Unit-III (ME-201)

Deformation of Bars under Axial Load

Elongation of Uniform Bar under Axial Load:

According to Hooke's law

$$\frac{\sigma}{e} = E$$

$$\frac{\left(\frac{P}{A}\right)}{\left(\frac{\delta L}{L}\right)} = E$$

$$or; \quad \delta L = \left(\frac{P.L}{A.E}\right)$$

Elongation of Uniform Circular Taper Bar under Axial Load:

The stress at any cross section can be found by dividing the load by the area of cross section and extension can be found by integrating extensions of a small length over whole length of the bar. We shall consider the following cases of variable cross section.

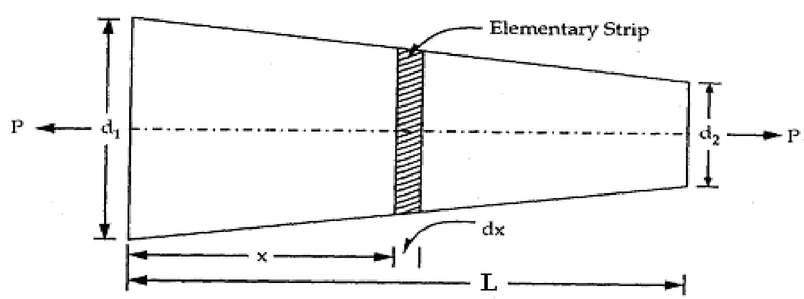

Fig-3

Let us consider a circular bar of length L tapering uniformly from diameter d_1 at the bigger end to diameter d_2 at the smaller end, and subjected to axial tensile load P as shown in fig 3.

Considering a small strip of length dx at a distance x from the bigger end.

Diameter of the elementary strip:

$$= d_1 - [(d_1 - d_2)x]/L$$

$$= d_1 - kx; \text{ where } k = (d_1 - d_2)/L$$

Cross-sectional area of the strip,

$$A_x = \frac{\pi}{4} d_x^2 = \frac{\pi}{4} (d_1 - k x)^2$$

Stress in the strip,

$$\sigma_x = \frac{P}{A_x} = \frac{P}{\frac{\pi}{4} (d_1 - k x)^2} = \frac{4 P}{\pi (d_1 - k x)^2}$$

Strain in the strip

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{4 P}{\pi (d_1 - k x)^2 E}$$

Elongation of the strip

$$\delta l_x = \varepsilon_x \, dx = \frac{4 P \, dx}{\pi (d_1 - k x)^2 E}$$

The total elongation of this tapering bar can be worked out by integrating the above expression between the limits x = 0 to x =L

$$\delta l = \int_0^l \frac{4 P \, dx}{\pi (d_1 - k x)^2 E} = \frac{4 P}{\pi E} \int_0^l \frac{dx}{(d_1 - k x)^2}$$

$$= \frac{4 P}{\pi E} \left[\frac{(d_1 - k x)^{-1}}{(-1) \times (-k)} \right]_0^l = \frac{4 P}{\pi E K} \left[\frac{1}{d_1 - k x} \right]_0^l$$

Putting the value of $k = (d_1 - d_2) / L$ in the above expression, we obtain

$$\delta l = \frac{4 P l}{\pi E (d_1 - d_2)} \left[\frac{1}{d_1 - \frac{(d_1 - d_2) l}{l}} - \frac{1}{d_1} \right]$$

$$= \frac{4 P l}{\pi E (d_1 - d_2)} \left[\frac{1}{d_2} - \frac{1}{d_1} \right]$$

$$= \frac{4 P l}{\pi E (d_1 - d_2)} \times \frac{d_1 - d_2}{d_1 d_2} = \frac{4 P l}{\pi E d_1 d_2}$$

Principle of Superposition for Elongation of a Composite Bar:

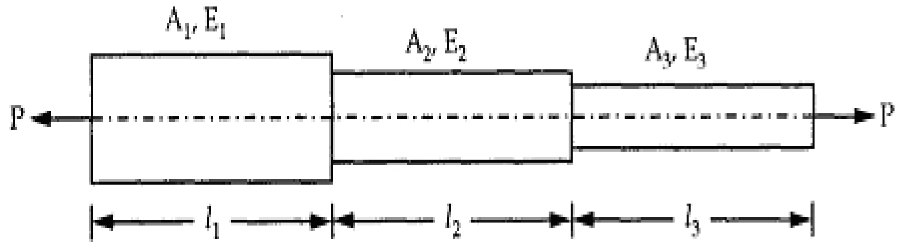

Fig 4

A machine member is subjected to a number of forces acting on its outer edges as well as at some intermediate sections along its length. The forces are then split up and their effects are considered on individual sections.

The resulting deformation is then given by the algebraic sum of the deformation of the individual sections. This is the principle of superposition which may be stated as;

"The total elongation in any stepped bar due to a load is the algebraic sum of elongations in individual parts of the bar".

Mathematically

$$\delta l = \sum_{i=1}^{i=n} \delta l_i$$

Consider a bar made up of different lengths and having different cross-sections as shown in Fig.4

For such a bar, the following conditions apply:

(i) Each section is subjected to the same external pull or push

(ii) Total change in length is equal to the sum of changes of individual lengths

That is:

$$\begin{aligned} P_1 &= P_2 = P_3 = P \text{ and} \\ \delta l &= \delta l_1 + \delta l_2 + \delta l_3 \\ &= \frac{\sigma_1 l_1}{E_1} + \frac{\sigma_2 l_2}{E_2} + \frac{\sigma_3 l_3}{E_3} \\ &= \frac{P_1 l_1}{A_1 E_1} + \frac{P_2 l_2}{A_2 E_2} + \frac{P_2}{A_3 E_3} \end{aligned}$$

If the bar segments are made of same material, then in that case

$$E_1 = E_2 = E_3 = E.$$

$$\delta l = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$

Strain Energy and Resilience:

When an external force acts on an elastic material and deforms it, internal resistance is developed in the material. The internal resistance does some work which is stored within the material as energy called **strain energy**, and this strain energy within elastic limit is known as **resilience**. What-ever energy is absorbed during loading, same energy is recovered during unloading and the material springs back to its original dimension.

Proof Resilience and Modulus of Resilience:

The maximum strain energy absorbed by a body up to its elastic limit is termed as Proof Resilience and this proof resilience per unit volume is called **Modulus of Resilience**.

Proof resilience = $(\sigma_e^2/2E)$ x volume

Where, σ_e is the stress at elastic limit.

Modulus of resilience = $\sigma_e^2/2E$