

UNIT-3

Topics covered :-

- (I) Necessity and advantages of 3 phase system
- (II) Star - Delta connections
- (III) Balanced supply and balanced load
- (IV) Line and phase voltage / current relations
- (V) Three phase power and its measurement

3.1 Necessity and advantages of 3 phase system

Three phase systems are important for at least three reasons -

- ① → First, nearly all electric power is generated and distributed in three phase, at the operating frequency of 50Hz or 60Hz. When one phase or two phase inputs are required, they are taken from the three phase system rather than generated independently.
 - ② → Second, the instantaneous power in a three phase system can be constant not pulsating. This results in uniform power transmission and less vibration of three phase machines.
 - ③ → Third, for the same amount of power, the three phase system is more economical than single phase. The amount of wire required for a three phase system is less than that required for an equivalent single phase system.
- * The output of 3 phase machine is always greater than single phase machine of same size.

① Different Type of supply connections:-

(i) Single phase supply system:-

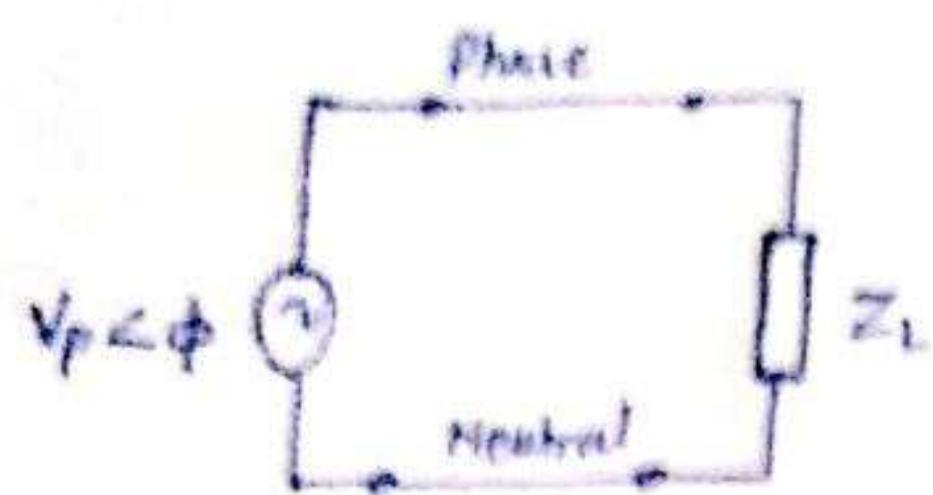


Fig. 1(a) Two wire type

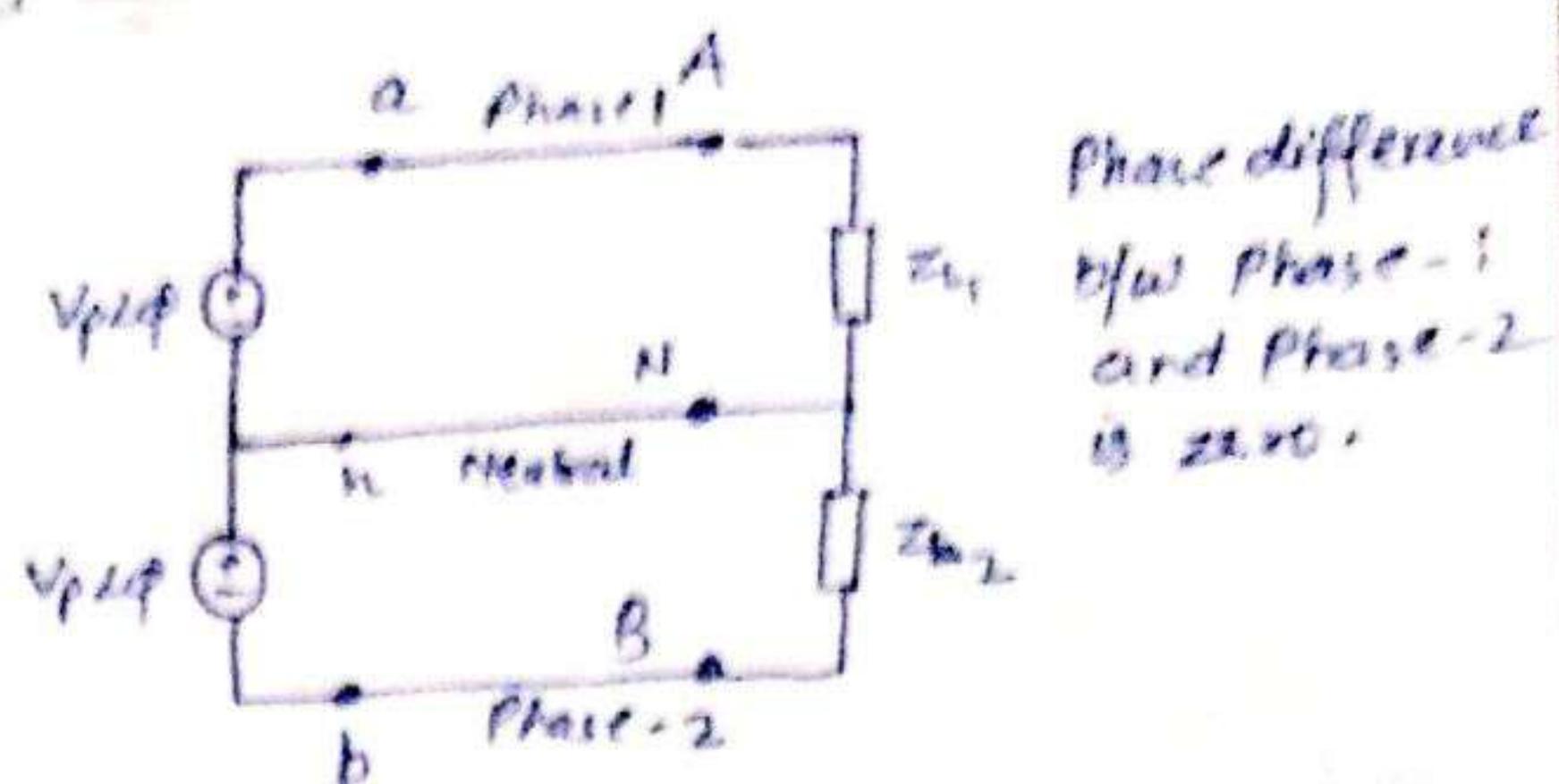


Fig. 1(c) Three wire type

(2) Two phase supply system

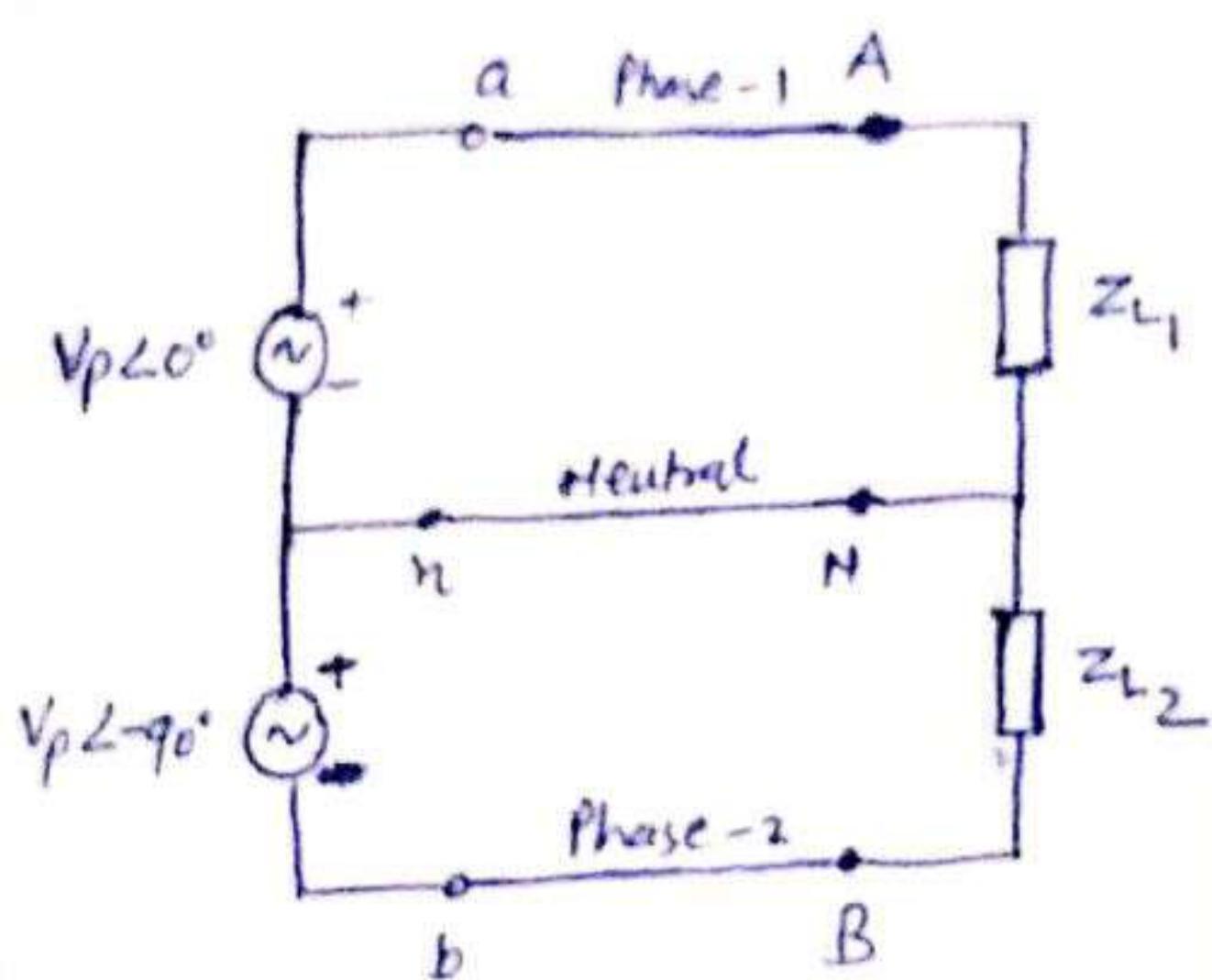
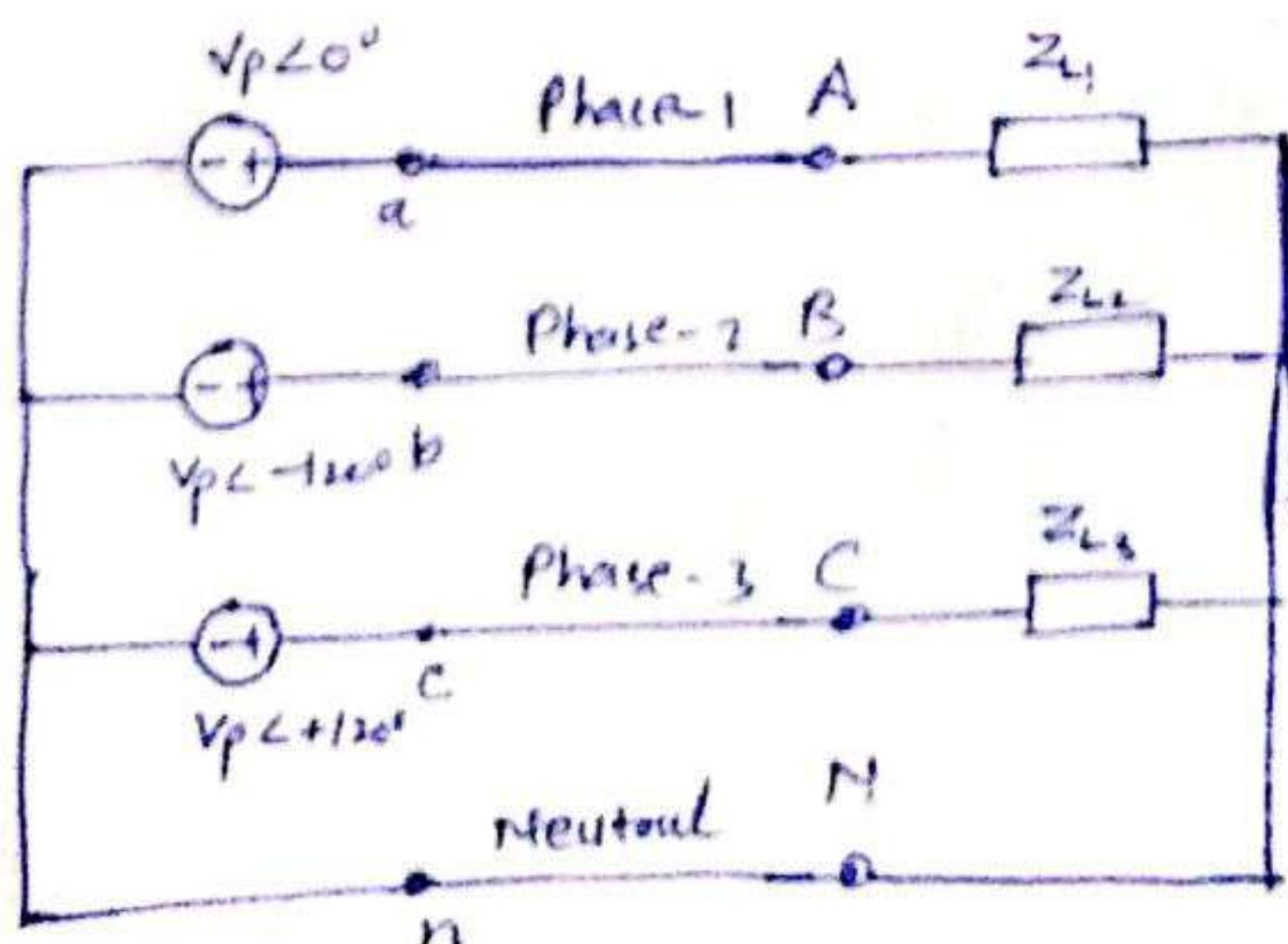


Fig. 2 Two phase Three wire system

(3) Three Phase Supply System:-



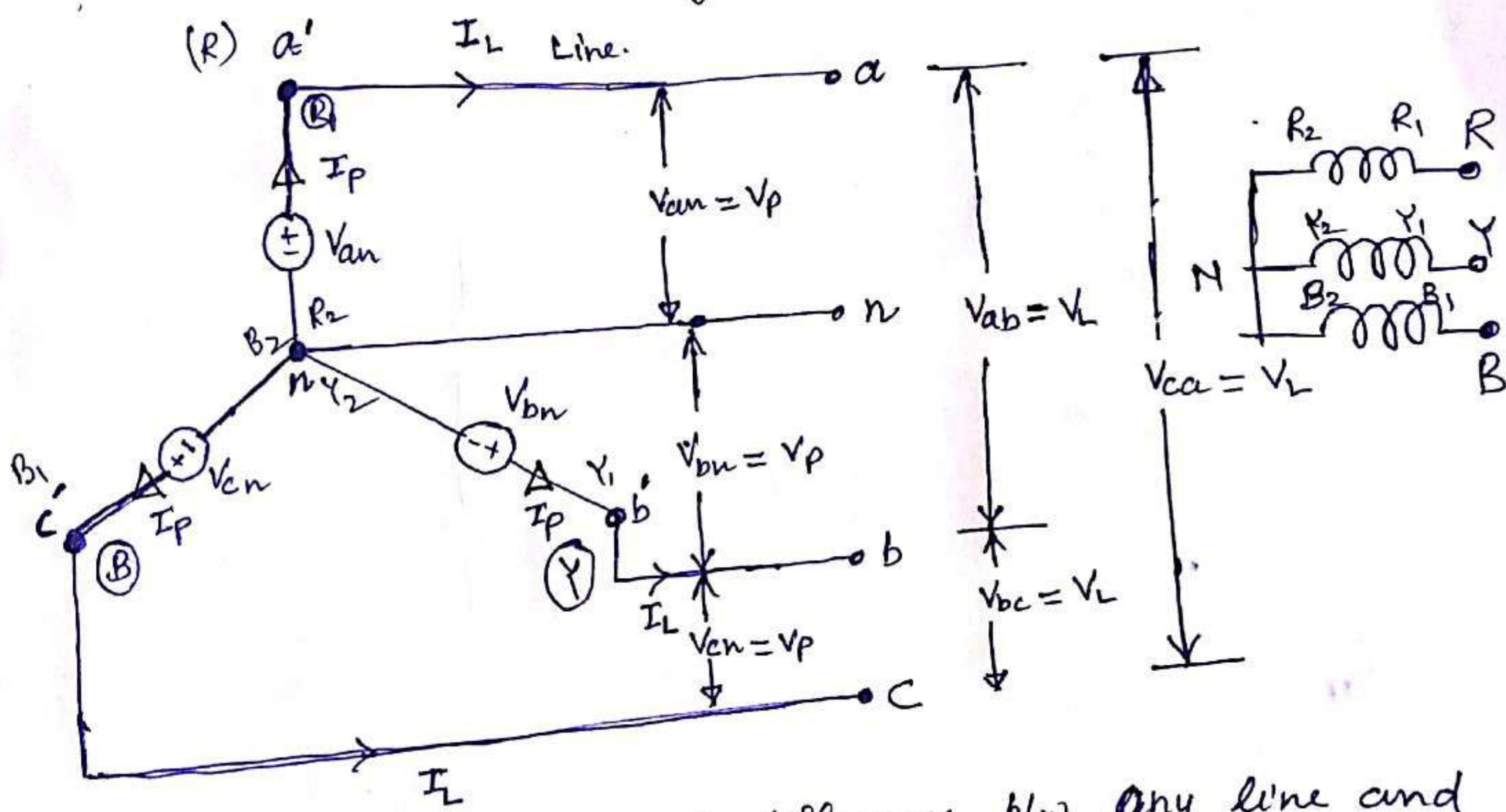
Phase difference b/w phase-1, Phase-2 and Phase-3 is 120° .

Fig. 3 3 phase 4 wire system

(b) Three phase supply connections :-

- (i) A typical three phase system consists of Three voltage sources connected to loads by three or four wires.
- (ii) A Three phase system is equivalent to three single Phase Circuits.
- (iii) The voltage sources can be either Star connected or Delta connected.

→ ① Star connected voltage sources :- [Star Connection]



Phase voltage: The potential difference b/w any line and Neutral wire is called phase voltage. [V_{an} , V_{bn} & V_{cn} → Phase voltages]

Phase current:- The current flowing in each phase winding is

Voltage: called phase current.

Line ~~voltage~~:- The potential difference between any two lines of supply is called line voltage.

