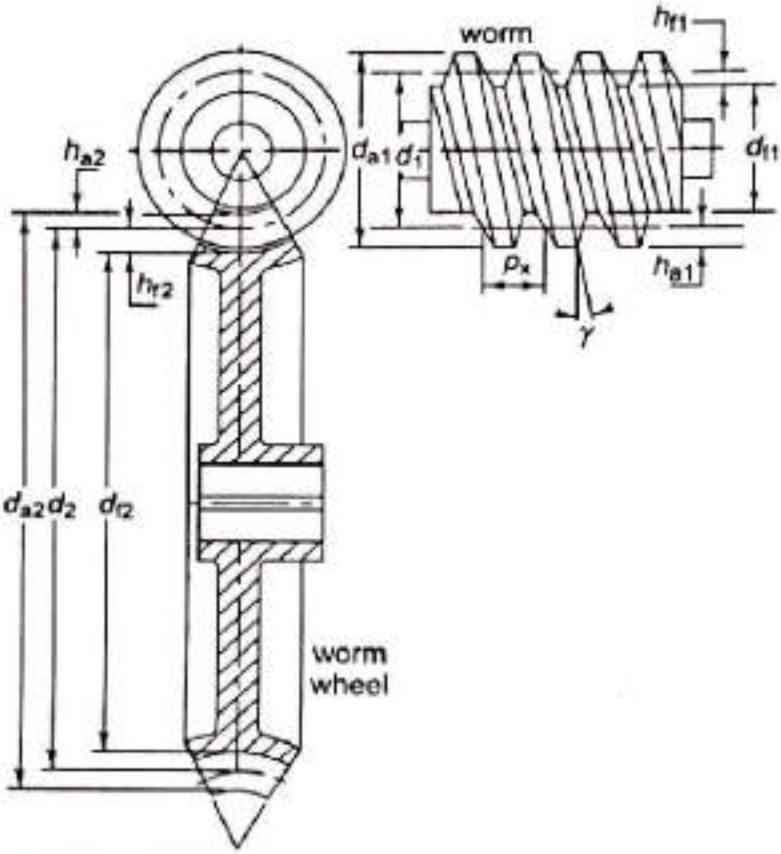
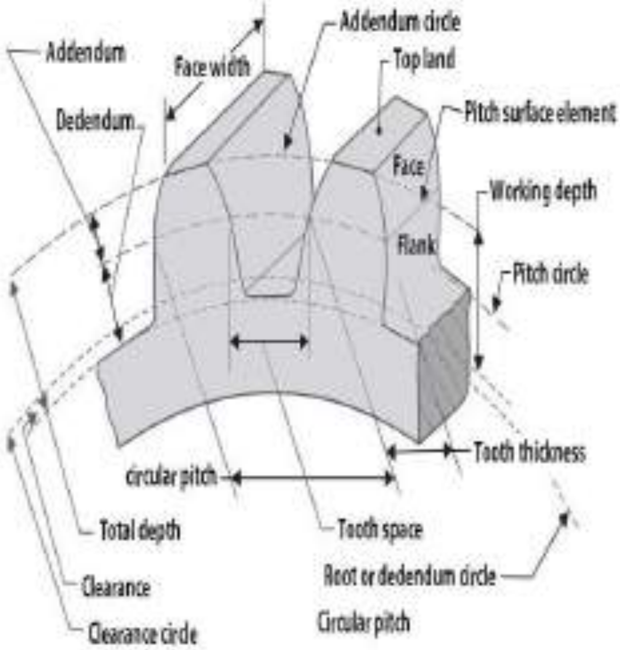


Fig. 20.2 *Dimension of worm gears*



20.3 PROPORTIONS OF WORM GEARS

The basic dimensions of the worm and the worm wheel are shown in Fig. 20.2. For an involute helicoidal tooth form,

$$h_{a1} = m \quad (20.11)$$

$$h_{f1} = (2.2 \cos \gamma - 1)\, m \quad (20.12)$$

$$c = 0.2\, m \cos \gamma \quad (20.13)$$

where,

h_{a1} = addendum (mm)

h_{f1} = dedendum (mm)

c = clearance (mm)

