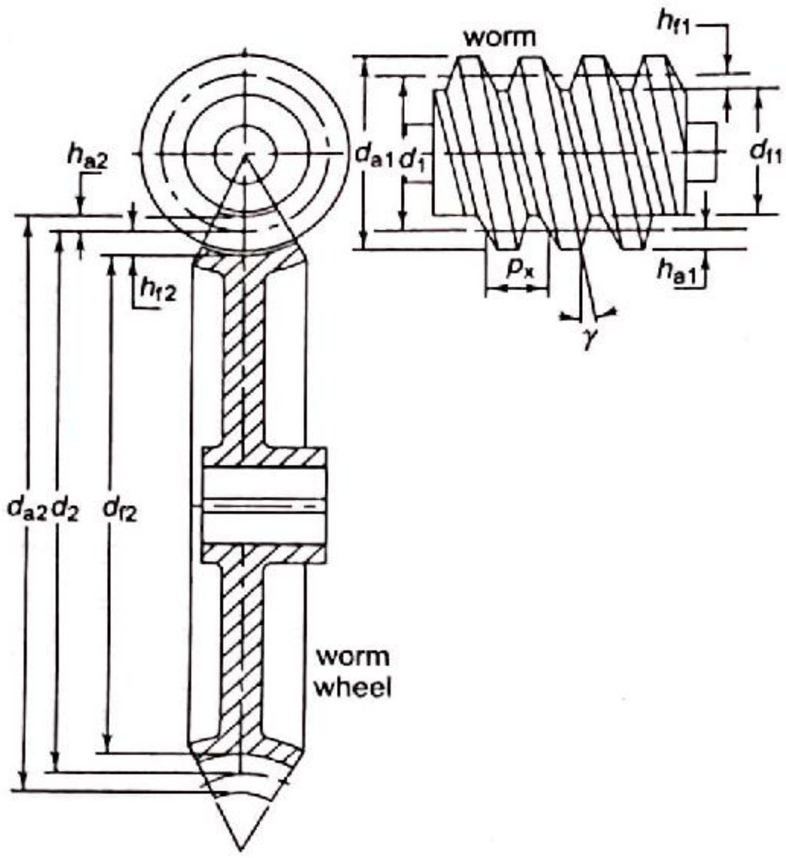
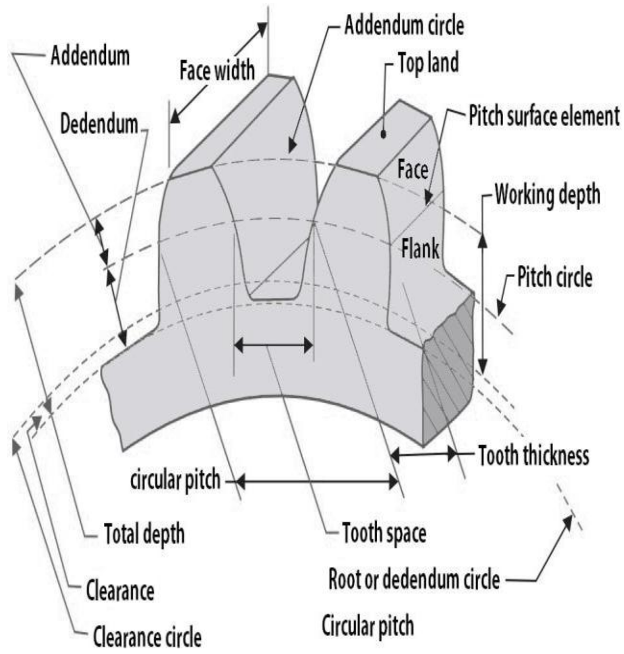



**Fig. 20.2** *Dimension of worm gears*




## 20.3 PROPORTIONS OF WORM GEARS

The basic dimensions of the worm and the worm wheel are shown in Fig. 20.2. For an involute helicoidal tooth form,

$$h_{a1} = m \tag{20.11}$$

$$h_{f1} = (2.2 \cos \gamma - 1)\, m \tag{20.12}$$

$$c = 0.2\, m \cos \gamma \tag{20.13}$$

where,

$h_{a1}$ = addendum (mm)