V_{ab} , V_{bc} , V_{ca} → line voltages

Line Current:- The current passing through any line is called line current.

If the phase voltages V_{an} , V_{bn} and V_{cn} have the same magnitude and frequency, and these voltages are 120° out of phase with each other, then the voltages are said to be balanced. This implies that

$$V_{an} + V_{bn} + V_{cn} = 0$$

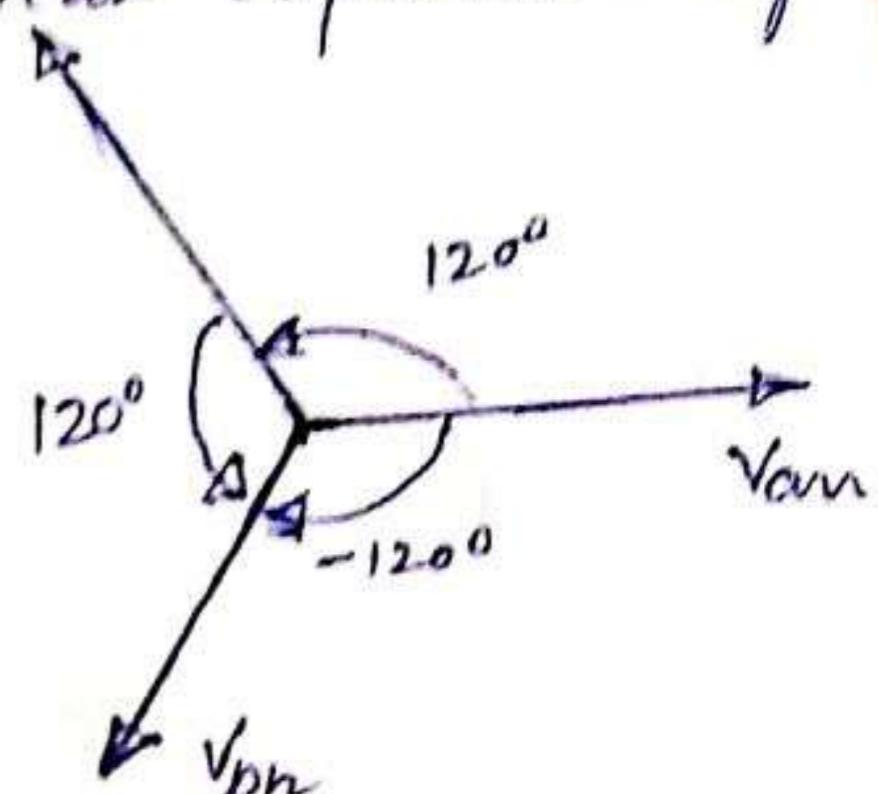
$$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$$

Balanced Supply - If the magnitude of all the three phase voltages are equal and these voltages are 120° out of phase with each other, then the supply voltages are called balanced. The mathematical expression is given below

$$V_{an} = V_p \angle 0^\circ$$

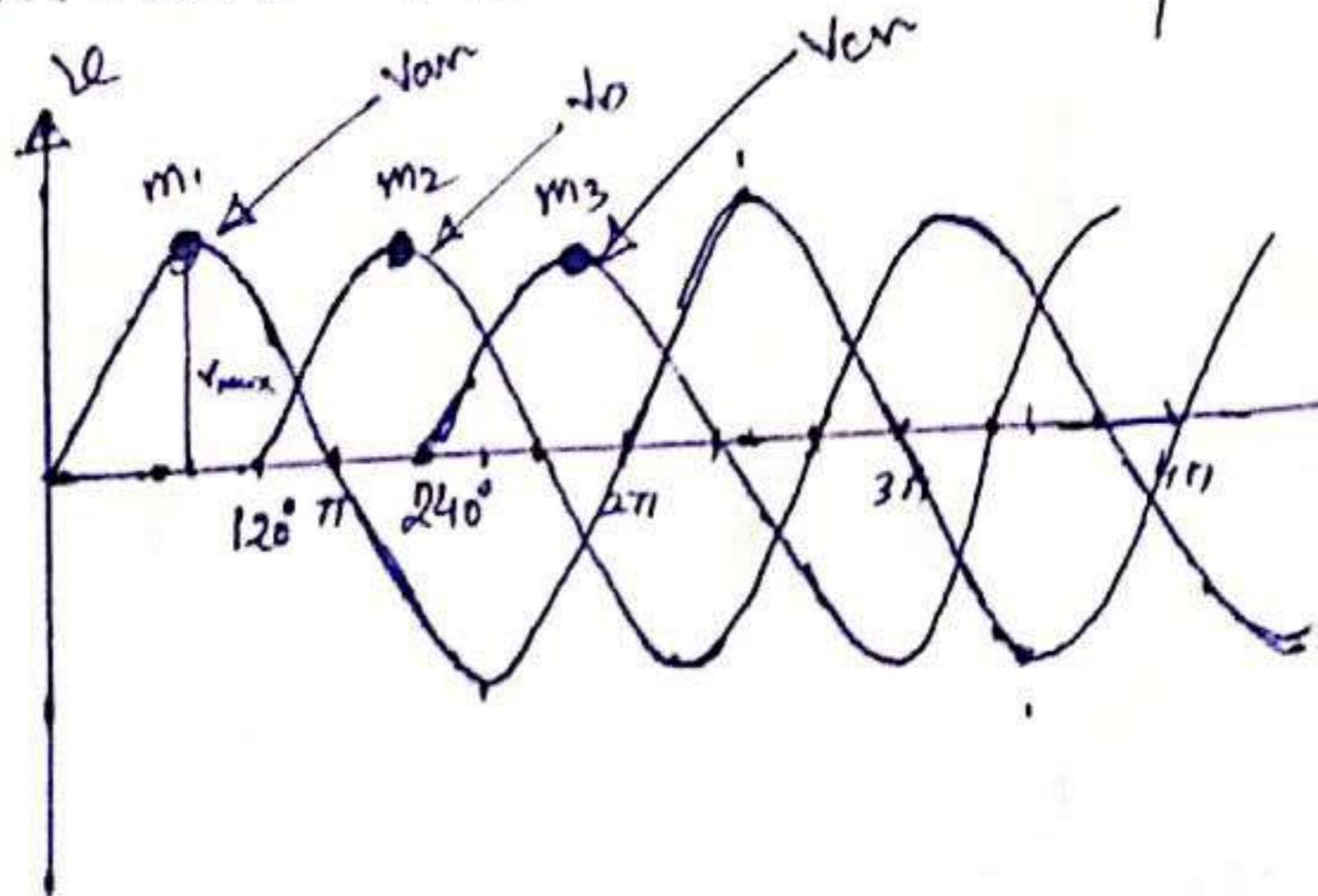
$$V_{bn} = V_p \angle -120^\circ$$

$$V_{cn} = V_p \angle -240^\circ = V_p \angle +120^\circ$$



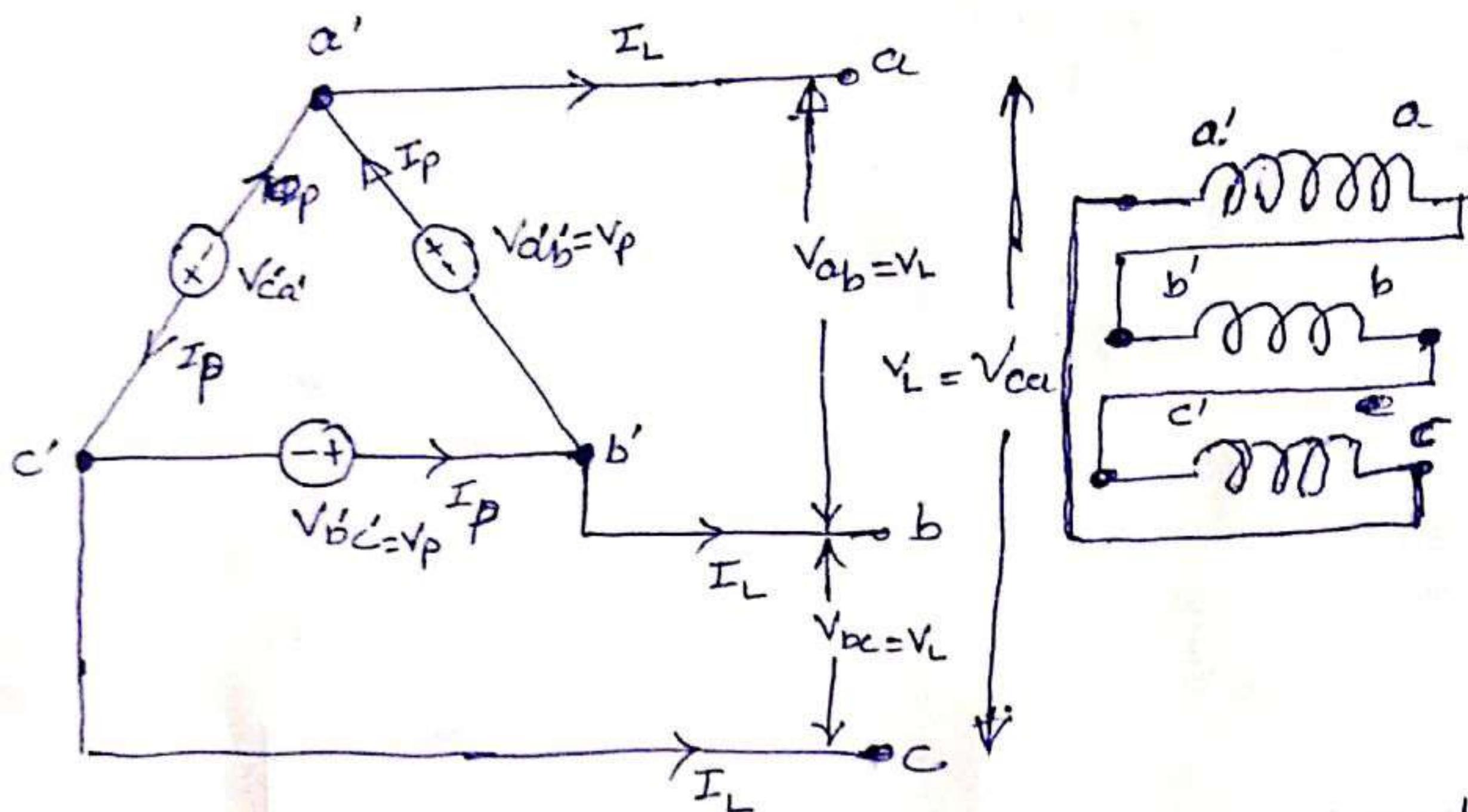
Phase Sequence:-

The ~~three~~ phase sequence may be regarded as the order in which the phase voltages reach their maximum values with respect to time.