The outside and root diameters of the worm are, therefore, expressed as follows:

$$d_{a1} = d_1 + 2h_{a1} = qm + 2m$$

or $$d_{a1} = m(q + 2) \quad (20.14)$$

$$d_{f1} = d_1 - 2h_{f1} = qm - 2m(2.2 \cos \gamma - 1)$$

$$d_{f1} = m(q + 2 - 4.4 \cos \gamma) \quad (20.15)$$

where

d_{a1} = outside diameter of the worm (mm)

d_{f1} = root diameter of the worm (mm)

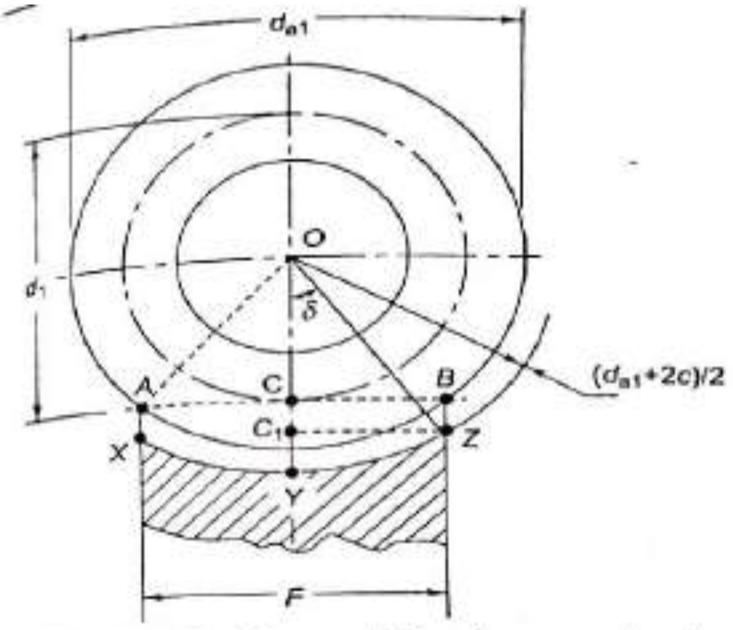


Fig. 20.3 *Face width of worm wheel*

From triangle AOC,

$$(\overline{AC})^2 = (\overline{AO})^2 - (\overline{OC})^2$$

or $$\left(\frac{F}{2}\right)^2 = \left(\frac{d_{a1}}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2 = \left[\frac{m(q+2)}{2}\right]^2 - \left[\frac{qm}{2}\right]^2$$

$$\therefore \quad F = 2m\sqrt{(q+1)} \qquad (20.20)$$

From triangle OZC_1,

$$\sin\delta = \frac{C_1Z}{OZ} = \frac{F/2}{(d_{a1}+2c)/2}$$

or $$\delta = \sin^{-1}\left(\frac{F}{d_{a1}+2c}\right)$$

The length of the root of the worm wheel teeth is arc XYZ, that is denoted by (l_r).

$$l_r = \text{arc } XYZ = \left(\frac{2\delta}{2\pi}\right)[\pi(d_{a1}+2c)]$$

$$= (d_{a1}+2c)\,\delta$$

$$l_r = (d_{a1}+2c)\sin^{-1}\left[\frac{F}{(d_{a1}+2c)}\right] \quad (20.21)$$

Example 20.1 A pair of worm gears is designated as,

1/30/10/8

Calculate:

(i) the centre distance;
(ii) the speed reduction;
(iii) the dimensions of the worm; and
(iv) the dimensions of the worm wheel

Solution For the given pair,

$z_1 = 1 \quad z_2 = 30$ teeth $\quad q = 10 \quad m = 8$ mm

From Eq. (20.9) and (20.10),

$$a = \frac{1}{2}m(q+z_2) = \frac{1}{2}(8)(10+30) = 160 \text{ mm} \qquad \text{(i)}$$

$$i = \frac{z_2}{z_1} = 30 \qquad \text{(ii)}$$

Dimensions of worm:

$$d_1 = qm = 10(8) = 80 \text{ mm} \qquad \text{(a)}$$

$$d_{a1} = m(q+2) = 8(10+2) = 96 \text{ mm} \qquad \text{(b)}$$

$$\tan\gamma = \frac{z_1}{q} = \frac{1}{10} \quad \text{or} \quad \gamma = 5.71°$$

$$d_{f1} = m(q+2-4.4\cos\gamma) = 8[10+2-4.4\cos(5.71°)]$$
$$= 60.9747 \qquad \text{(c)}$$

