FOET
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Subject – Network Analysis
Topic – Network Function and Synthesis
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Contents

Network Functions

As the name suggests, in **theory of network synthesis** we are going to study about the synthesis of various networks which consists of both the active (Energy source) and passive elements (resistor, inductors and capacitors).

what is a network function?

In the frequency domain, **network functions** are defined as the quotient obtained by dividing the phasor corresponding to the circuit output by the phasor corresponding to the circuit input.

In simple words, **network functions** are the ratio of output phasor to the input phasor when phasors exists in frequency domain.

Network Functions

The general form of network functions are given below:

$$F(s) = \frac{a_m s^m + a_{m-1} s^{m-1} + \cdots + a_1 s^1 + a_0}{b_n s^n + b_{n-1} s^{n-1} + \cdots + b_1 s^1 + b_0}$$

There are three main necessary conditions for the stability of these network functions and they are written below:

- The degree of the numerator of F(s) should not exceed the degree of denominator by more than unity. In other words (m - n) should be less than or equal to one.
- F(s) should not have multiple poles on the jω-axis or the y-axis of the pole-zero plot.
- F(s) should not have poles on the right half of the s-plane.

Hurwitz Polynomial

If we have a stable network function then the denominator of the F(s) is called the **Hurwitz Polynomial**.

$$Let,\ F(s) = \frac{P(s)}{Q(s)}$$

Where, Q(s) is a **Hurwitz polynomial**.

Properties of Hurwitz Polynomials

- For all real values of s value of the function P(s) should be real.
- The real part of every root should be either zero or negative.
- Let us consider the coefficients of denominator of F(s) is b_n, $b_{(n-1)}$, $b_{(n-2)}$. . . . b_0. Here it should be noted that b_n, $b_{(n-1)}$, b_0 must be positive and b_n and $b_{(n-1)}$ should not be equal to zero simultaneously.

Properties of Hurwitz Polynomials

- The continued fraction expansion of Even/Odd or Odd/Even part of the **Hurwitz polynomial** should give all positive quotient terms.
- In case of purely even or purely odd polynomial, we must do continued fraction with the derivative of the purely even or purely odd polynomial and rest of the procedure is same as mentioned in the point number (4).

From the above discussion we conclude one very simple result,
If all the coefficients of the quadratic polynomial are real and positive then that quadratic polynomial is always a Hurwitz polynomial.

Positive Real Functions

Any function which is in the form of F(s) will be called as a ***positive real function*** *if fulfill these four important conditions:*

- F(s) should give real values for all real values of s.
- P(s) should be a Hurwitz polynomial.
- If we substitute s = jω then on separating the real and imaginary parts, the real part of the function should be greater than or equal to zero, means it should be non negative. This most important condition and we will frequently use this condition in order to find out the whether the function is positive real or not.
- On substituting s = jω, F(s) should posses simple poles and the residues should be real and positive.

Properties of Positive Real Function

- The numerator and denominator of F(s) should be Hurwitz polynomials.
- The degree of the numerator of F(s) should not exceed the degree of denominator by more than unity. In other words (m-n) should be less than or equal to one.
- If F(s) is positive real function then reciprocal of F(s) should also be positive real function.
- Remember the summation of two or more positive real function is also a positive real function but in case of the difference it may or may not be positive real function.

Property 1. L-C immittance function

- 1. Z_{LC} (s) or Y_{LC} (s) is the ratio of odd to even or even to odd polynomials.
  - Consider the impedance Z(s) of passive one-port network.

$$Z(s) = \frac{M_1(s) + N_1(s)}{M_2(s) + N_2(s)}$$ (M is even N is odd)

As we know, when the input current is *I*, the average power dissipated by one-port network is zero:

Average Power = $\frac{1}{2}\mathrm{Re}[Z(j\mu)]|I|^2 = 0$
(for pure reactive network)

Property 1. L-C immittance function

$$EvZ(s) = \frac{M_1(s)M_2(s) - N_1(s)N_2(s)}{M_2(s)^2 + N_2(s)^2} = 0$$

$$M_1(j\mu)M_2(j\mu) - N_1(j\mu)N_2(j\mu) = 0$$

$$M_1 = 0 = N_2 \quad \text{OR} \quad M_2 = 0 = N_1$$

$$Z(s) = \frac{M_1}{N_2}, Z(s) = \frac{N_1}{M_2}$$

Z(s) or Y(s) is the ratio of even to odd or odd to even!!