m_1 & m_2 & m_3
are maximum
values of V_{an}
 V_{bn} & V_{cn} .

b-2 Delta Connected Voltage Sources :-



$$|V_{ab}| = |V_{bc}| = |V_{ca}| = V_p$$

$I_p \rightarrow$ Phase current
 $I_L \rightarrow$ Line current

$$\text{Ans} \quad |V_{a'b'}| = |V_{b'c'}| = |V_{c'a'}| = V_p$$

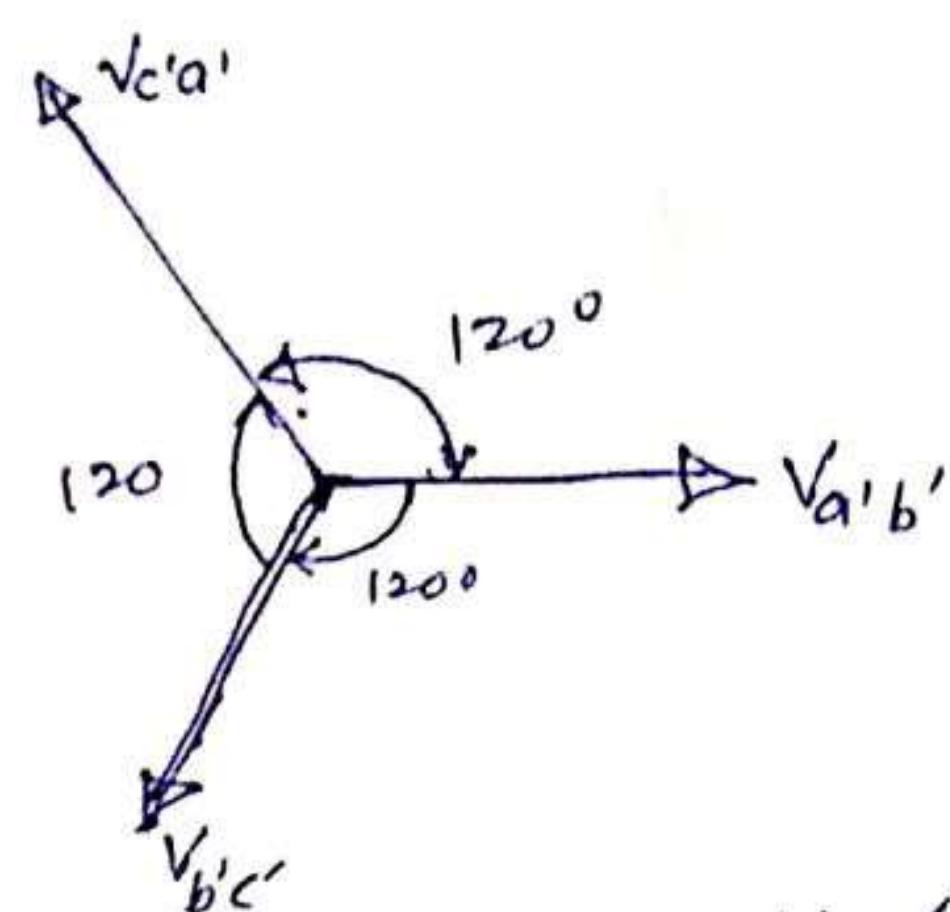
Here V_{ab} , V_{bc} & $V_{ca} \rightarrow$ are line voltages

$V_{a'b'} \& V_{b'c'}, V_{c'a'} \rightarrow$ phase voltages.

$$V_{a'b'} = V_p \angle 0^\circ$$

$$V_{b'c'} = V_p \angle -120^\circ$$

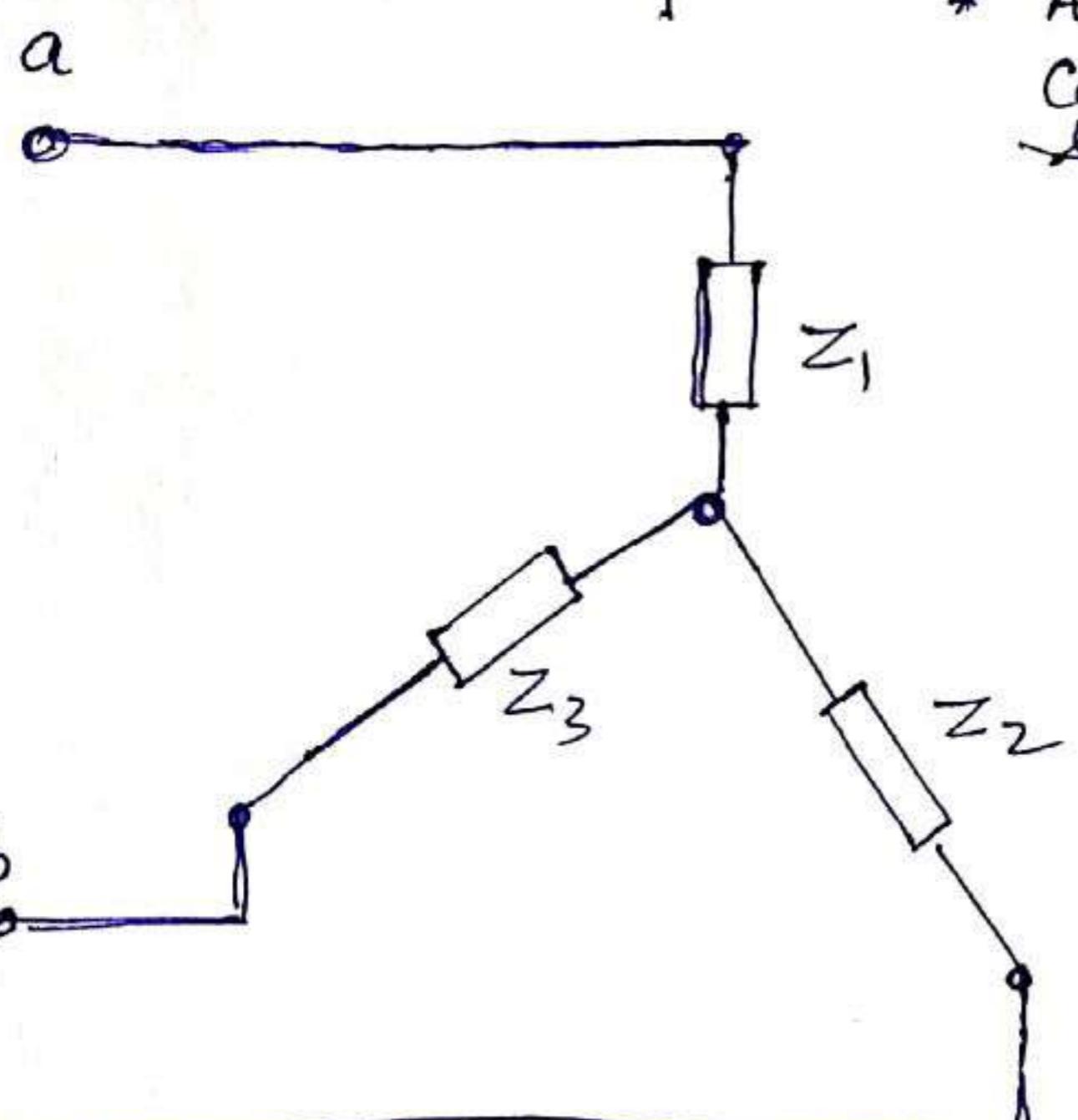
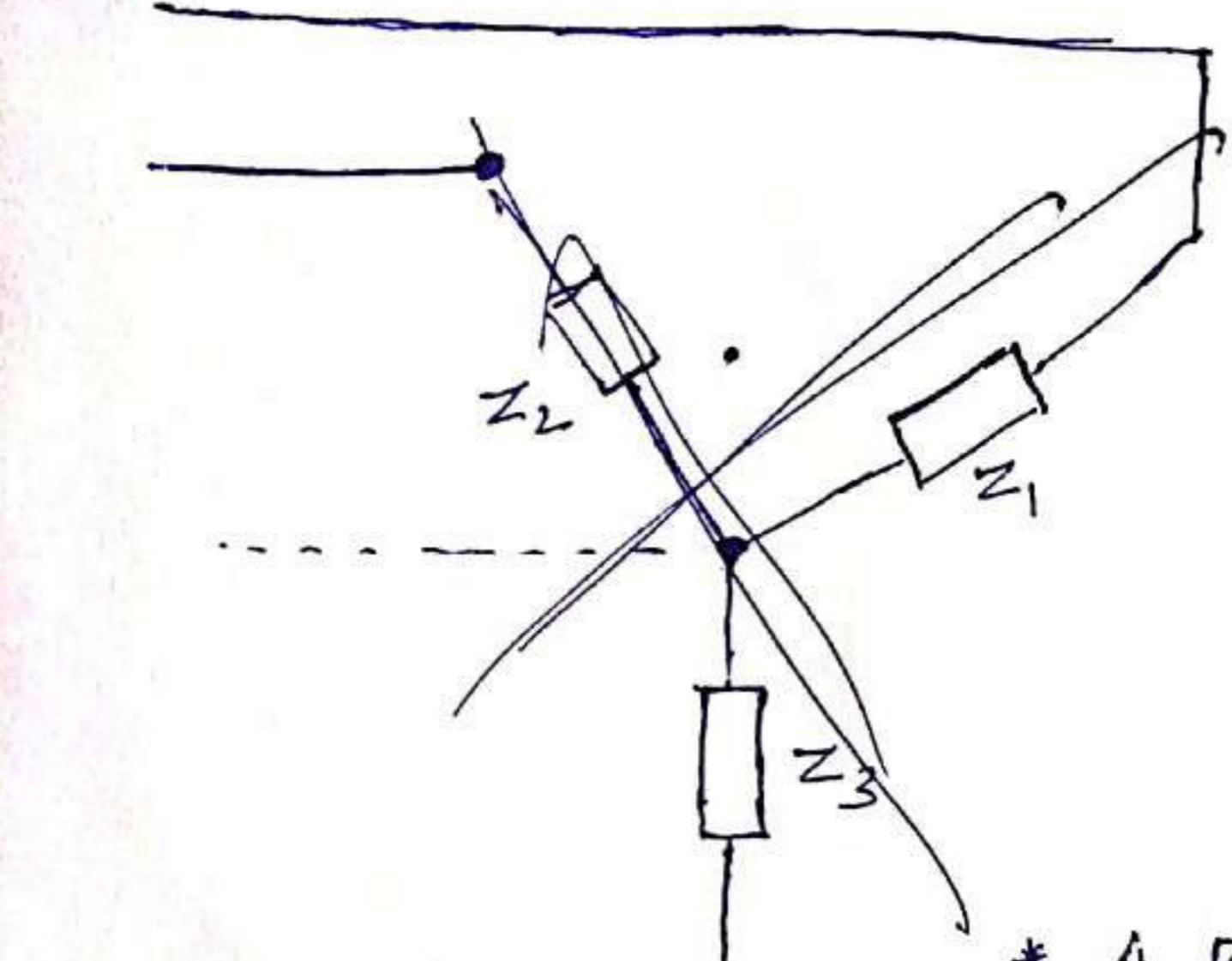
$$\begin{aligned} V_{c'a'} &= V_p \angle -240^\circ \\ &= V_p \angle +120^\circ \end{aligned}$$



