Gear Trains :-

Sometimes, two or more than two gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear trains or train of the toothed wheels. It becomes necessary when it is required to obtained speed reduction within a small space.

Types of Gear trains :-

1) Simple gear train.
2) Compound gear train.
3) Reverted gear train.
4) Planetary or epicyclic gear train.

1) **Simple gear train** :- When there is only one gear on each shaft, then it is known as simple gear train.

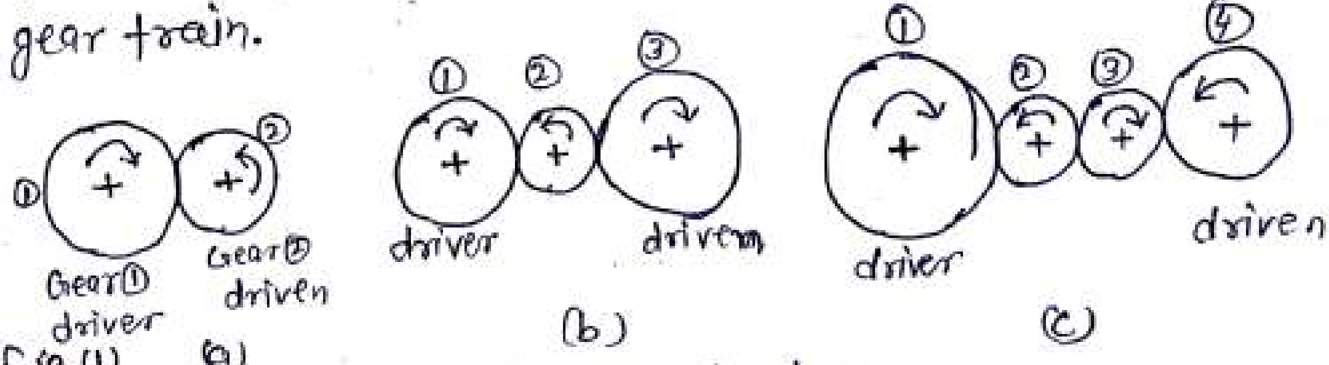


Fig.(1)

In case of simple gear train:

1. Two external gears of a pair always move in opposite direction.

(vi) All odd numbered gear move one direction and all even numbered gears move in opposite direction. For example, as in fig(c) gears 1,3 moves in clock wise direction but gear 2,4 moves in counterclockwise direction.

(iii) <u>Speed ratio</u> :- The ratio of speed of the driven shaft to that of driving shaft is speed ratio or velocity ratio.

It is −ve when input and output gears rotates in the opposite directions and it is +ve when the two rotates in the same direction.

<u>Train value</u> :- Reciprocal of speed ratio is known as train value.

For fig(c)

$$\frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\frac{N_3}{N_2} = \frac{T_2}{T_3}, \quad \frac{N_4}{N_3} = \frac{T_3}{T_4}$$

where
T − No. of teeth on a gear
N − gear speed in rpm

Now by multiplying

$$\frac{N_2}{N_1} \times \frac{N_3}{N_2} \times \frac{N_4}{N_3} = \frac{T_1}{T_2} \times \frac{T_2}{T_3} \times \frac{T_3}{T_4}$$

speed ratio $\boxed{\frac{N_4}{N_1} = \frac{T_1}{T_4}}$

From the above equation it is clear that the intermediate gears have no effect on the speed ratio.

~~2) Compound gear train~~ :-

* When centre distance between the gears is large and we need constant velocity ratio, the motion from one gear to another, in such cases may be transmitted by the following two methods

(i) By providing the large size gear

(ii) By providing one or more intermediate gears.

The first method is very inconvenient and uneconomical and hence generally second method is adopted which is convenient and economical.

* Intermediate gears are called idle gears as they donot effect the speed ratio or train value of the system. It is also used to obtain desired direction of motion of the driven gear.

* When the no. of intermediate gears are odd, the motion of driver and driven is like.