$h_{f1}$ = dedendum (mm)

$c$ = clearance (mm)

The outside and root diameters of the worm are, therefore, expressed as follows:

$$d_{a1} = d_1 + 2h_{a1} = qm + 2m$$

or $$d_{a1} = m(q+2) \tag{20.14}$$

$$d_{f1} = d_1 - 2h_{f1} = qm - 2m(2.2 \cos \gamma - 1)$$

$$d_{f1} = m(q + 2 - 4.4 \cos \gamma) \tag{20.15}$$

where

$d_{a1}$ = outside diameter of the worm (mm)

$d_{f1}$ = root diameter of the worm (mm)

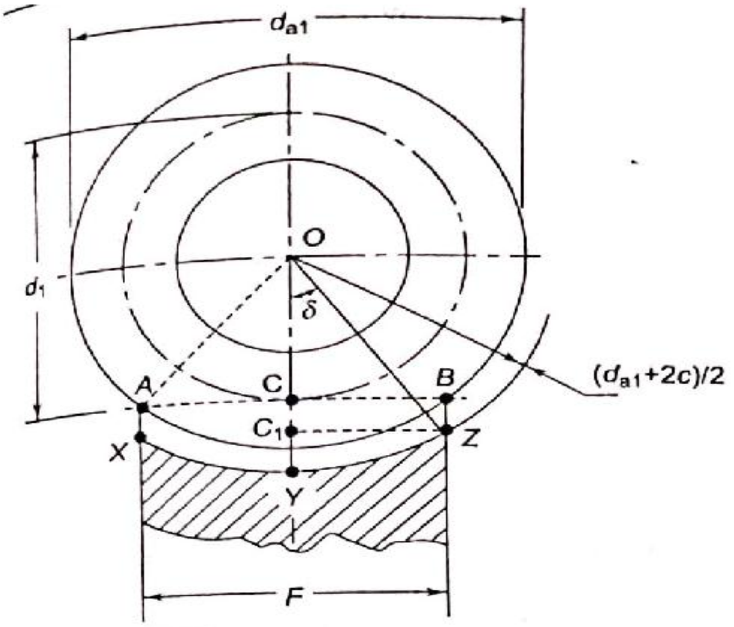



**Fig. 20.3** *Face width of worm wheel*

From triangle $AOC$,

$$(\overline{AC})^2 = (\overline{AO})^2 - (\overline{OC})^2$$

or
$$\left(\frac{F}{2}\right)^2 = \left(\frac{d_{a1}}{2}\right)^2 - \left(\frac{d_1}{2}\right)^2 = \left[\frac{m(q+2)}{2}\right]^2 - \left[\frac{qm}{2}\right]^2$$

$$\therefore \quad F = 2m\sqrt{(q+1)} \qquad (20.20)$$

From triangle $OZC_1$,

$$\sin\delta = \frac{C_1Z}{OZ} = \frac{F/2}{(d_{a1}+2c)/2}$$

or
$$\delta = \sin^{-1}\left(\frac{F}{d_{a1}+2c}\right)$$

The length of the root of the worm wheel teeth is arc $XYZ$, that is denoted by $(l_r)$,

$$l_r = \text{arc } XYZ = \left(\frac{2\delta}{2\pi}\right)[\pi(d_{a1}+2c)]$$

$$= (d_{a1}+2c)\,\delta$$

$$l_r = (d_{a1}+2c)\sin^{-1}\left[\frac{F}{(d_{a1}+2c)}\right] \qquad (20.21)$$

**Example 20.1** A pair of worm gears is designated as,

1/30/10/8

Calculate:

(i) the centre distance;
(ii) the speed reduction;
(iii) the dimensions of the worm; and
(iv) the dimensions of the worm wheel