$$p_x = \pi m = \pi(8) = 25.1327 \text{ mm} \qquad \text{(d)}$$

Dimensions of worm wheel:

$$d_2 = mz_2 = 8(30) = 240 \text{ mm} \qquad \text{(a)}$$

$$d_{a2} = m(z_2 + 4\cos\gamma - 2) = 8[30 + 4\cos(5.71°) - 2]$$
$$= 255.8412 \text{ mn} \qquad \text{(b)}$$

$$d_{f2} = m(z_2 - 2 - 0.4\cos\gamma) = 8[30 - 2 - 0.4\cos(5.71°)]$$
$$= 220.8159 \text{ mm} \qquad \text{(c)}$$

20.4 FORCE ANALYSIS

The analysis of three components of the resultant tooth force between meshing teeth of worm and worm wheel is based on following assumptions:

(i) The worm is the driving element, while the worm wheel is the driven element.

(ii) The worm has right-handed threads.

(iii) The worm rotates in anti-clockwise directions as shown in Fig. 20.4.

The three components of the gear tooth force between the worm and the worm wheel are shown in Fig. 20.4. Suffix 1 is used for the worm, while suffix 2 for the worm wheel. The components of the resultant force acting on the worm are as follows:

$(P_1)_t$ = tangential component on the worm (N)

$(P_1)_a$ = axial component on the worm (N)

$(P_1)_r$ = radial component on the worm (N)

The components $(P_2)_t$, $(P_2)_a$ and $(P_2)_r$ acting on the worm wheel are defined in a similar way. The force acting on the worm wheel is equal and opposite reaction of the force acting on the worm.

Therefore,

$$(P_2)_t = -(P_1)_a \qquad (20.22)$$

$$(P_2)_a = -(P_1)_t \qquad (20.23)$$

$$(P_2)_r = -(P_1)_r \qquad (20.24)$$

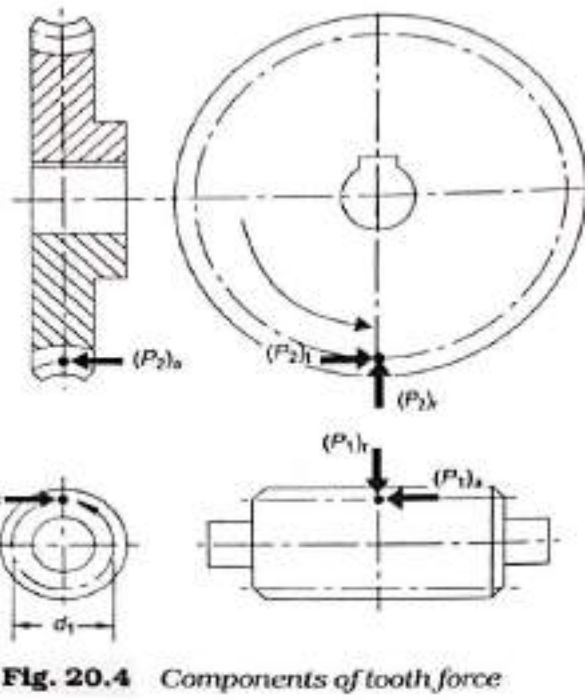


Fig. 20.4 *Components of tooth force*

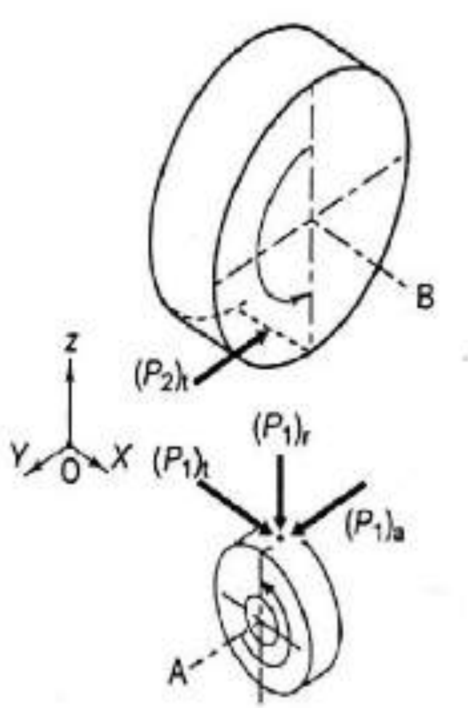


Fig. 20.5 *Direction of force components*

FORCE COMPONENTS ON WORM

The components of the normal reaction P acting on the worm are shown in Fig. 20.6 α is normal pressure angle, while γ is the lead angle. Note that angle α is in plane $ABCD$ shaded by dots, while angle γ is in top plane $AEBF$. Resolving the normal reaction P in the plane $ABCD$ shown in Fig. 20.6(b),

