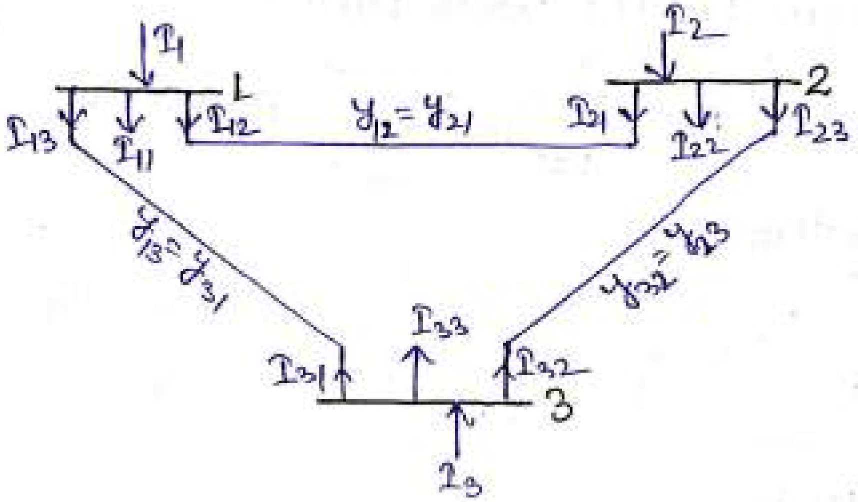
Nodal Admittance Matrix :-

The load flow equations, using nodal admittance formulation for a three-bus system are developed first and then they are generalized for an n-bus system



Three-bus System

At node 1

$$I_1 = I_{11} + I_{12} + I_{13}$$

$$= V_1 y_{11} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13}$$

$$= V_1 y_{11} + V_1 y_{12} - V_2 y_{12} + V_3 y_{13} - V_3 y_{13}$$

$$= V_1 (y_{11} + y_{12} + y_{13}) - V_2 y_{12} - V_3 y_{13}$$

$$I_1 = V_1 Y_{11} + V_2 Y_{12} + V_3 Y_{13} \longrightarrow (1)$$

Here y_{11} is the shunt charging admittance at bus 1 and ground.

$$Y_{11} = y_{11} + y_{12} + y_{13}$$

$$Y_{12} = -y_{12}, \quad Y_{13} = -y_{13}$$

Similarly nodal current equations for the other nodes can be written as follows —

$$I_2 = V_1 Y_{21} + V_2 Y_{22} + V_3 Y_{23} \longrightarrow (2)$$

$$I_3 = V_1 Y_{31} + V_2 Y_{32} + V_3 Y_{33} \longrightarrow (3)$$

These equations canbe written in a matrix form asfollows -

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \longrightarrow (4)$$

$Y_{12} = y_{12}$
$Y_{31} = Y_{13} = -y_{13}$
$Y_{21} = -y_{12}$
$Y_{32} = Y_{23} = -y_{23}$

Generalise equ. for the node current

$$\boxed{I_p = \sum_{q=1}^{3} Y_{pq} V_q ,} \quad p = 1 \text{ to } 3 \longrightarrow (5)$$

For n-Bus system

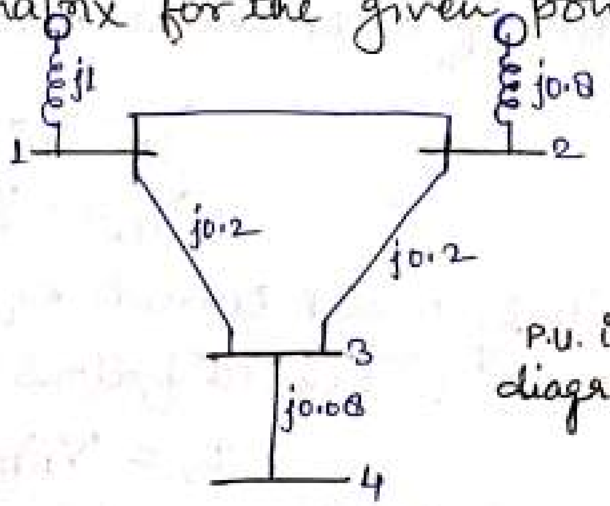
$$\boxed{I_p = \sum_{q=1}^{n} Y_{pq} V_q} \quad p = 1, 2 \ldots n$$

In matrix form

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & & Y_{2n} \\ \vdots & \vdots & & \vdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

It can be shown that the nodal admittance matrix is a sparse matrix (a few no. of elements are non-zero) for an actual power system.

Find the admittance matrix for the given power system network.