* If the no. of intermediate gears are even, the motion of the driven ~~one~~ will be in the opposite direction of the driver.

$$\text{Speed ratio} = \frac{\text{speed of driven}}{\text{speed of driver}} = \frac{\text{No. of teeth on driver}}{\text{No. of teeth on driven}}$$

(2) <u>Compound Gear train</u> :- When a series of gears are connected in such a way that two or more gears rotates about an axis with the same angular velocity (i.e. more than one gear on a shaft), it is known as compound gear train.

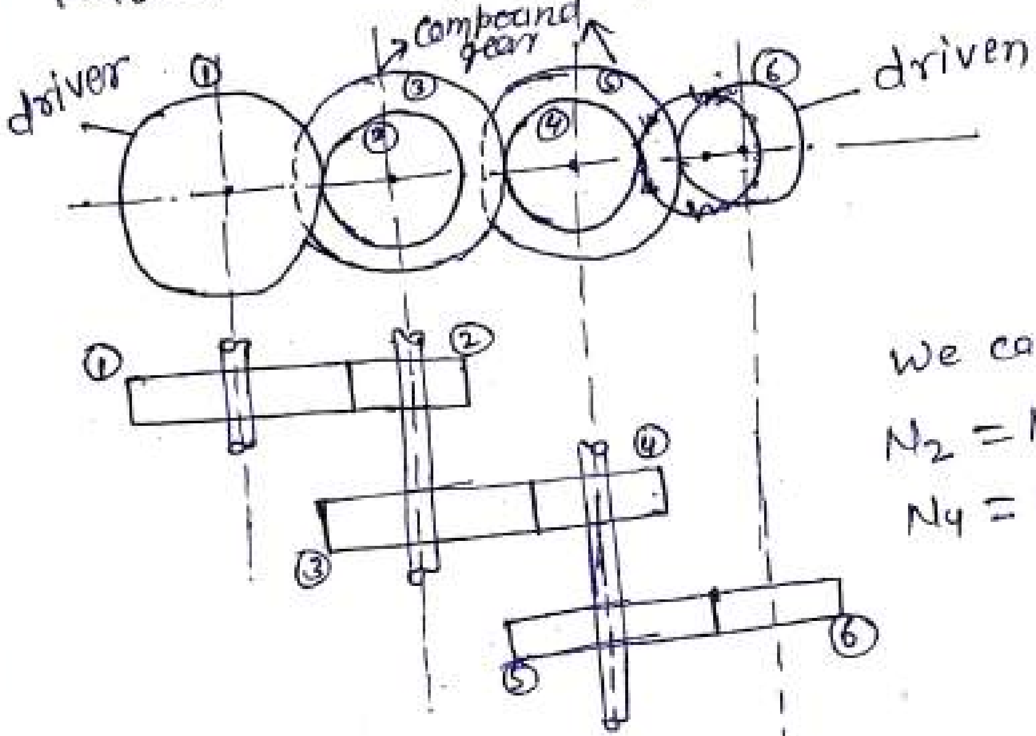


Fig.(2)

We can observe that

$N_2 = N_3$

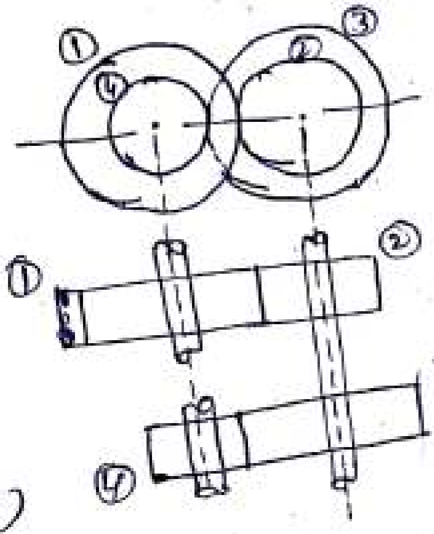
$N_4 = N_5$

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} \times \frac{N_6}{N_5} = \frac{T_1}{T_2} \times \frac{T_3}{T_4} \times \frac{T_5}{T_6}$$

$$\frac{N_6}{N_1} = \frac{T_1 \times T_3 \times T_5}{T_2 \times T_4 \times T_6}$$

(3) **Reverted Gear Train** :- If the axes of first and last gear of a compound gear coinside i.e. the first of driver and last driven gear co-inside, it is called a reverted gear train.