$$P_N = P \cos \alpha \qquad \text{(a)}$$

$$P_r = P \sin \alpha \qquad \text{(b)}$$

Resolving the component P_N in the plane $AEBF$ shown in Fig. 20.6(c),

$$P_a = P_N \cos \gamma \qquad \text{(c)}$$

$$P_t = P_N \sin \gamma \qquad \text{(d)}$$

From relationships (a), (b), (c) and (d),

$$P_t = P \cos \alpha \sin \gamma$$

$$P_a = P \cos \alpha \cos \gamma$$

$$P_r = P \sin \alpha \qquad (20.25)$$

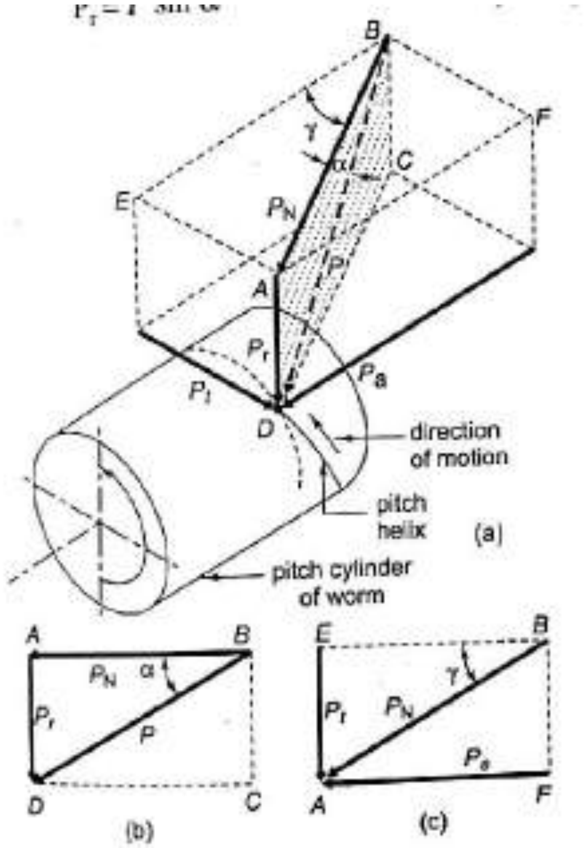
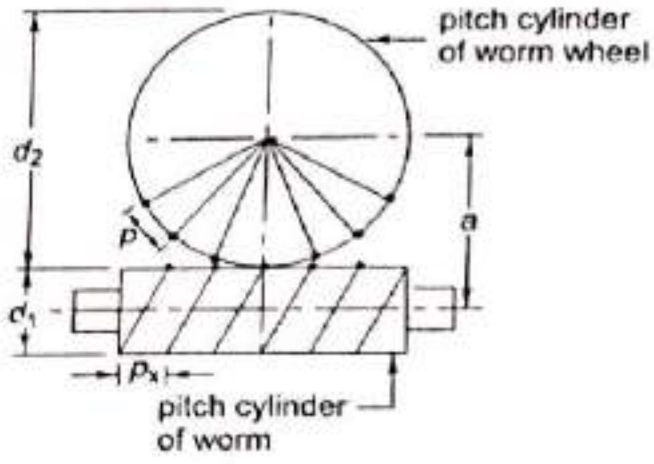
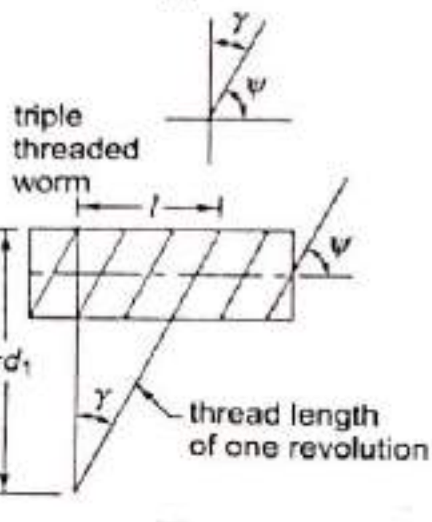