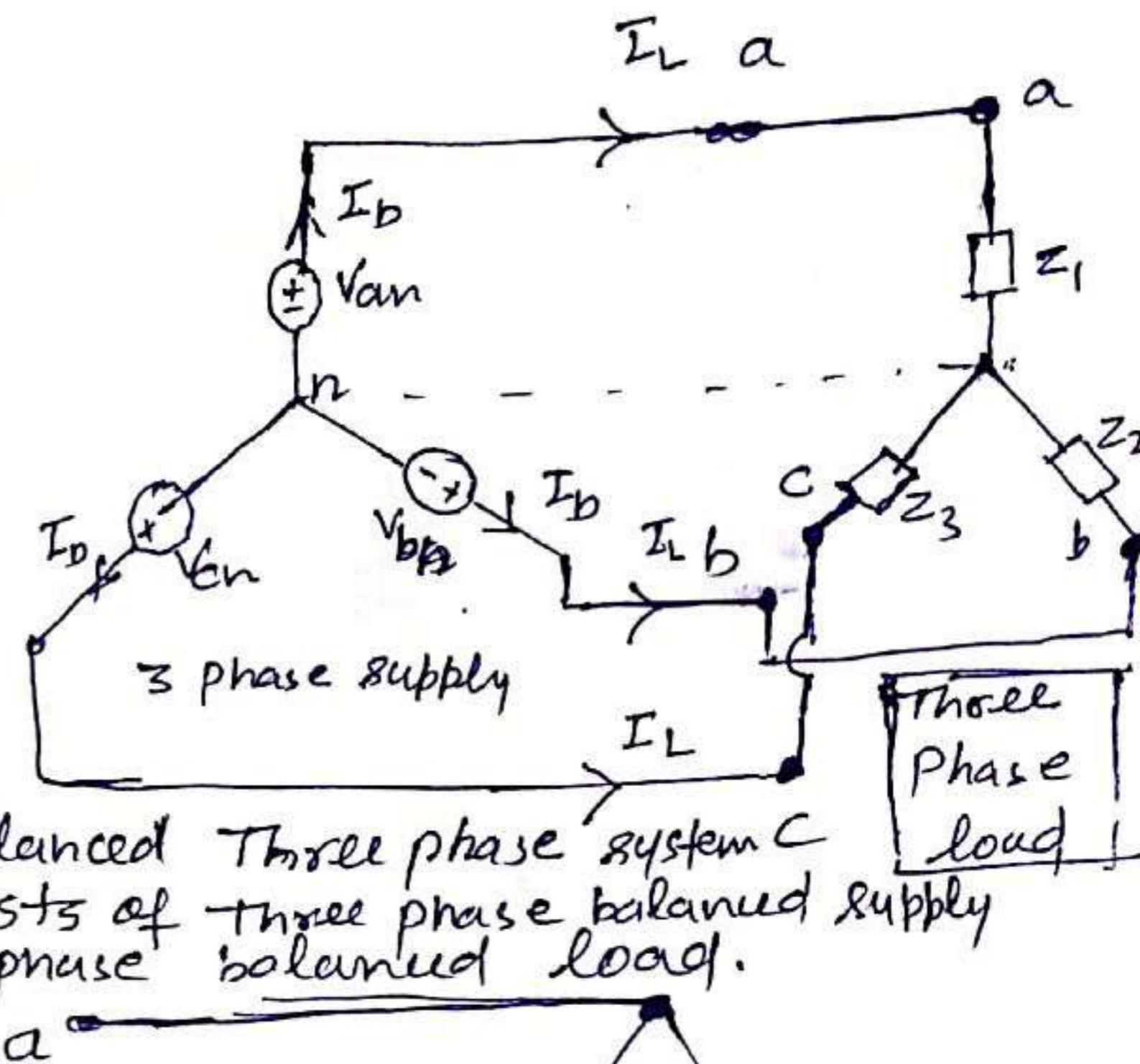
Balanced phase voltages are equal in magnitude and are out of phase with each other by 120° .

C) Three Phase load Connection!:-

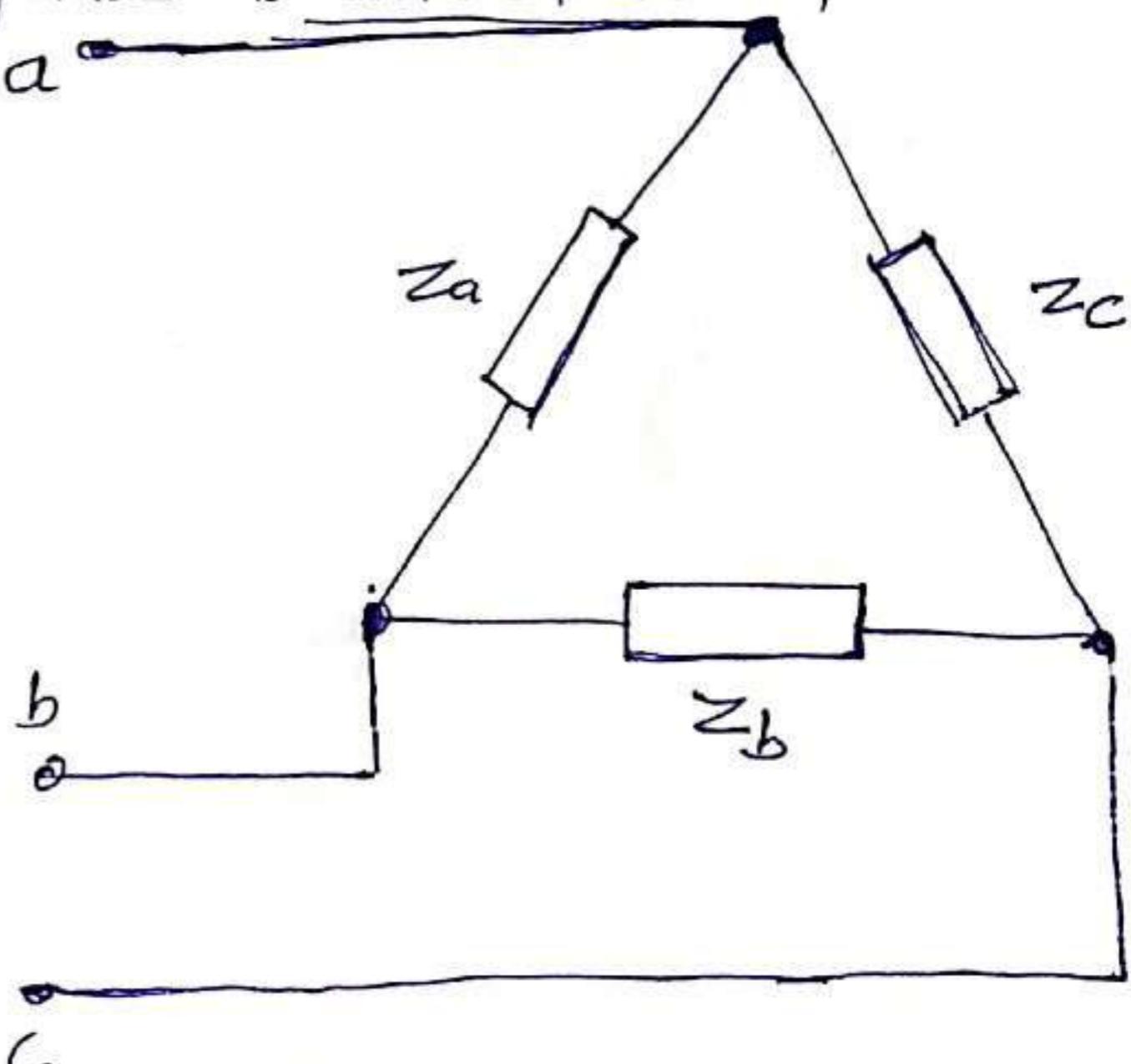
- A three phase load can be either star connected or delta connected.
- A three phase load is equivalent to three single phase load.