*Solution* For the given pair,

$z_1 = 1 \quad z_2 = 30$ teeth $\quad q = 10 \quad m = 8$ mm

From Eq. (20.9) and (20.10),

$$a = \frac{1}{2}m(q+z_2) = \frac{1}{2}(8)(10+30) = 160 \text{ mm} \qquad \text{(i)}$$

$$i = \frac{z_2}{z_1} = 30 \qquad \text{(ii)}$$

Dimensions of worm:

$$d_1 = qm = 10(8) = 80 \text{ mm} \qquad \text{(a)}$$

$$d_{a1} = m(q+2) = 8(10+2) = 96 \text{ mm} \qquad \text{(b)}$$

$$\tan\gamma = \frac{z_1}{q} = \frac{1}{10} \quad \text{or} \quad \gamma = 5.71°$$

$$d_{f1} = m(q+2-4.4\cos\gamma) = 8[10+2-4.4\cos(5.71°)]$$
$$= 60.9747 \qquad \text{(c)}$$

$$p_x = \pi m = \pi(8) = 25.1327 \text{ mm} \qquad \text{(d)}$$

Dimensions of worm wheel:

$$d_2 = mz_2 = 8(30) = 240 \text{ mm} \qquad \text{(a)}$$

$$d_{a2} = m(z_2 + 4\cos\gamma - 2) = 8[30 + 4\cos(5.71°) - 2]$$
$$= 255.8412 \text{ mn} \qquad \text{(b)}$$

$$d_{f2} = m(z_2 - 2 - 0.4\cos\gamma) = 8[30 - 2 - 0.4\cos(5.71°)]$$
$$= 220.8159 \text{ mm} \qquad \text{(c)}$$

## 20.4 FORCE ANALYSIS

The analysis of three components of the resultant tooth force between meshing teeth of worm and worm wheel is based on following assumptions:

(i) The worm is the driving element, while the worm wheel is the driven element.

(ii) The worm has right-handed threads.

(iii) The worm rotates in anti-clockwise directions as shown in Fig. 20.4.

The three components of the gear tooth force between the worm and the worm wheel are shown in Fig. 20.4. Suffix 1 is used for the worm, while suffix 2 for the worm wheel. The components of the resultant force acting on the worm are as follows:

$(P_1)_t$ = tangential component on the worm (N)

$(P_1)_a$ = axial component on the worm (N)

$(P_1)_r$ = radial component on the worm (N)

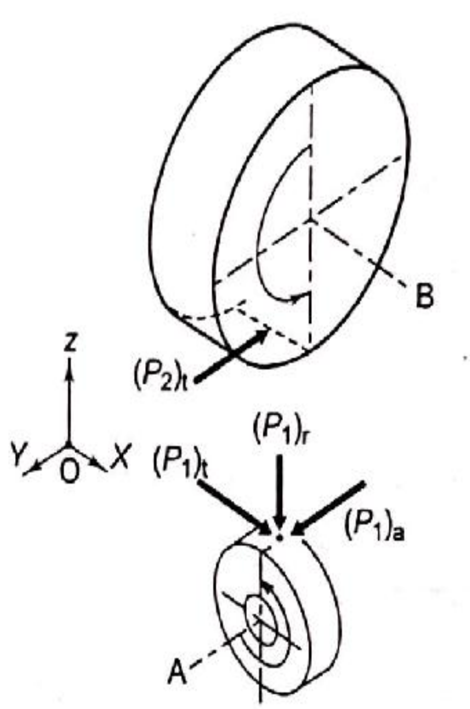



**Fig. 20.5** *Direction of force components*

The components $(P_2)_t$, $(P_2)_a$ and $(P_2)_r$ acting on the worm wheel are defined in a similar way. The force acting on the worm wheel is equal and opposite reaction of the force acting on the worm.