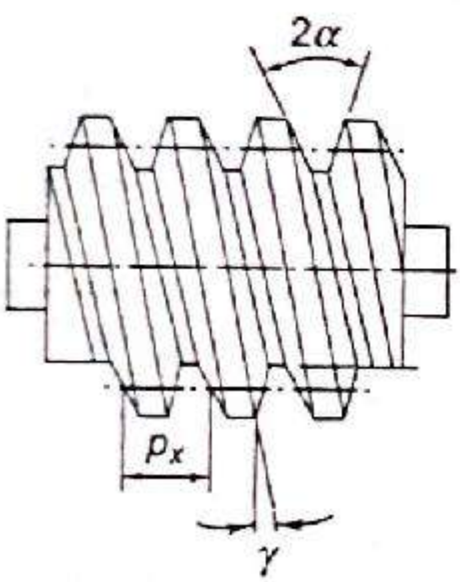
Fig. 20.6 *Components of normal reaction*



(a)



(b)



FRICTION FORCE COMPONENTS

The resultant frictional force is (μP) where μ is the coefficient of friction. The direction of the frictional force will be along the pitch helix and opposite to the direction of rotation, as shown in Fig. 20.7. There are two components of the frictional force:

(i) Component ($\mu P \cos \gamma$) in the tangential direction. The direction of this component is same as that of P_t.

(ii) Component ($\mu P \sin \gamma$) in the axial direction. The direction of this component is opposite to that of P_a.

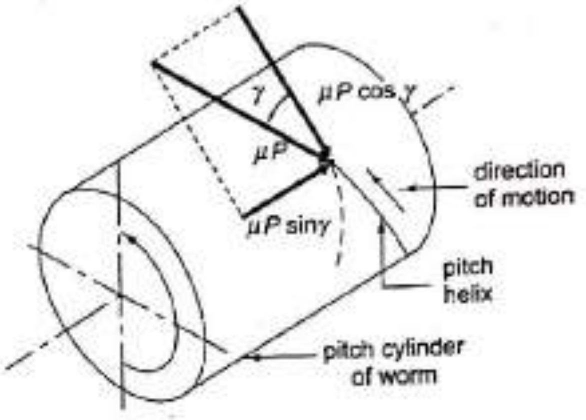


Fig. 20.7 *Components of frictional force*

Superimposing the components of normal reaction and frictional force, we have

$$(P_1)_t = P \cos \alpha \sin \gamma + \mu P \cos \gamma$$

$$\therefore \quad (P_1)_t = P(\cos \alpha \sin \gamma + \mu \cos \gamma) \qquad (20.26)$$

Similarly, $(P_1)_a = P\cos \alpha \cos \gamma - \mu(\sin \gamma)$

$$\therefore \quad (P_1)_a = P(\cos \alpha \cos \gamma - \mu \sin \gamma \qquad (20.27)$$

and $\quad (P_1)_r = P \sin \alpha \qquad (20.28)$

In practice, the tangential component $(P_1)_t$ on the worm is determined from the torque that is transmitted from the worm to the worm wheel. Therefore,

$$(P_1)_t = \frac{2 M_t}{d_1} \qquad (20.29)$$

From Eqs (20.26) and (20.27),

$$(P_1)_a = (P_1)_t \times \frac{(\cos \alpha \cos \gamma - \mu \sin \gamma)}{(\cos \alpha \sin \gamma + \mu \cos \gamma)} \qquad (20.30)$$

From Eqs (20.26) and (20.28),

$$(P_1)_r = (P_1)_t \times \frac{\sin \alpha}{(\cos \alpha \sin \gamma + \mu \cos \gamma)} \qquad (20.31)$$

Equations (20.29), (20.30) and (20.31) are used to determine the magnitude of components of the resultant tooth force.

NUMERICAL

Example 20.2 A pair of worm and worm wheel is designated as

3/60/10/6

The worm is transmitting 5 kW power at 1440 rpm to the worm wheel. The coefficient of friction is 0.1 and the normal pressure angle is 20°. Determine the components of the gear tooth force acting on the worm and the worm wheel.