Application: in clocks and in the simple lathes.



$$\boxed{N_2 = N_3}$$

fig. (3)

In a reverted gear train, the motion of the first gear and last gear is like.

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$

$$\frac{N_4}{N_1} = \frac{T_1 \times T_3}{T_2 \times T_4}$$

from Geometry : $r_1 + r_2 = r_3 + r_4$

⇒ Planetary or Epicyclic Gear train:-

In a epicyclic gear trains, the axis of the shaft on which the gears are mounted may move relative to a fixed ~~arm~~ axis.

A simple epicyclic gear train is shown in fig.

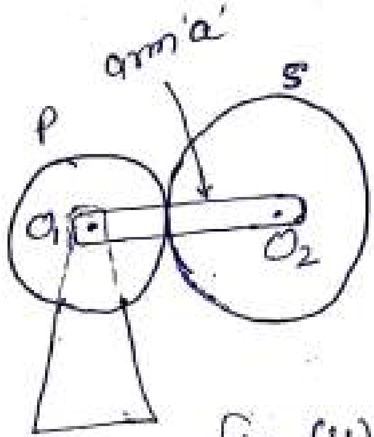


fig. (4)

- In this gear P and are 'a' have a common axis at O_1 about which they can rotate.
- The gear S meshes with gear P and has its axis on the arm at O_2, about which gear S can rotate.
- If the arm is fixed, the gear train is simple and gear P can drive ~~a~~ gear S and vice-versa.
- If gear P is fixed and the arm is rotated about the axis of gear P, then the gear B is forced to rotate upon and around P.

Application:-

- The epicyclic gear trains are useful for transmitting the high velocity ratios with gear of moderate size in a comparatively lesser space.
- Used in back gear of lathe, differential gear of the automobiles, hoists, wrist watches etc.

Note:- Epicyclic gear train may simple or compound.

⇒ Analysis of epicyclic gear train :-

Let us consider fig. 4.

Assume

* clock wise rotation +ve and anticlockwise -ve

Let arm 'a' is fixed and gear P turn x-revolutions in clock wise direction.

revolution made by $a = 0$

revolution made by $P = x$

revolution made by $S = -\left(\frac{T_P}{T_S}\right)x$

~~Now if the mechanism is locked together and turned~~

Now assume the complete system is locked and this system is turned through 'y-revolutions in clockwise direction. Then

Revolutions made by $a = y$

Revolutions made by $P = y + x$

Revolutions made by $S = y - \frac{T_P}{T_S} \cdot x$

⇒ The complete procedure can be summarised in tabular form:-

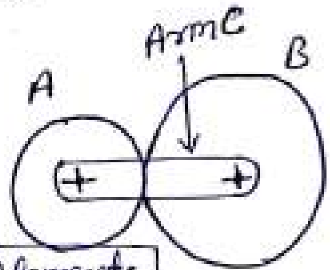
Line	Conditions of Motion	Revolutions of elements: Arm a	Gear P	Gear S
1.	Arm 'a' is fixed and gear P rotates 1 revolution in clock wise direction (+ve)	0	+1	$-\frac{T_P}{T_S}$
2.	Arm 'a' fixed & Gear P rotates +x rev.	0	+x	$-x \cdot \frac{T_P}{T_S}$
3.	+y rev. to all elements	+y	y+x	$y - x\frac{T_P}{T_S}$

Numerical Problem :-

Problem ①: In an epicyclic gear train, on arm carries two gears A and B having teeth 36 and 45 respectively

(i) If the arm rotates at 150 rpm in the anticlockwise direction about the centre of the gear A which is fixed. Determine the speed of gear B.

(ii) If the gear A instead of being fixed, makes 300 rpm in the clockwise direction, what will be the speed of gear B.?



