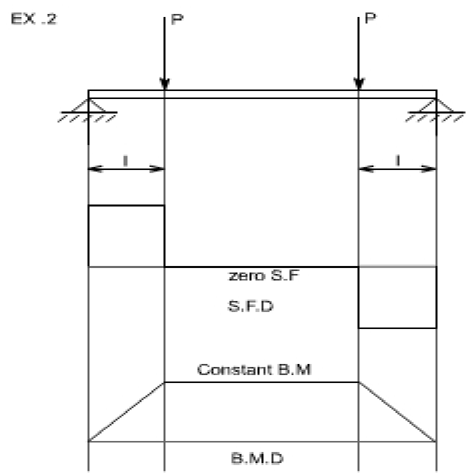
UNIT-III
Pure Bending of Beams

Pure bending:

If a beam is loaded in such a fashion that the shear forces are zero on any cross-section perpendicular to the longitudinal axis of the beam and hence it is subjected to constant bending moment then the beam is said to be in the state of pure bending.

That means F = 0

since $\frac{dM}{dX} = F = 0$ or M = constant.



Neutral surface and Neutral axis:

We know that when a beam is under bending the top fibers of the beam will be subjected to tension/compression and the bottom fibers to compression/tension. It is reasonable to suppose that there is a fiber somewhere between the top and bottom fibers at which the stress is zero. This fiber is known as **neutral fiber** or **neutral layer** or **neutral surface**.

The line of intersection of neutral surface and any transverse section is called the **neutral axis (NA)**.

Assumptions of Pure Bending:

1. Beam is made up a no of layers which are free to expand or contract independently.

2. Beam is initially straight, and unstressed.

3. Material of beam is homogeneous and isotropic.

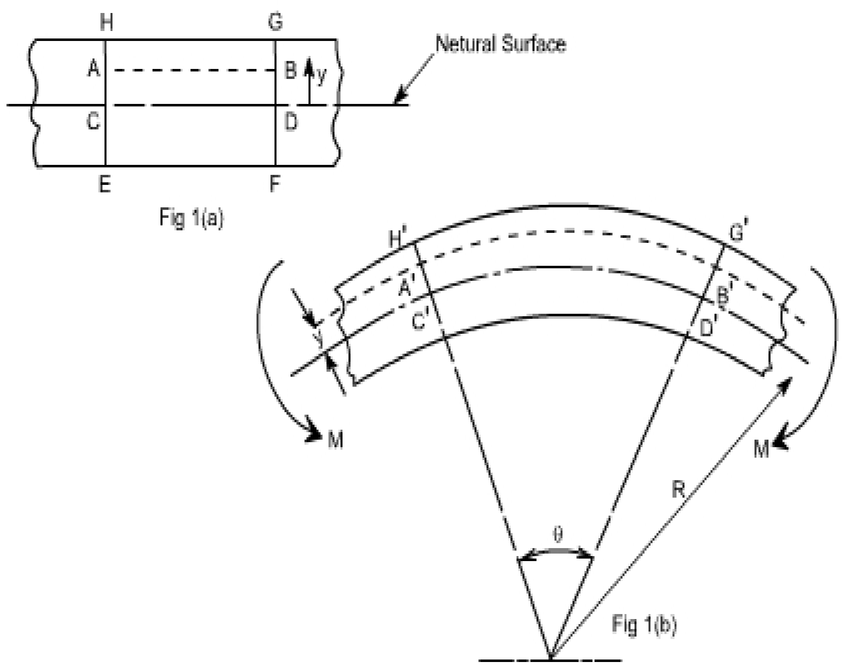
4. Loading is within elastic limit.

5. Plane cross - sections remain plane before and after bending.

6. Young's modulus for beam material is same in tension and compression.

7. Radius of curvature of beam is large enough in comparison to its cross sectional dimensions.

Bending Equation:



Let us consider a beam initially unstressed as shown in fig 1(a). Now the beam is subjected to a constant bending moment (i.e. 'Zero Shearing Force') along its length as would be obtained by applying equal couples at both the ends. The beam will bend to the radius R as shown in Fig 1(b).

