

Unit-II

Magnetic Properties

It is convenient to group the magnetic properties of solids under the following-

1. diamagnetism
2. paramagnetism
3. ferromagnetism, antiferromagnetism, ferrimagnetism.

When a substance is placed in a magnetic field H , a magnetic moment M per unit volume results; M is called the **magnetization**. For isotropic materials, M and H are **parallel vectors** and the susceptibility χ defined by-

$$M = \chi H \quad \dots\dots(1)$$

is then a scalar quantity. In anisotropic substances, χ is a **tensor**. In case M refers to a gram molecule, one may introduce the molar susceptibility χ_m . All atoms or ions produce a diamagnetic contribution to the total susceptibility, although it may be masked by the other types; it is a consequence of the magnetic moment induced in the atoms by an external field. In this respect, diamagnetism may be compared with the electronic polarization in an electric field. Both are essentially independent of temperature. There exists, however

an essential difference: in the electrical case the induced moment lies along the direction of an applied field, leading to a positive electrical susceptibility; in the magnetic case the induced moment produces a negative susceptibility.

Paramagnetism requires the existence of permanent magnetic dipoles and the paramagnetic susceptibility is the analogue of the orientational susceptibility associated with permanent electric dipoles. In both cases the susceptibility is positive and temperature dependent. The properties corresponding to group (3) above also require the existence of permanent magnetic dipoles, and moreover, a relatively strong interaction between them.

The magnetic induction **B** may be defined as-

$$B = H + 4\pi M = \mu H \dots \dots (2)$$

where, μ is called the **permeability**: it should not be confused with the same symbol used below for the magnetic dipole moment of an atom. We shall assume **H, M and B** to be parallel vectors, so that μ is a scalar. For para and diamagnetic materials, the permeability is a constant, unless saturation conditions are approached. For the properties mentioned under (3) the relation btw B and H is much more complicated and shows hysteresis

From (1) and (2), it follows that -

$$\mu = 1 + 4\pi \chi \dots \dots (3)$$

This relation is the analogue of the expression for the dielectric constant ϵ . when χ represents the ratio of the electric moment per unit volume and the applied electric field.

It is convenient to normalize the potential energy of a dipole μ in a magnetic field H in such a way that -

$$E_{\text{pot}} = -\mu \cdot H \dots \dots (4)$$

The Origin Of Permanent Magnetic Dipoles -

As stated in the hypothesis of Ampere, magnetic dipoles have their origin in the flow of electric currents. From electricity theory, it is well known, for example, that a stationary loop current flowing in a plane produce a magnetic field which at large distances may be described as resulting from a magnetic dipole,

$$\mu = \frac{IS}{c}$$

where I is the current and S is the area of the loop. The dipole direction is perpendicular to the plane of loop. Employing this relation, let us consider the magnetic dipole moment associated with an electron describing a circular

orbit of radius r , the angular velocity of the electron being ω . The loop current in this case is $-e\omega r/2\pi$ so that, according to the eqnⁿ(6), the magnetic dipole moment associated with the electron orbit is -

$$\mu = -\frac{e\omega r^2}{2c} \dots \dots (7)$$

It is of interest to relate the magnetic dipole moment to the angular momentum of the electron, which in this case is $m\omega r^2$. According to the eqnⁿ(7), we have -

$$\mu = -\left(\frac{e}{2mc}\right) \times \text{angular momentum} \dots \dots (8)$$

The minus sign indicates that the dipole moment points in a direction opposite to the vector representing the angular momentum.

Relation (8) is valid for any electron orbit, it is not valid, however, for the spin of an electron or nucleus.

The Use Of Quantum Numbers-

- a) The principle quantum number 'n' determines the energy of the orbit; it can accept only the integer values, $n=1, 2, 3, \dots$. The corresponding electronic shells are called the K, L, M, N, ... shells.

b) The **angular momentum** of the orbit is determined by the quantum number L , which is restricted to the set of values

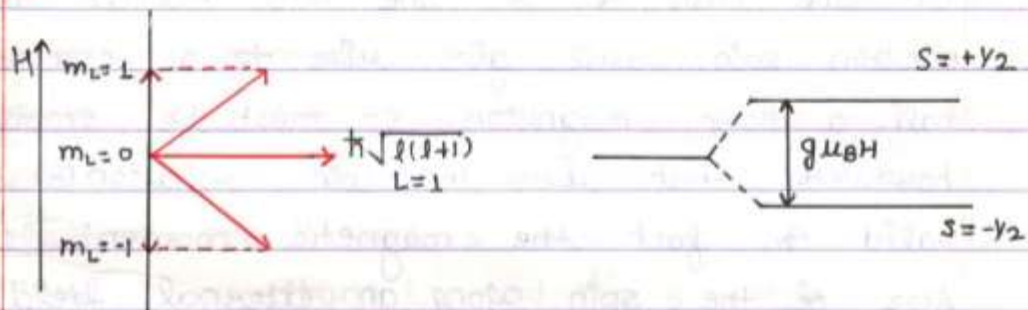
$$L = 0, 1, 2, \dots, (n-1) \quad \dots \dots (9)$$