'Star Connected Load'



* A Balanced Three phase system consists of three phase balanced supply & 3 phase balanced load.



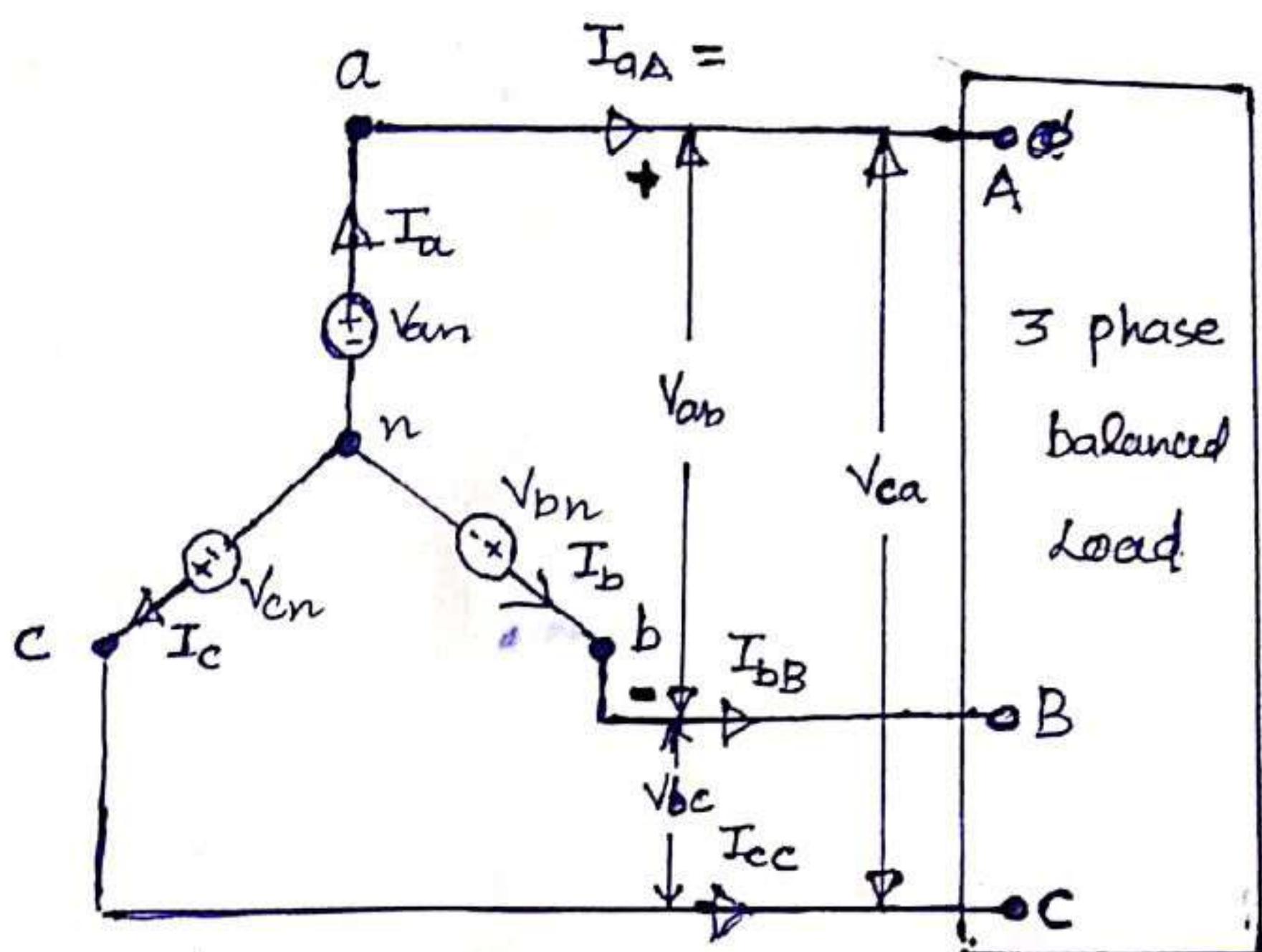
'Delta Connected load'

Balanced Load :- A balanced load is one in which the phase impedances are equal in magnitude and in phase.

$$z_1 = z_2 = z_3 = Z \angle \phi$$

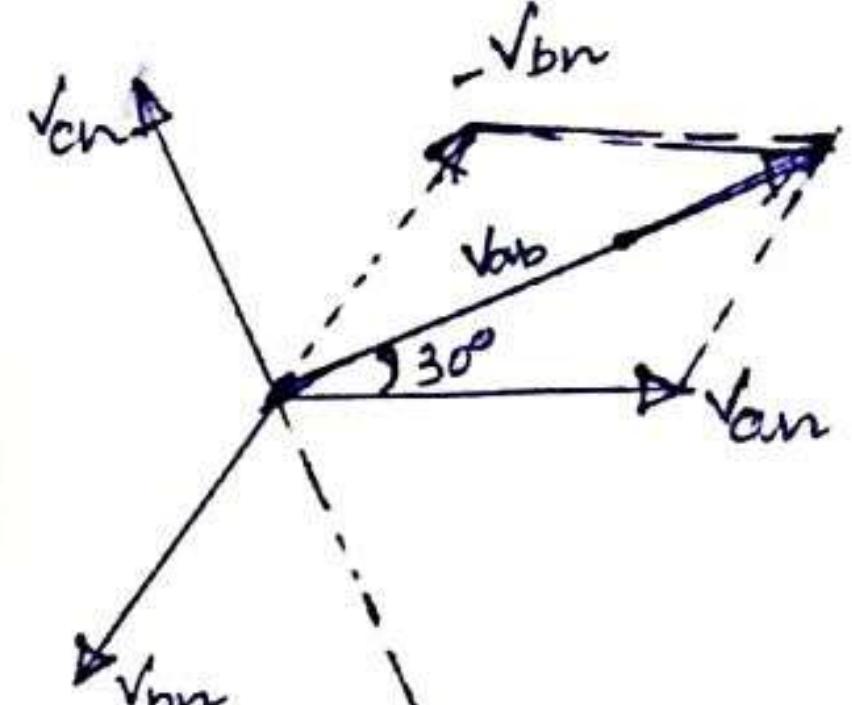
$$z_a = z_b = z_c = Z \angle \phi$$

(d) Line and phase voltages / current relations in Star connection :- [Derivation of V_L & I_L]



a, b & c supply
voltage terminal

A, B, C load
terminals



V_{an} ,
 V_{bn} ,
 V_{cn} → Phase voltage,
(V_p)

Let $|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$,

I_a ,
 I_b ,
 I_c → Phase currents.
(I_p)

V_{ab} ,
 V_{bc} ,
 V_{ca} → Line voltages (V_L)

Let $|V_{ab}| = |V_{bc}| = |V_{ca}| = V_L$

I_{aa} ,
 I_{bb} ,
 I_{cc} → Line currents (I_L)

$V_{an} = V_p \angle 0^\circ$

$V_{bn} = V_p \angle -120^\circ$

$V_{cn} = V_p \angle +120^\circ$

$V_{ab} = \sqrt{3} V_p \angle 30^\circ$

$|V_{ab}| = \sqrt{3} V_p$

$V_L = \sqrt{3} V_p$

Applying KVL -

- $V_{an} + V_{ab} + V_{bn} = 0$

$V_{ab} = V_{an} - V_{bn}$

$V_{ab} = V_p \angle 0^\circ - V_p \angle -120^\circ$

$= V_p + j0 - (-\frac{1}{2} - j\frac{\sqrt{3}}{2})V_p$

$= V_p \left[1 + \frac{1}{2} + j\frac{\sqrt{3}}{2} \right]$

$= V_p \left[\frac{3}{2} + j\frac{\sqrt{3}}{2} \right]$

$= \sqrt{3} V_p \angle 30^\circ$

Gr. Ward AU

Similarly we can obtain

$V_{bc} = V_{bn} - V_{cn} = \sqrt{3} V_p \angle -90^\circ$

$V_{ca} = V_{cn} - V_{an} = \sqrt{3} V_p \angle -210^\circ$

Thus the magnitude of the line voltages V_L is $\sqrt{3}$ times the magnitude of phase voltages V_p .