Therefore,

$$(P_2)_t = -(P_1)_a \quad (20.22)$$

$$(P_2)_a = -(P_1)_t \quad (20.23)$$

$$(P_2)_r = -(P_1)_r \quad (20.24)$$

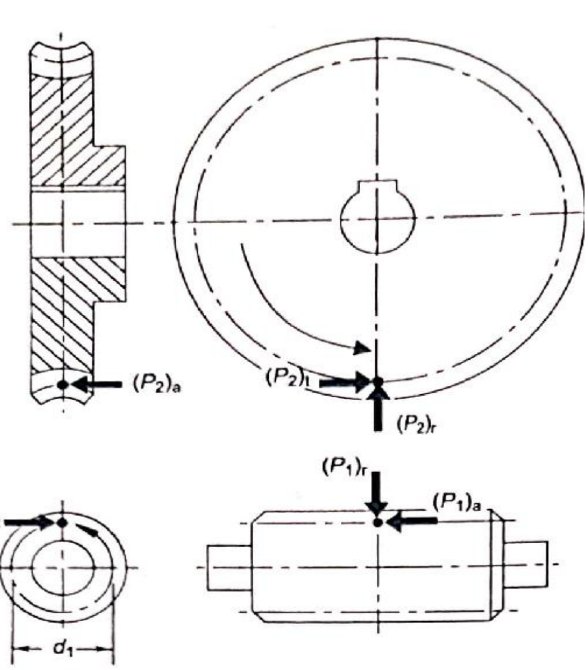



**Fig. 20.4** *Components of tooth force*

# FORCE COMPONENTS ON WORM

The components of the normal reaction $P$ acting on the worm are shown in Fig. 20.6 $\alpha$ is normal pressure angle, while $\gamma$ is the lead angle. Note that angle $\alpha$ is in plane $ABCD$ shaded by dots, while angle $\gamma$ is in top plane $AEBF$. Resolving the normal reaction $P$ in the plane $ABCD$ shown in Fig. 20.6(b),

$$P_N = P\cos\alpha \qquad \text{(a)}$$

$$P_r = P\sin\alpha \qquad \text{(b)}$$

Resolving the component $P_N$ in the plane $AEBF$ shown in Fig. 20.6(c),

$$P_a = P_N\cos\gamma \qquad \text{(c)}$$

$$P_t = P_N\sin\gamma \qquad \text{(d)}$$

From relationships (a), (b), (c) and (d),

$$P_t = P\cos\alpha\sin\gamma$$

$$P_a = P\cos\alpha\cos\gamma$$

$$P_r = P\sin\alpha \qquad (20.25)$$

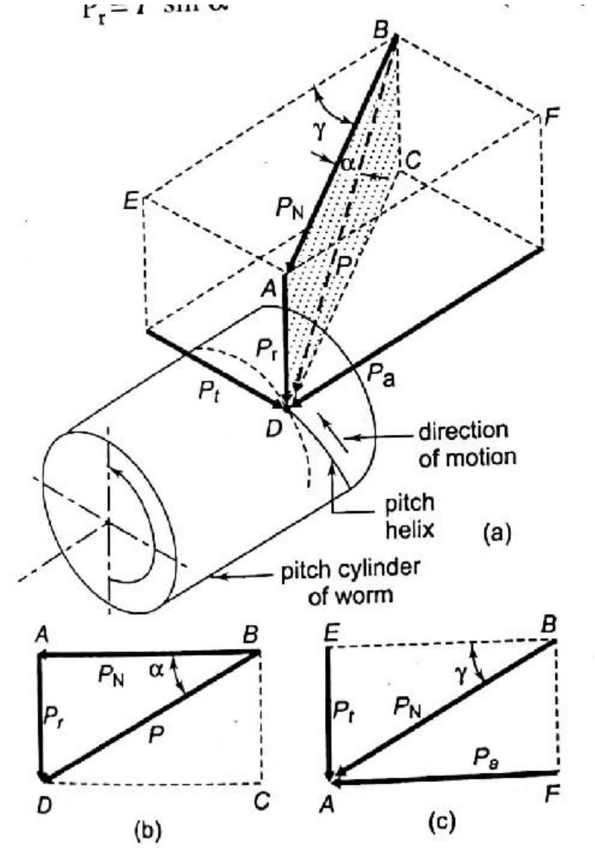
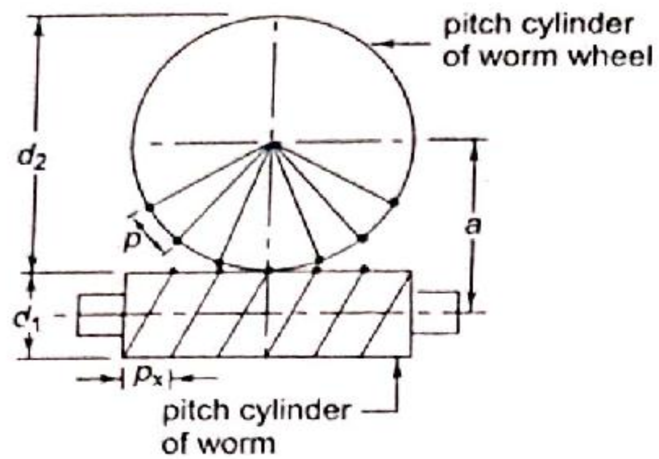
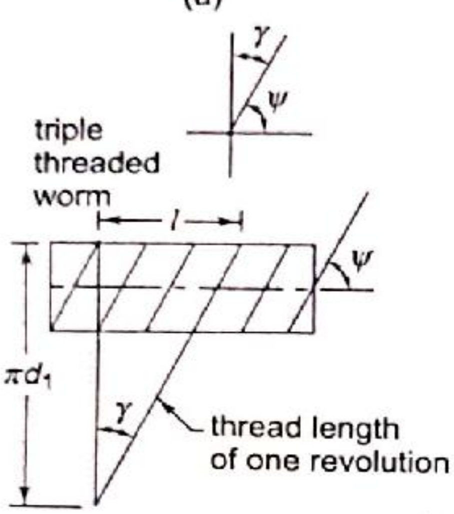