The total angular momentum associated with a given value of L is - $\hbar [L(L+1)]^{1/2} \quad \dots \dots (10)$

Electrons associated with states $L = 0, 1, 2, 3, \dots$ are called, respectively, s, p, d, f, g... electrons.

Note that electrons in an s state always have zero angular momentum and thus a vanishing magnetic moment.

c) The possible components of the angular momentum along any specified direction (such as the direction of an external magnetic field H)



are determined by the magnetic quantum no m , where m is restricted to the set of values -

$$m_L = L, (L-1) \dots 0, \dots -L(L-1), -L \quad \dots \dots (11)$$

For example - a p electron has the possible components of angular momentum along the direction of a magnetic field $\hbar, 0, -\hbar$.

Consequently, the possible magnetic moment components along the direction of an applied magnetic field are - $\frac{e\hbar}{2mc}$, 0, $\frac{e\hbar}{2mc}$

The quantity $\frac{e\hbar}{2mc} = 0.927 \times 10^{-20}$ erg/oersted is called

the **Bohr magneton**; it will be denoted by μ_B .

d) So far we have described an electron simply as a particle of charge e and mass m . However, the electron itself has an angular momentum known as the spin. The possible angular momentum components of the spin along an external field direction are $\pm\hbar/2$. This has led to the introduction of the spin quantum no. $s = \pm 1/2$.

On the basis of (8) one thus expects that the electron spin will give rise to a component of half a Bohr magneton. It must be emphasized, however, that for the spin, relation (8) is not valid. In fact the magnetic moment component μ_{sz} of the spin along an external field is given by -

$$\mu_{sz} = g \left(\frac{e}{2mc} \right) \left(\frac{\hbar}{2} \right) \dots \dots (12)$$

where, g is called the **spectroscopic splitting factor**, or the **gyromagnetic ratio**. For the electron spin, $g = 2.0023$, i.e. the **electron spin**

gives rise to very nearly one Bohr magneton in the direction (or opposite) of an external field H . The reason for the name "splitting factor" is the following: consider an electron with a spin $1/2$ and without orbital angular momentum, under influence of a magnetic field H . This gives rise to two energy levels separated by an energy.

$$\Delta E = 2|\mu_s z| H = g \left(\frac{e}{2mc} \right) \hbar H = g \mu_B H \dots \dots (13)$$

where, we used equⁿ(4) and equⁿ(12). Thus g determines the amount by which the original level is split up.

e) The orbital angular momentum and the spin may be combined vectorially to give the total angular momentum; the latter is determined by the quantum number, j . Thus, for an electron with a certain L and a spin of $1/2$, j can accept the value $L \pm 1/2$. In atoms containing a number of electrons, the l vectors may be combined to form a resultant L , and the s vectors are combined to form a resultant S . This type of combination is called Russell - Saunders coupling. The resultants L and S then combine to form the total angular momentum, J of the whole electron system of the atom. For such atoms, the spectroscopic splitting factor is given by the Lorde formula.

$$g = \frac{L + J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \dots \dots (14)$$

Hund rules. In order to predict the magnetic dipole moment associated with the electronic system of a given atom, the above considerations must be combined with the Pauli principle and Hund's rules. According to the **Pauli principle**, only one electron can occupy a state defined by the set of quantum numbers **n, l, m_l and s** . Filled electron shells do not contribute to the magnetic moment of an atom. Thus, the magnetic moment in atoms must result from incompletely filled shells. With regard to the latter, Hund's rules state that for the ground state of such atoms.

- i) The electron spins add to give the maximum possible S consistent with the Pauli principle.
- ii) The orbital momenta combine to give the maximum value for L that is consistent with (i)
- iii) For an incompletely filled shell, we have-

$J = L - S$ for a shell less than half occupied.

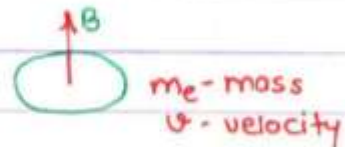
$J = L + S$ for a shell more than half occupied.