Thus

$V_L = \sqrt{3} V_p$

Derivation of V_L & I_L continued ---

Applying KCL at Node - a -

$$I_a = I_{aA}$$

Similarly -

$$I_b = I_{bB}$$

$$I_c = I_{cC}$$

Let - Now $|I_a| = |I_b| = |I_c| = I_p$

$$\Rightarrow |I_{aA}| = |I_{bB}| = |I_{cC}| = I_L$$

'phasor-diagram'

Then -

$$I_p = I_L$$

Hence in Star-connection phase \leftrightarrow current is equal to line current.

→ Calculation of load impedance :-

In 3 phase system load is calculated per phase.

$$Z_p = \frac{V_p \angle \phi_1}{I_p \angle \phi_2} = \frac{|V_p|}{|I_p|} \angle (\phi_1 - \phi_2)$$

→ 3 phase power:-

$$\text{Active Power } P = 3 V_p I_p \cos \phi$$

$$P = 3 \frac{V_L}{\sqrt{3}} I_L \cos \phi$$

$$P = \sqrt{3} V_L I_L \cos \phi$$

Watt

$$\text{Reactive Power } Q = \sqrt{3} V_L I_L \sin \phi$$

VA

$$\text{Apparent Power } S = \sqrt{3} V_L I_L$$

VA

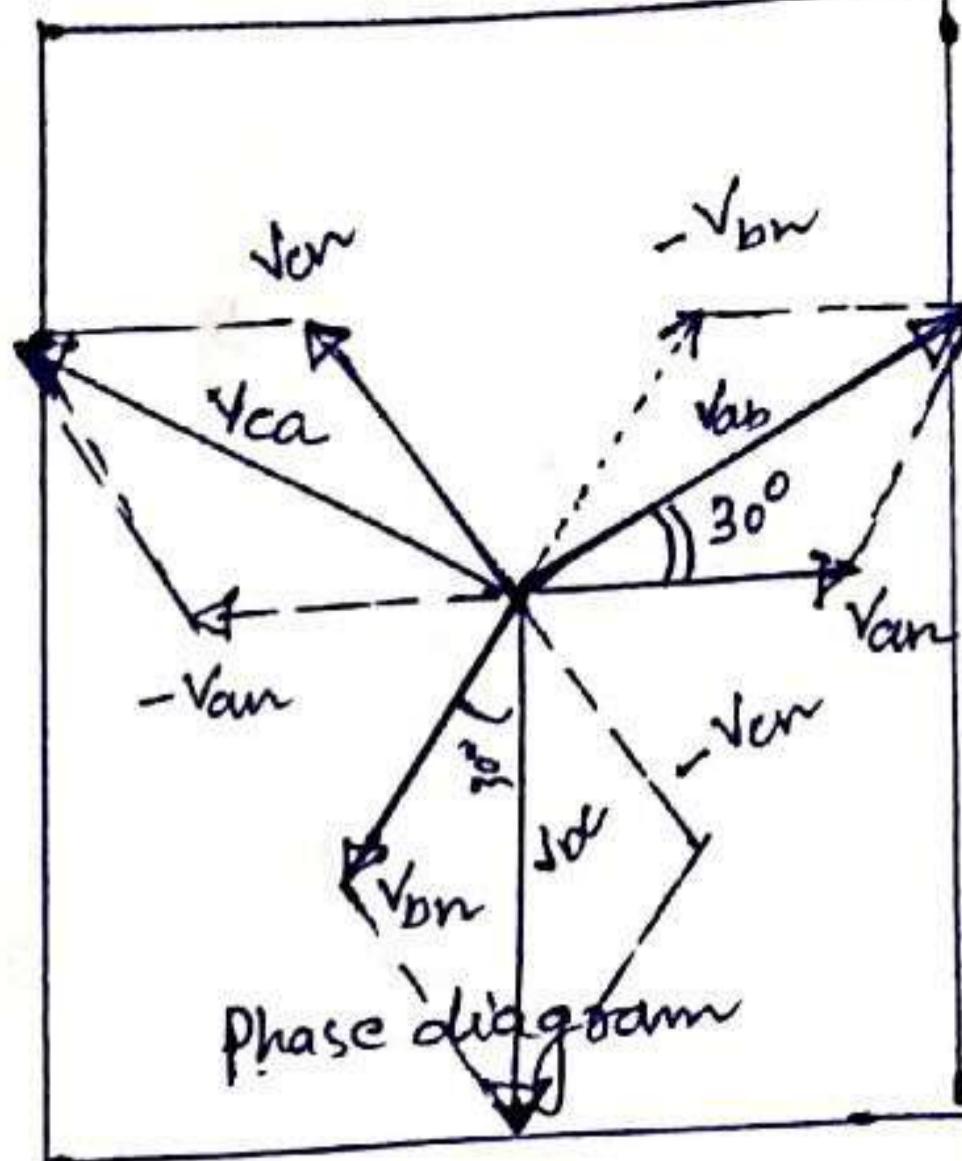
Note:- ① Line voltage leads their respective phase voltage by 30°

Ex. consider after $\boxed{V_L = \sqrt{3} V_p}$

③

$$I_L = I_p$$

④



Q.1

The three inductive coil having resistance of $16\ \Omega$ and reactance $12\ \Omega$ are connected in star across a $400V$, 3 phase $50Hz$ supply. Calculate -

- (a) Power factor
- (b) Phase current
- (c) Line current
- (d) Power absorbed.

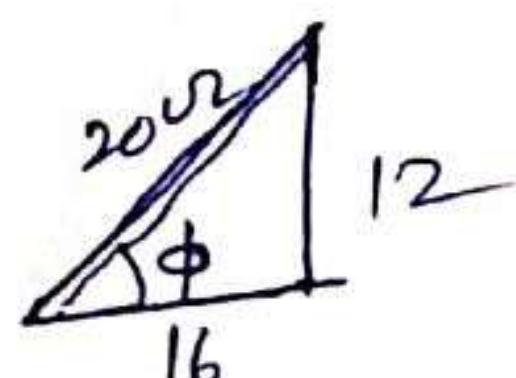
Sol.

$$\text{Here } Z_1 = (16 + j12)\ \Omega$$

$$Z_1 = Z_2 = Z_3 = (16 + j12)\ \Omega = Z_p$$

$$|Z_p| = \sqrt{16^2 + 12^2} = 20\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{12}{16}\right) = 36.86^\circ$$



(a) Power factor $\cos\phi = \frac{16}{20} = 0.8$ lagging

(b) $V_L = 400\ V$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.94\ V$$

$$I_p = \frac{V_p}{Z_p} = \frac{230.94 \angle 0^\circ}{20 \angle 36.86^\circ} = \frac{230.94 \angle 0^\circ}{20 \angle 36.86^\circ}$$

$$I_p = 11.547 A \angle -36.86^\circ$$

(c) In star connection $I_L = I_p =$

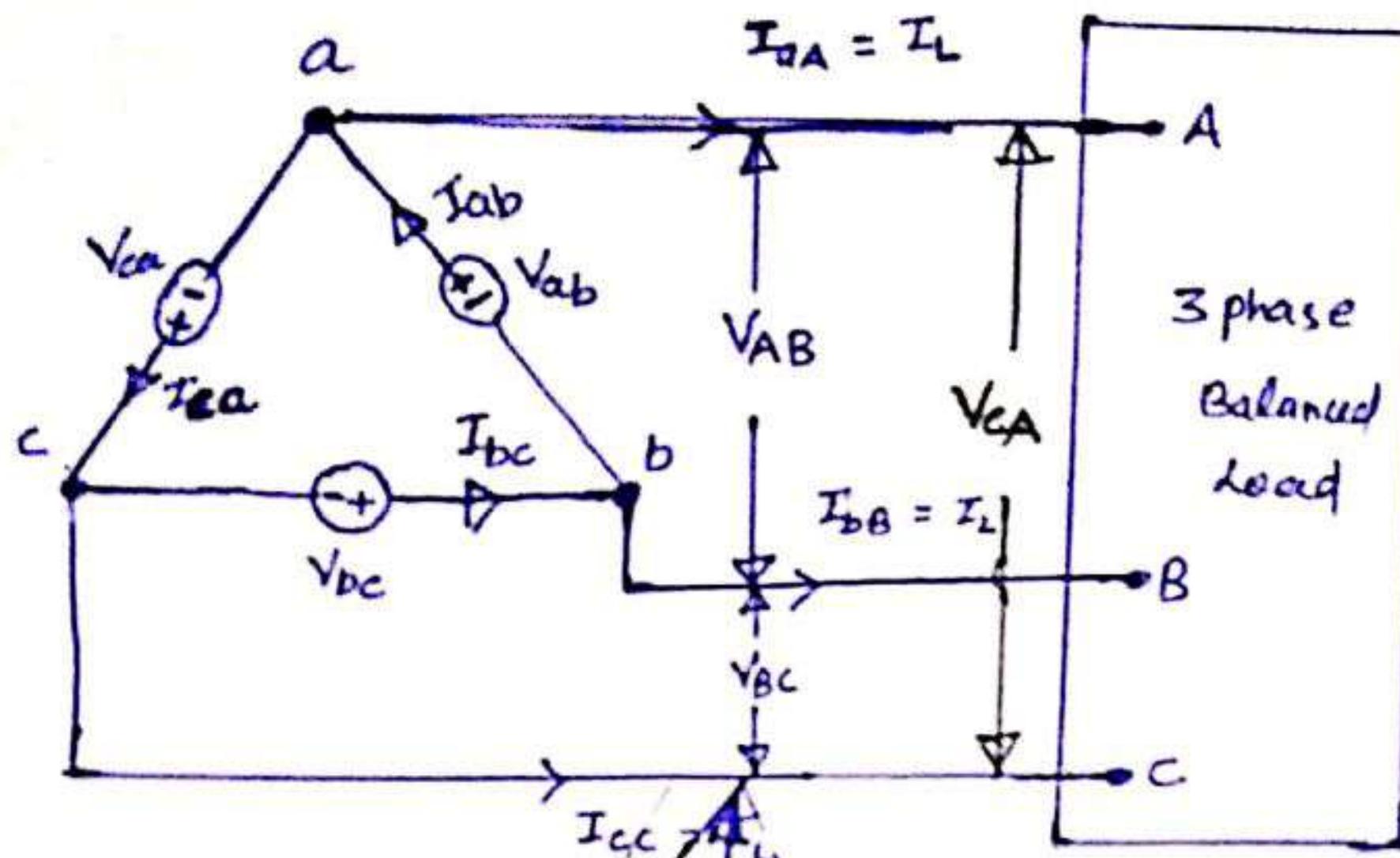
$$\text{line current } I_L = 11.547 \angle -36.86^\circ A$$

