9} \left[\frac{0.3 + j0.1}{1 - j0} - (-1 + j3)(1.0559 + j0.0341) - (-2 + j6)(1.0347 - j0.0893) \right]$$

$$= \frac{4.0630 - j9.4204}{3 - j9} = 1.0775 + j0.0923 \text{ pu.}$$