Property 2. L-C immittance function

- 2.The poles and zeros are simple and lie on the $j\mu$ axis.

$$Z(s) = \frac{M_1}{N_2}, Z(s) = \frac{N_1}{M_2}$$

  - Since both ***M and N are Hurwitz***, they have only imaginary roots, and it follows that the poles and zeros of Z(s) or Y(s) are on the imaginary axis.
  - Consider the example $Z(s) = \frac{a_4 s^4 + a_2 s^2 + a_0}{b_5 s^5 + b_3 s^3 + b_1 s}$

Property 2. L-C immittance function

$$Z(s) = \frac{a_4 s^4 + a_2 s^2 + a_0}{b_5 s^5 + b_3 s^3 + b_1 s}$$

In order for the impedance to be positive real → the coefficients must be real and positive.

Impedance function **cannot have multiple poles or zeros** on the axis.

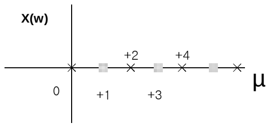
The highest powers of the numerator and the denominator polynomials can differ by, at most, unity.

Ex) highest order of the numerator : 2n -1> highest order of the denominator can either be 2n-1 (simple pole at s= ∞) or

the order can be 2n+1 (simple zero at s= ∞).

Property 3. L-C immittance function

- **3.The poles and zeros interlace on the $j\mu$ axis.**



Highest power: 2n -> next highest power must be 2n-2

They cannot be missing term. Unless?

if $b_5 s^5 + b_1 s = 0$ → **s=0,** $s_k = (\frac{b_1}{b_5})^{1/4} e^{i(2k-1)\pi/4}$

Property 3. L-C immittance function

We can write a general L-C impedance or admittance as

$$Z(s) = \frac{K(s^2+\mu_1^2)(s^2+\mu_3^2)...(s^2+\mu_i^2)...}{s(s^2+\mu_2^2)(s^2+\mu_4^2)...(s^2+\mu_j^2)...}$$

$$Z(s) = \frac{K_0}{s} + \frac{2K_2 s}{s^2+\mu_2^2} + \frac{2K_4 s}{s^2+\mu_4^2} + ... + K_\infty s$$

Since these poles are all on the jw axis, the residues must be real and positive in order for Z(s) to be positive real .

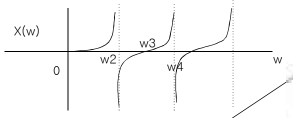
S=jw → Z(jw)=jX(w) (no real part)

Property 3. L-C immittance function

$$\frac{dX(\mu)}{d\mu} = \frac{K_0}{\mu^2} + K_\infty + \frac{2K_2(\mu^2+\mu_2^2)}{(\mu_2^2-\mu^2)^2} + ...$$

Since all the residues Ki are positive, it is eay to see that for an L-C function $\frac{dX(\mu)}{d\mu} \geq 0$

Ex) $Z(s) = \frac{Ks(s^2+\mu_3^2)}{(s^2+\mu_2^2)(s^2+\mu_4^2)}$ $jX(\mu) = +j\frac{K\mu(-\mu^2+\mu_3^2)}{(-\mu^2+\mu_2^2)(-\mu^2+\mu_4^2)}$



Properties 4 and 5. L-C immittance function

- The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.
- There must be either a zero or a pole at the origin and infinity.