Sol^n: $T_A = 36$, $T_B = 45$

$N_C = 150$ rpm

Step	Condition of motion	Revolutions of Elements		
		Arm C	Gear A	Gear B
1	Arm fixed + Gear A +1 rev.	0	+1	$-\frac{T_A}{T_B}$
2	Arm fixed + Gear A +x rev.	0	+x	$-x\frac{T_A}{T_B}$
3	Y revo. of all elements.	y	y+x	$y-x\frac{T_A}{T_B}$

(i) $N_C = 150$ rpm Anticlockwise

$N_C = -150$ rpm $\Rightarrow y = -150$ rpm

Gear A - fixed $\Rightarrow x + y = 0$

$x = -y = -(-150) = +150$ rpm

Speed of gear B

$$N_B = y - x\frac{T_A}{T_B} = -150 - 150 \cdot \frac{36}{45} = -270 \text{ rpm}$$

$$\boxed{N_B = 270 \text{ rpm Anticlockwise}}$$

(ii) Gear A → 300 rpm clockwise

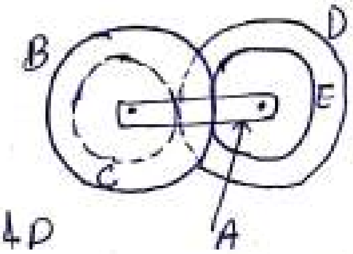
$$x + y = 300$$
$$x = 300 - y = 300 - (-150) = +450 \text{ rpm}$$

speed of gear B

$$N_B = y - x \frac{T_A}{T_B} = -150 - 450 \cdot \frac{36}{45} = -510 \text{ rpm}$$

$$\boxed{N_B = 510 \text{ rpm Anticlockwise}}$$

Problem ②: In a reverted epicyclic gear train, the arm A carriers two gears B and C and a compound gear D-E. The gear B meshes with gear E and the gear C meshes with gear D.



The no. of teeth on gear B, C & D are 75, 30 and 90, respectively. Find the speed and direction of gear C when gear B is fixed and the arm A makes 100 rpm clockwise.

Solⁿ: $T_B = 75$, $T_C = 30$, $T_D = 90$, $N_A = 100$ rpm (C)

From geometry $r_B + r_E = r_C + r_D$ —①

Since the no. of teeth on each gear for the same module, are proportional to their pitch circle dia.

$$m = \frac{d}{T} \Rightarrow d = mT \Rightarrow d_1 = mT_1 \ \& \ d_2 = mT_2 \ldots$$

equa. ① $r_B + r_E = r_C + r_D \Rightarrow d_B + d_E = d_C + d_D$

$$mT_B + mT_E = mT_C + mT_D$$

$$T_B + T_E = T_C + T_D$$

$$T_E = T_C + T_D - T_B = 30 + 90 - 75 = 45$$

$$\boxed{T_E = 45}$$

step	Condition of motion	Revolution of elements.			
		Arm A	Compound gear D-E	Gear B	Gear C
1.	Arm fixed & gear D-E rotated through +1 rev.	0	+1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2.	Arm fixed & gear D-E +x rev.	0	+x	$-x\frac{T_E}{T_B}$	$-x\frac{T_D}{T_C}$
3.	+y rev. to all elements	+y	y+x	$y-x\frac{T_E}{T_B}$	$y-x\frac{T_D}{T_C}$

Gear B is fixed

$$y - x\frac{T_E}{T_B} = 0 \Rightarrow y = x \cdot \frac{45}{75} \Rightarrow \boxed{y = 0.6x}$$

the arm A makes 100 rpm clockwise $\Rightarrow y = +100$ rpm

$$x = \frac{100}{0.6} \Rightarrow \boxed{x = +166.67 \text{ rpm}}$$

speed of gear C

$$N_C = y - x\frac{T_D}{T_C} \Rightarrow N_C = 100 - 166.67 \cdot \frac{90}{30}$$

$$N_C = -400 \text{ rpm}$$

$$\boxed{N_C = 400 \text{ rpm Anticlockwise}}$$