Langevin Diamagnetic Equations - It is a **diamagnetic property equation**.

The equation about the magnetic susceptibility for the diamagnetic material.

Since, we know that, the magnetic flux is given by -

$$\phi = BA$$



∴ According to Lenz's law -

$$\text{induced emf} = \frac{d\phi}{dt} = A \cdot \frac{dB}{dt} = \xi_p = \oint \vec{E} \cdot d\vec{l}$$

$$\Rightarrow \xi_p = \oint \vec{E} \cdot d\vec{l}$$

$$\xi_p = E \cdot \oint dl$$

$$\xi_p = E \cdot 2\pi r$$

$$E = \frac{\xi_p}{2\pi r}$$

∴ We know that -

$$F = m \frac{dv}{dt} = eE$$

$$\Rightarrow m \frac{dv}{dt} = e \frac{\xi_p}{2\pi r}$$

$$m \frac{dv}{dt} = \frac{e}{2\pi r} \frac{d\phi}{dt} \quad \left\{ \because \xi_p = \frac{d\phi}{dt} \right\}$$

$$m \int_{v_0}^{v_0 + \Delta v_0} \frac{dv}{dt} dt = \frac{e}{2\pi r} \int_0^{\phi} \frac{d\phi}{dt} dt$$

$$m \int_{v_0}^{v_0 + \Delta v_0} dv = \frac{e}{2\pi r} \int_0^{\phi} d\phi$$

$$m_e \nabla V_0 = \frac{e}{2\pi\mu} \phi$$

$$\Delta V = \frac{e}{2\pi m_e \mu} \phi$$

Hence, $\Delta \omega = \frac{\Delta V}{\mu}$ ($\because V = \omega \mu$)

Since,

$$\frac{\Delta V}{\mu} = \frac{e}{2\pi \mu^2 m_e} \phi$$

$$\frac{\Delta V}{\mu} = \frac{e}{2\pi \mu^2 m_e} B \cdot A$$

$$\frac{\Delta V}{\mu} = \frac{e}{2\pi \mu^2 m_e} B \cdot (\pi \mu^2) \quad (\because A = \pi \mu^2)$$

$$\frac{\Delta V}{\mu} = \frac{e}{2m_e} B \quad \text{in S.I units}$$

Therefore,

$$\Delta \omega_L = \frac{\Delta V}{\mu} = \frac{e}{2m_e} B \quad \text{in S.I units}$$

and

$$\Delta \omega_L = \frac{e}{2m_e c} B \quad \text{in C.G.S units.}$$

The change in angular velocity is called Larmor angular velocity and the frequency associated is called the Larmor frequency.

Due to this orbit precession, this Larmor precession is equivalent to a current

\Rightarrow Larmor precession = current

Hence,

$I = \text{charge} \times \text{revolution per unit time}$

$$I = (-ze) \times \left(\frac{1}{2\pi} \cdot \frac{eB}{2me} \right)$$

\downarrow
 $\omega/2\pi$

Now,

Magnetic dipole moment = $I \times A$ \rightarrow Area
 \downarrow
Current

$$\mu = \frac{-ze^2 B}{4\pi me} \langle \rho^2 \rangle \quad \#$$

$$\mu = \frac{-ze^2 B}{4me c} \langle \rho^2 \rangle$$

where, $\langle \rho^2 \rangle$ is the mean square perpendicular distance of e^- from the field axis (z-axis)

Hence,

$$\langle \rho^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$$

The mean square radius for the spherically symmetric distribution is -

$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$

Since, we know that for the spherically symmetric.

$$\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$$

Hence,

$$\langle \rho^2 \rangle = \frac{2}{3} \langle r^2 \rangle$$

Therefore, on substituting we get -

$$\mu = \frac{-ze^2 B}{4\pi m_e c} \left(\frac{2}{3}\right) \pi \langle r^2 \rangle$$

Where,

$\langle r^2 \rangle$ is the distance from the nucleus to the electron.

Hence,

$$\mu = \frac{-ze^2}{6m_e c} \langle r^2 \rangle$$

Let N be the number of atoms per unit volume, and M be the induced magnetic moment per unit volume. Therefore,

$$M = \mu N = \chi B$$

or,

$$\chi = \frac{M}{B} = \frac{\mu N}{B}$$

Therefore,

$$\chi = \frac{-ze^2 N}{6m_e c} \langle r^2 \rangle \quad \text{In S.I. unit}$$

or,

$$\chi = \frac{-ze^2 N}{6m_e c} \langle r^2 \rangle \quad \text{In C.G.S. unit}$$

This is called the Langevin Diamagnetic equation.