As a result of this bending, the top fibers of the beam will be subjected to tension and the bottom fibers to compression. It is reasonable to suppose that there is a fiber somewhere between the top and bottom fibers at which the stress is zero. This fiber is known as neutral fiber or neutral layer or neutral surface.

In order to compute the value of bending stresses developed in a loaded beam, let us consider the two cross-sections of a beam **HE** and **GF**, originally parallel as shown in fig 1(a).when the beam is to bend it is assumed that these sections remain parallel i.e. **H'E'** and **G'F'**, the final position of the sections, are still straight lines, they then subtend some angle θ.

Consider now fiber AB in the material, at a distance y from the N.A, when the beam bends this will stretch to A'B'

Therefore,

$$\text{strain in fibre AB} = \frac{\text{change in length}}{\text{orginal length}}$$

$$= \frac{A'B' - AB}{AB} \qquad \text{But } AB = CD \text{ and } CD = C'D'$$

refer to fig1(a) and fig1(b)

$$\therefore \text{strain} = \frac{A'B' - C'D'}{C'D'}$$

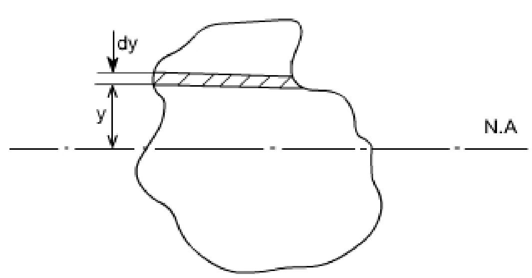
Since CD and C'D' are on the neutral axis and it is assumed that the Stress on the neutral axis zero. Therefore, there won't be any strain on the neutral axis

$$= \frac{(R+y)\theta - R\theta}{R\theta} = \frac{R\theta + y\theta - R\theta}{R\theta} = \frac{y}{R}$$

$$\text{However } \frac{\text{stress}}{\text{strain}} = E \qquad \text{where } E = \text{Young's Modulus of elasticity}$$

Therefore, equating the two strains as obtained from the two relations i.e,

$$\frac{\sigma}{E} = \frac{y}{R} \text{ or } \frac{\sigma}{y} = \frac{E}{R} \qquad \ldots\ldots\ldots\ldots(1)$$



Consider any arbitrary cross-section of beam, as shown above now the strain on a fiber at a distance ‘y' from the N.A, is given by the expression

$$\sigma = \frac{E}{R} y$$

if the shaded strip is of area'dA'
then the force on the strip is

$$F = \sigma\, \delta A = \frac{E}{R} y\, \delta A$$

Moment about the neutral axis would be $= F.y = \frac{E}{R}\, y^2 \delta A$

The toatl moment for the whole
cross-section is therefore equal to

$$M = \sum \frac{E}{R} y^2\ \delta A = \frac{E}{R} \sum y^2 \delta A$$

Now the term $\sum y^2 \delta A$ is the property of the material and is called as a second moment of area of the cross-section and is denoted by a symbol I.

Therefore

$$M = \frac{E}{R} I \qquad ....(2)$$

combining equation 1 and 2 we get

$$\boxed{\frac{\sigma}{y} = \frac{M}{T} = \frac{E}{R}}$$

This is the Bending Equation for pure bending.

Section Modulus:

From the simple bending theory equation, the maximum stress obtained in any cross-section is given as

$$\sigma_{max^m} = \frac{M}{I}\, y_{max^m}$$

For any given allowable stress the maximum moment which can be accepted by a particular shape of cross-section is therefore

$$M = \frac{I}{y_{max^m}}\, \sigma_{max^m}$$

For ready comparison of the strength of various beam cross-section this relationship is sometimes written in the form

$$M = Z\, \sigma_{max^m} \quad \text{where} \quad Z = \frac{I}{y_{max^m}}$$

is termed as section modulus.

The higher the value of Z for a particular cross-section, the greater will be the bending moment which it can withstand for a given maximum stress.