(d) Power consumed $P = \sqrt{3} V_L I_L \cos\phi$

$$P = \sqrt{3} \times 400 \times 11.547 \times 0.8$$

$$P = 6400 \text{ Watt}$$

Derivation of $V_L \times I_L$ in Delta Connection :-



Alternate method from phasor diagram

$$I_{OA} = \sqrt{(I_{ab})^2 + |I_{ca}|^2}$$

$$I_L = \sqrt{I_p^2 + I_p^2 + 2I_p^2 \times \frac{1}{2}}$$

$$I_L = \sqrt{3} I_p$$

$I_{ab}, I_{bc}, I_{ca} \rightarrow$ Phase current

$I_{OA}, I_{OB}, I_{OC} \rightarrow$ Line current

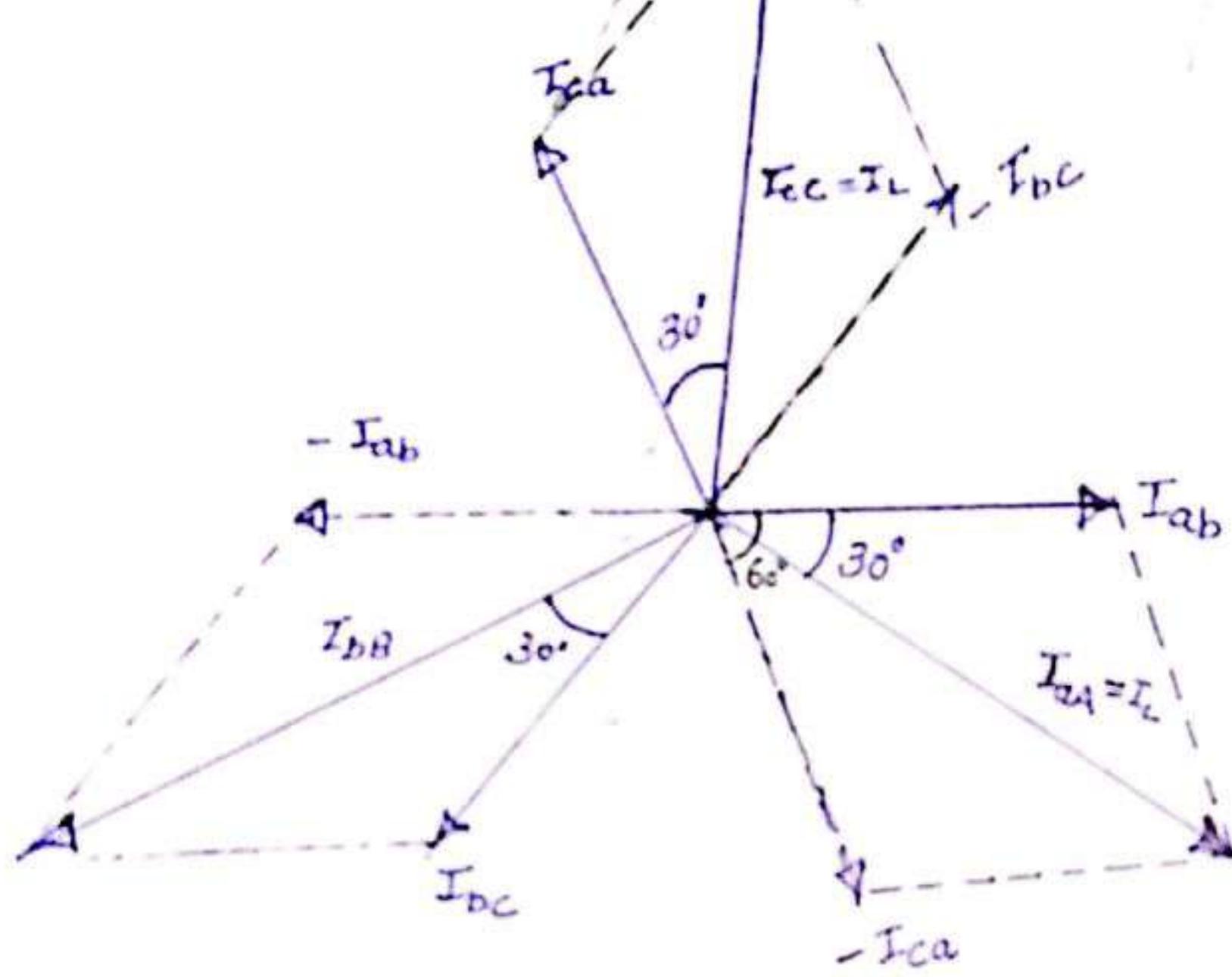
$V_{ab}, V_{bc}, V_{ca} \rightarrow$ Phase voltage

$V_{AB}, V_{BC}, V_{CA} \rightarrow$ Line voltage

$$I_{ab} = I_p \angle 0^\circ$$

$$I_{bc} = I_p \angle -120^\circ$$

$$I_{ca} = I_p \angle -240^\circ = I_p \angle +120^\circ$$



'Phasor diagram'

Applying KCL at Node - a

$$I_{OA} + I_{ca} = I_{ab}$$

$$I_{OA} = I_{ab} - I_{ca} \quad \text{--- ①}$$

$$I_{OA} = I_p \angle 0^\circ - I_p \angle +120^\circ$$

$$= I_p + j0 - (-\frac{1}{2} + j\frac{\sqrt{3}}{2}) I_p$$

$$= I_p + (\frac{1}{2} - j\frac{\sqrt{3}}{2}) I_p$$

$$= I_p [1 + \frac{1}{2} - j\frac{\sqrt{3}}{2}]$$

$$= I_p [\frac{3}{2} - j\frac{\sqrt{3}}{2}]$$

$$= \sqrt{3} I_p \angle -30^\circ$$

Applying KCL at Node - b

$$I_{OB} = I_{bc} - I_{ab}$$

$$I_{OB} = I_p \angle -120^\circ - I_p \angle 0^\circ$$

$$I_{OB} = I_p \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right) - I_p$$

$$I_{OB} = I_p \left[-\frac{1}{2} - 1 - j\frac{\sqrt{3}}{2} \right]$$

$$I_{OB} = I_p \left[-\frac{3}{2} - j\frac{\sqrt{3}}{2} \right]$$

$$= \sqrt{3} I_p \angle 30^\circ$$

Similarly -

$$I_{OC} = I_{ca} - I_{bc} = \sqrt{3} I_p$$

V_L & I_L Derivation continued -

If $|I_{aA}| = |I_{bB}| = |I_{cC}| = I_L$

Then
$$I_L = \sqrt{3} I_p$$

By Applying KVL -

~~V_{ab}~~ $V_{AB} = V_{ab}$

$V_{BC} = V_{bc}$

$V_{CA} = V_{ca}$

if $|V_{ab}| = |V_{bc}| = |V_{ca}| = V_p$

$\therefore |V_{AB}| = |V_{BC}| = |V_{CA}| = V_L$

Then
$$V_L = V_p$$

→ Power:-

Active power $P = \sqrt{3} V_L I_L \cos \phi$

reactive power $Q = \sqrt{3} V_L I_L \sin \phi$

Apparent Power $S = \sqrt{3} V_L I_L$

→ Impedance :-

$$Z_p = \frac{V_p}{I_p} = \frac{|V_p| \angle \phi_1}{|I_p| \phi_2}$$

→ Summary for Delta connection! -

(I) Line voltage (V_L) = phase voltage (V_p)

(II) Line current (I_L) = $\sqrt{3} \times$ phase current
 $= \sqrt{3} I_p$