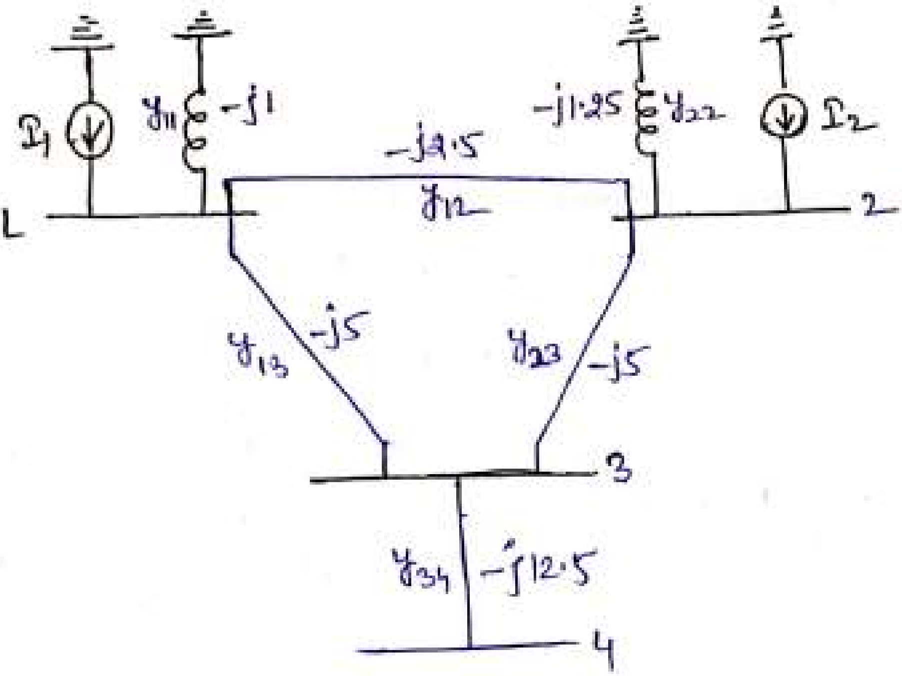


P.U. impedance diagram of P.S. N/W

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix}$$

Since the nodal solution is based upon ~~Mr~~ Kirchhoff's current law, impedance are converted to admittance i.e.

$$y_{ij} = \frac{1}{Z_{ij}} = \frac{1}{r_{ij} + jx_{ij}}$$



$$Y_{11} = y_{11} + y_{12} + y_{13} + y_{14}$$
$$= -j1 + (-j2.5) - j5 + 0$$
$$= -j8.5$$

$$Y_{22} = y_{21} + y_{22} + y_{23} + y_{24}$$
$$= -j2.5 - j1.25 - j5 + 0$$
$$= -j8.75$$

$$Y_{33} = y_{31} + y_{32} + y_{33} + y_{34}$$
$$= -j5 - j5 + 0 - j12.5$$
$$= -j22.5$$

$$Y_{44} = y_{41} + y_{42} + y_{43} + y_{44}$$
$$= 0 + 0 - j12.5 + 0$$
$$= -j12.5$$

$Y_{12} = +j2.5 \quad Y_{13} = +j5, \; Y_{14} = 0$

$Y_{21} = +j2.5 \quad Y_{23} = +j5, \; Y_{24} = 0$

$Y_{31} = +j5 \quad Y_{32} = +j5, \; Y_{34} = +j12.5$

$Y_{41} = 0 \quad Y_{42} = 0, \; Y_{43} = +j12.5.$

$$Y_{Bus} = \begin{bmatrix} -j8.5 & +j2.5 & +j5 & 0 \\ +j2.5 & -j8.75 & +j5 & 0 \\ +j5 & +j5 & -j22.5 & +j12.5 \\ 0 & 0 & +j12.5 & -j12.5 \end{bmatrix}$$

LOAD FLOW PROBLEM FORMULATION —

The complex power injected by the source into the i^{th} bus of a power system is —

$$S_i = P_i + jQ_i = V_i I_i^* \qquad i = 1, 2 \ldots n \longrightarrow (1)$$

Where $V_i \rightarrow$ voltage at i^{th} Bus w.r.t ground

$I_i \rightarrow$ source current injected into the Bus.

The load flow problem is handled more conveniently by use of I_i rather than I_i^*. Therefore, taking the complex conjugate of equ. (1)

$$P_i - jQ_i = V_i^* I_i \longrightarrow (2)$$

As we know that

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k \longrightarrow (3)$$

Use equ. no. (3) in (2)

$$P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k \qquad i = 1, 2 \ldots n \longrightarrow (4)$$

Equating real and imaginary parts

$$P_i = \mathrm{Re}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\} \longrightarrow (5)$$

$$Q_i = -\mathrm{Im}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\} \longrightarrow (6)$$