Solution

$z_1 = 3 \quad z_2 = 60 \text{ teeth} \qquad q = 10 \quad m = 6 \text{ mm}$

$d_1 = qm = 10(6) = 60 \text{ mm}$

$$\tan\gamma = \frac{z_1}{q} = \frac{3}{10} = 0.3 \text{ or } \qquad \gamma = 16.7°$$

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_1} = \frac{60 \times 10^6 (5)}{2\pi (1440)}$$

$$= 33157.28 \text{ N} - \text{mm}$$

From Eq. (20.29),

$$(P_1)_t = \frac{2M_t}{d_1} = \frac{2(33157.28)}{60} = 1105.24 \text{ N} \quad \text{(a)}$$

From Eqs (20.30),

$$(P_1)_a = (P_1)_t \times \frac{(\cos\alpha \cos\gamma - \mu \sin\gamma)}{(\cos\alpha \sin\gamma + \mu \cos\gamma)}$$

$$= 1105.24 \times \frac{[\cos(20)\cos(16.7) - 0.1\sin(16.7)]}{[(\cos(20)\sin(16.7) + 0.1\cos(16.7)]}$$

$$= 2632.55\text{N} \quad \text{(b)}$$

From Eq. (20.31),

$$(P_1)_r = (P_1)_t \times \frac{\sin\alpha}{(\cos\alpha \sin\gamma + \mu\cos\gamma)}$$

$$= 1105.24 \times \frac{\sin(20)}{[\cos(20)\sin(16.7) + 0.1\cos(16.7)]}$$

$$= 1033{\cdot}35 \text{ N} \quad \text{(c)}$$

The force components acting on the worm wheel are as follows (Eqs. 20.22 to 20.24).

$(P_2)_t = (P_1)_a = 2632.55 \text{ N}$

$(P_2)_a = (P_1)_t = 1105.24 \text{ N}$

$(P_2)_r = (P_1)_r = 1033.35 \text{ N}$

Efficiency of worm gear

The values of the coefficient of friction in this figure are based on the following two assumptions:

(i) The worm wheel is made of phosphor-bronze, while the worm is made of case-hardened steel.

(ii) The gears are lubricated with a mineral oil having a viscosity of 16 to 130 centiStokes at 60°C.

The efficiency of the worm gear drive is given by,

$$\eta = \frac{\text{power output}}{\text{power input}} = \frac{(P_2)_t \times (d_2)/2 \times (n_2)}{(P_1)_t \times (d_1)/2 \times (n_1)} \quad \text{(a)}$$

Since

$$\frac{n_2}{n_1} = \frac{1}{i} \quad \text{(b)}$$

and

$$\frac{d_2}{d_1} = \frac{m z_2}{m q} = \frac{z_2}{q} = \frac{z_2/z_1}{q/z_1} = i \tan\gamma \quad \text{(c)}$$

From (a), (b) and (c),

$$\eta = \frac{(P_2)_t}{(P_1)_t} \times \tan\gamma = \frac{(P_1)_a}{(P_1)_t} = \times \tan\gamma$$

From Eq. (20.30),

$$\eta = \tan\gamma \times \frac{(\cos\alpha \cos\gamma - \mu \sin\gamma)}{(\cos\alpha \sin\gamma + \mu \cos\gamma)}$$

$$\therefore \quad \eta = \frac{(\cos\alpha - \mu \tan\gamma)}{(\cos\alpha + \mu \cot\gamma)} \quad (20.34)$$

The efficiency of spur or helical gears is very high and virtually constant in the range of 98% to 99%. On the other hand, the efficiency of worm gears is low and varies considerably in the range of 50% to 98%. In general, the efficiency is inversely proportional to speed ratio, provided the coefficient of friction is constant.