Summary of properties

1. $Z_{LC}(s)$ or $Y_{LC}(s)$ is the ratio of odd to even or even to odd polynomials.
2. The poles and zeros are simple and lie on the $j\mu$ axis
3. The poles and zeros interlace on the $j\mu$ axis.
4. The highest powers of the numerator and denominator must differ by unity; the lowest powers also differ by unity.
5. There must be either a zero or a pole at the origin and infinity.

Examples

$$Z(s)=\frac{Ks(s^2+4)}{(s^2+1)(s^2+3)} \qquad Z(s)=\frac{s^5+4s^3+5s}{3s^4+6s^2}$$

$$Z(s)=\frac{K(s^2+1)(s^2+9)}{(s^2+2)(s^2+10)}$$

$$Z(s)=\frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

Synthesis of L-C Driving point immittances

- L-C immittance is a positive real function with poles and zeros on the $j\mu$ axis only.

$$Z(s)=\frac{K_0}{s}+\frac{2K_2 s}{s^2+\mu_2{}^2}+\frac{2K_4 s}{s^2+\mu_4{}^2}+\dots+K_\infty s$$

- The synthesis is accomplished directly from the partial fraction .
- F(s) is impedance -> then the term : K_0/s capacitor of 1/K farads

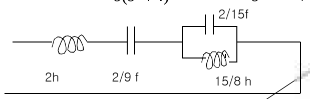
  the K(infinite)s is an inductance of K(infinite) henrys.

For Z(s) partial fraction

$2K_i s/(s_2+\mu_i{}_2)$ Is a parallel tank capacitance and inductance.

$1/2K_i$, $2K_i/\mu_{i2}$

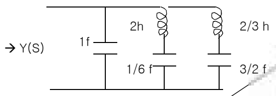
$$Z(s)=\frac{2(s^2+1)(s^2+9)}{s(s^2+4)}=2s+\frac{\frac{9}{2}}{s}+\frac{\frac{15}{2}s}{s^2+4}$$



For Y(s) partial fraction

- In admittance

$$Y(s)=\frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}=s+\frac{\frac{1}{2}s}{s^2+3}+\frac{\frac{3}{2}s}{s^2+1}$$



Another methodology

- Using property 4 "The highest powers if numerator and denominator must differ by unity; the lowest powers also differ by unity."
  - Therefore, there is always a zero or a pole at s=infinite .
  - suppose Z(s) numerator:2n ,denominator:2n-1
  - this network has pole at infinite. -> we can remove this pole by removing an impedance $L_1 s$

    $Z_2(s) = Z(s) - L_1 s$
- Degree of denominator : 2n-1 numerator:2n-2
  - $Z_2(s)$ has zero at s=infinite.
  - $Y_2(s) = 1/Z_2(s)$, → $Y_3(s) = Y_2(s) - C_2 s$

Another methodology

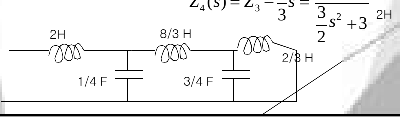
- This infinite term removing process continue until the remainder is zero.
- Each time we remove the pole, we remove an inductor or capacitor depending upon whether the function is an impedance or an admittance.
- Final synthesized is a ladder whose series arms are inductors and shunt arms are capacitors.

$$Z(s) = \frac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3}$$

$$Z_2(s) = \frac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3} - 2s = \frac{4s^3 + 10s}{s^4 + 4s^2 + 3}$$

$$Y_3(s) = \frac{s^4 + 4s^2 + 3}{4s^3 + 10s} - \frac{1}{4}s = \frac{\frac{3}{2}s^2 + 3}{4s^3 + 10s}$$

$$Z_4(s) = Z_3 - \frac{8}{3}s = \frac{2s}{\frac{3}{2}s^2 + 3}$$



CAUER Form

- This circuit (Ladder) called as Cauer because Cauer discovered the continues fraction method.
- Without going into the proof of the statement m in can be said that both the Foster and Cauer form gice the minimum number of elements for a specified L-C network.

Example of Cauer Method

$$Z(s) = \frac{(s^2 + 1)(s^2 + 3)}{s(s^2 + 2)}$$