Unit-III (ME-201)

Elastic Constants

Elastic Limit:

When an external force acts on a body, the body tends to undergo some deformation. If the external force is removed and the body comes back to its origin shape and size, the body is known as elastic body. This property, by virtue of which certain materials return back to their original position after the removal of the external force, is called elasticity.

The body will regain its previous shape and size only when the deformation caused by the external force, is within a certain limit. Thus there is a limiting value of force up to and within which, the deformation completely disappears on the removal of the force. The value of stress corresponding to this limiting force is known as the elastic limit of the material.

Hooke's law:

It states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit.

Stress /Strain = Constant

This constant is known as elastic constant.

Types of Elastic Constants:

There are three elastic constants;

Normal stress/ Normal strain = **Young's modulus** or Modulus of elasticity (E)

Shear stress/ Shear strain = **Shear modulus** or Modulus of Rigidity (G)

Direct stress/ Volumetric strain = **Bulk modulus** (K)

Young's Modulus or Modulus of elasticity (E):

It is defined as the ratio of normal stress (σ) to the longitudinal strain (e).

$$E = (\sigma_n) / (e)$$

Modulus of Rigidity or Shear Modulus (G or C):

It is the ratio between shear stress (**τ**) and shear strain (**e_s**). It is denoted by G or C.

$$G= \tau/e_s$$

Bulk Modulus or Volume Modulus of Elasticity (K):

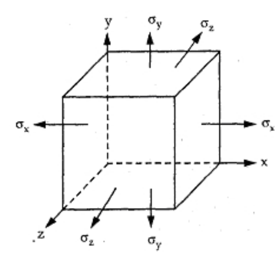
It may be defined as the ratio of normal stress (on each face of a solid cube) to volumetric strain. It is denoted by K. Bulk modulus is a measure of the resistance of a material to change of volume without change of shape or form.

$$K = \text{Direct Stress / Volumetric strain}$$

$$= \sigma/e_v$$

Relation between E, K and Poisson's Ratio (μ or 1/m):

Consider a cubical element subjected to volumetric stress σ which acts simultaneously along the mutually perpendicular x, y and z-direction.



The resultant strains along the three directions can be worked out by taking the effect of individual stresses.

Strain in the x-direction,

e_x = strain in x-direction due to σ_x - strain in x-direction due to σ_Y - strain in x-direction due to σ_Z

$= \sigma_x/E – \sigma_y/mE – \sigma_z/mE$ ------------------(1)

$$\text{But } \sigma_X = \sigma_Y = \sigma_Z = \sigma$$

$$\text{So } \sigma_X = \sigma/E – \sigma/mE – \sigma/mE$$

$$\sigma_X = \sigma/E(1–2/m)$$

$$\text{Similarly } \sigma_Y = \sigma/E (1–2/m) \text{ and } \sigma_Z = \sigma/E (1–2/m)$$

Now Volumetric strain

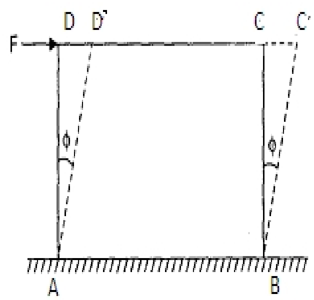
$$e_V = e_X + e_Y + e_Z = 3\sigma/E(1–2/m) \text{ ----------(2)}$$

Since bulk modulus $K = \sigma/ e_V$

$$K = \sigma/ \{3\sigma/E\ (1–2/m)\ \}$$

On simplification **E = 3K (1-2/m) or E = 3K (1-2μ)**

Relation between E, G and Poisson's Ratio (1/m or μ): Consider a cubic element *ABCD* fixed at the bottom face and subjected to shearing force at the top face. The block experiences the following effects due to this shearing load:



1. Shearing stress τ is induced at the faces DC and *AB*
2. Complimentary shearing stress of the same magnitude is set up on the faces *AD* and BC.
3. The block distorts to a new configuration ABC'D'.
4. The diagonal AC elongates (tension) and diagonal *BD* shortens (compression).

Longitudinal strain 'e' in diagonal AC = (AC' – AC)/AC

= (AC' – AE)/AC

= EC'/AC -----------(1)

Where CE is perpendicular from C onto AC'.

Since CC' is too small, assume

Angle ACB = Angle ACB = 45^0

Therefore EC' = CC'cos45^0 = CC'/√2

Longitudinal strain 'e' = CC'/AC√2

= CC'/√2.BC.√2

= tanΦ/2 = Φ/2 = e_S/2------------------(2)

Where, Φ = CC'/BC represents the shear strain (e_S)

In terms of shear stress τ and modulus of rigidity G, shear strain (e_S) = τ/G ------------(3)

Putting shear strain (e_S) = 2. Longitudinal strain

Longitudinal strain of diagonal AC = τ/2 -----------(4)

The strain in diagonal AC is also given by

= strain due to tensile stress in AC - strain due to compressive stress in *BD*

$= \tau/E - (-\tau/mE) = \tau/E\ (1 + 1/m)$ -------------(5)

From equation (4) and (5), we get

$$\tau/2G = \tau/E(1 + 1/m)$$

or **E = 2G(1 + 1/m)** or **E = 2G(1+μ) -------------**(6)

Relation between E, G and K:

With reference to the relations (1) and (6) derived above,

$$E = 2G\ (1 + \mu) = 3K\ (1- 2\ \mu)$$

To eliminate 1/m from these two expressions for E, we have

$$\mathbf{E = 9KG/\ (G + 3K)}$$

Finally**; E = 2G (1 + μ) = 3K (1 – 2 μ) or E = 9KG/ (G + 3K)**