Numericals

Q-1 A vibrating system consists of a mass of 50 kg, a spring of stiffness 30 KN/m and a damper. The damping provided is only 20% of the critical value. Determine:

(i) Damping factor (ξ) (ii) The critical damping coefficient (iii) ω_d (iv) Logarithmic Decrement (v) The Ratio of two consecutive amplitudes

Given: $m = 50\,kg$ $\quad s = 30 \times 10^3\, N/m$ $\quad c = 0.2\, c_c$

(i) $$\xi = \frac{c}{c_c} = \frac{0.2\,c_c}{c_c} = 0.2 \quad \text{Answer}$$

(ii) $$\xi = \frac{c}{c_c} = \frac{c}{2\sqrt{sm}} \quad \therefore c_c = 2\sqrt{sm}$$

$$\therefore c_c = 2\sqrt{30 \times 10^3 \times 50} = 2.45\ N/mm/sec \quad \text{Answer}$$

(iii) $$\omega_d = \left(\sqrt{1-\xi^2}\right)\omega_n = \sqrt{1-(0.2)^2} \times \sqrt{\frac{30\times 10^3}{50}}$$

$$= 24\ rad/sec. \quad \text{Answer}$$

(iv) $$\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}} = \frac{2 \times 3.14 \times 0.2}{\sqrt{1-(0.2)^2}} = 1.28 \quad \text{Ans}$$

(v) $$\frac{x_n}{x_{n+1}} = ? \qquad \delta = \ln\left(\frac{x_n}{x_{n+1}}\right)$$

$$\therefore \frac{x_n}{x_{n+1}} = e^{\delta} = e^{1.28} = 3.6 \quad \text{Answer}$$

Q-2 In a single degree damped vibrating system, a suspended mass of 8 kg makes 30 oscillations in 18 seconds. The amplitude decreases to 0.25 of the initial value after 5 oscillations. Determine (i) The stiffness of spring (ii) δ (iii) ξ (iv) c

Soln: Given: $m = 8\,kg$ $N = 30$ $t = 180\,sec$

$$\text{frequency} = \frac{\text{No. of oscillation}}{\text{per second}} = \frac{30}{18} = 1.67\ Hz$$

$$\omega_n = 2\pi f_n = 2\pi \times 1.67$$
$$= 10.47\ rad/sec$$

(i) Stiffness

$$\omega_n = \sqrt{\frac{S}{m}} \Rightarrow 10.47 = \sqrt{\frac{S}{8}} \Rightarrow S = 877\ N/m$$

$$\boxed{S = 0.877\ N/mm}$$

(ii) Given amplitude decreases to 0.25 of initial after 5 oscillation

$$\therefore x_5 = 0.25\,x_0$$

$$\frac{x_0}{x_5} = \frac{x_0}{x_1} \times \frac{x_1}{x_2} \times \frac{x_2}{x_3} \times \frac{x_3}{x_4} \times \frac{x_4}{x_5}$$

$$\boxed{\frac{x_0}{x_5} = \frac{1}{0.25}}$$

we know that

$$\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \frac{x_4}{x_5}$$

$$\frac{x_0}{x_1} = \left(\frac{x_0}{x_5}\right)^{1/5}$$

$$\therefore \frac{x_0}{x_5} = \left(\frac{x_0}{x_1}\right)^5$$

$$\frac{x_0}{x_1} = \left(\frac{1}{0.25}\right)^{1/5} = 1.32$$

$$\delta = \ln\left(\frac{x_0}{x_1}\right) = \ln 1.32 = 0.278 \quad \text{Answer}$$

iii) $\xi =$

$$\frac{2\pi \xi}{\sqrt{1-\xi^2}} = 0.278$$

$$\frac{2\pi}{0.278}\xi = \sqrt{1-\xi^2}$$

$$22.6\,\xi = \sqrt{1-\xi^2} \Rightarrow 1-\xi^2 = 510.82\,\xi^2$$

$$\therefore \xi^2 = 0.00195$$

$$\boxed{\xi = 0.0442}$$ Answer

(iv)

$$\xi = \frac{c}{2\sqrt{sm}}$$

$$c = 0.0442 \times 2 \times \sqrt{877 \times 8}$$

$$\boxed{c = 7.4 \text{ N/m/sec}}$$ Answer

Q-3 Determine the time in which the mass in a damped vibrating system would settle down to 1/50th of it's initial deflection for the following data

$m = 200\,kg$, $\xi = 0.22$, $S = 40\,N/m$

Also find no. of oscillation to reach this value.

Solⁿ:- $\frac{x_0}{x_n} = e^{\xi \omega_n N T_d}$

$$\omega_n = \sqrt{\frac{s}{m}}$$

$$= \sqrt{\frac{40 \times 10^3}{200}} = 14.14 \text{ rad/sec}$$

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$$50 = e^{0.22 \times 14.14\, N T_d}$$

Total time $N T_d = 1.26$ sec Answer

$$T_d = \frac{2\pi}{\left(\sqrt{1-\xi^2}\right)\omega_n} = \frac{2\pi}{\sqrt{(1-0.22^2)} \times 14.14} = 0.455 \text{ sec}$$

$\therefore$ No. of oscillations $= \frac{1.26}{0.455} = 2.26$ Ans

18.4 A vibrating system consists of a mass of 20 kg, a spring of stiffness 20 kN/m and a damper. The damping provided is only 30% of the critical value. Determine the natural frequency of the damped vibration and the ratio of two consecutive amplitudes.

(30.2 rad/s ; 7.21)

18.5 The following data relate to a damped vibrating system:

m = 140 kg s = 50 N/mm   and $\zeta = 0.25$

Determine the time in which the mass would settle down to 1/80th of its initial deflection. Also, what will be the number of oscillations complete to reach this value?

(0.927 s ; 2.7)

18.6 In a single-degree damped vibrating system, the suspended mass of 4 kg makes 24 oscillations in 20 seconds. The amplitude decreases to 0.3 of the initial value after 4 oscillations. Find the stiffness of the spring, the logarithmic decrement, the damping factor and damping coefficient.

(227 N/m; 0.3; 0.0478; 2.88 N/m/s)