In general, the worm is the driver and the worm wheel is the driven member and the reverse motion is not possible. This is called *self-locking* drive, because the worm wheel cannot drive the worm. As for screw threads, the criterion for self-locking is the relationship between the coefficient of friction and lead angle. *A worm gear drive is said to be self-locking, if the coefficient of friction is greater than tangent of lead angle, i.e. the friction angle is more than the lead angle* This approximate condition is rewritten as,

$$\mu > \tan \gamma$$

There is another term *reversible* or *overrunning* or *back-driving* worm gear drive. In this type of drive, the worm and worm wheel can drive each other. In general, the worm is the driver and the worm wheel is driven member. If the driven machinery has large inertia and if the driving power supply is cut off suddenly, the worm is freely driven by the worm wheel. This prevents the damage to the drive and source of power. *A worm gear drive is said to be reversible, if the coefficient of friction is less than tangent of lead angle, i.e. the friction angle is less than the lead angle*. This approximate condition is rewritten as,

$$\mu < \tan \gamma$$

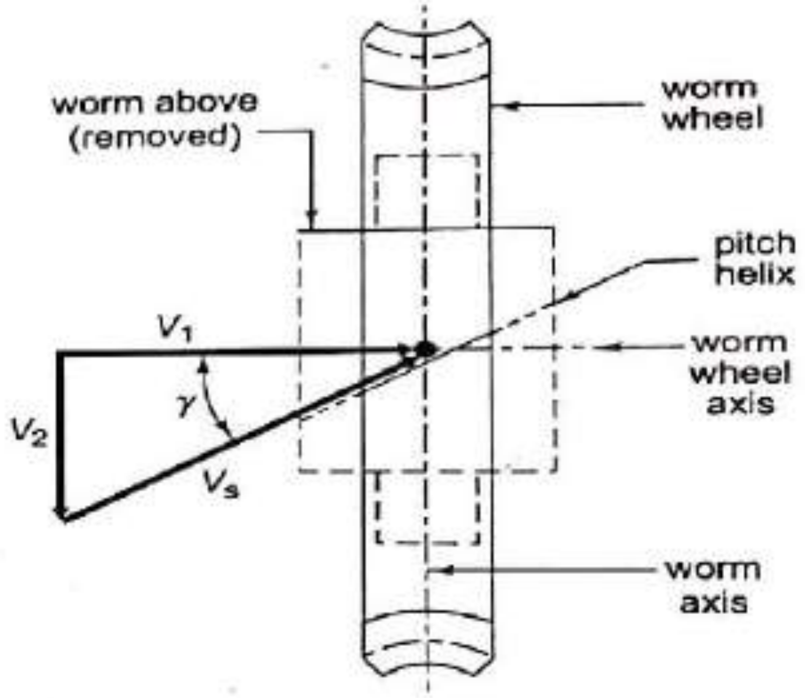


Fig. 20.8 *Valocity of sliding*

where,

V_1 = pitch line velocity of the worm (m/s)

V_2 = pitch line velocity of the worm wheel (m/s)

V_s = rubbing velocity (m/s)

The pitch line velocity of the worm is given by,

$$V_1 = \frac{\pi d_1 n_1}{60(1000)} \tag{20.32}$$

From the velocity triangle,

$$V_s = \frac{V_1}{\cos \gamma}$$

$$V_s = \frac{\pi d_1 n_1}{60\,000 \cos \gamma} \tag{20.33}$$

Example 20.3 1 kW power at 720 rpm is supplied to the worm shaft. The number of starts for threads of worm is four with a 50 mm pitch circle diameter. The worm wheel has 30 teeth with a 5 mm module. The normal pressure angle is 20°. Calculate the efficiency of the worm gear drive and the power lost in friction.

Solution From Eq. (20.5),

$$l = \pi m z_1 = \pi (5)(4) = (20\pi) \text{ mm}$$

$$\tan \gamma = \frac{l}{\pi d_1} = \frac{20\pi}{\pi (50)} = 0.4 \qquad \text{or} \qquad \gamma = 21.8^\circ$$

From Eq. (20.33),

$$V_s = \frac{\pi d_1 n_1}{60\,000 \cos \gamma} = \frac{\pi (50)(720)}{60\,000 \cos (21.8)} = 2.03 \text{ m/s}$$

From Fig. 20.9, the coefficient of friction is 0.035.

From Eq. (20.34),

$$\eta = \frac{(\cos \alpha - \mu \tan \gamma)}{(\cos \alpha + \mu \cot \gamma)} = \frac{[\cos(20) - 0.035 \tan(21.8)]}{[\cos(20) - 0.035 \cot(21.8)]}$$

$$= 0.9012 = 90.12\%$$

Power lost in friction $= (1 - \eta)$ kW $= (1 - 0.9012)(1)$

$= 0.0988$ kW or 98.8W

THANK YOU

Question Session

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