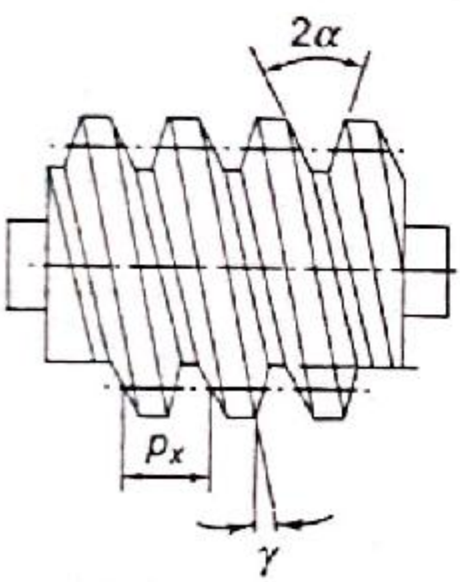

**Fig. 20.6** *Components of normal reaction*




(a)




(b)



# FRICTION FORCE COMPONENTS

The resultant frictional force is ($\mu P$) where $\mu$ is the coefficient of friction. The direction of the frictional force will be along the pitch helix and opposite to the direction of rotation, as shown in Fig. 20.7. There are two components of the frictional force:

(i) Component ($\mu P \cos \gamma$) in the tangential direction. The direction of this component is same as that of $P_t$.

(ii) Component ($\mu P \sin \gamma$) in the axial direction. The direction of this component is opposite to that of $P_a$.

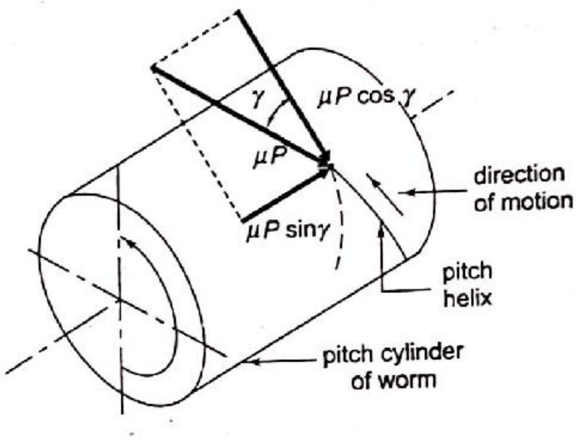



**Fig. 20.7** *Components of frictional force*

Superimposing the components of normal reaction and frictional force, we have

$$(P_1)_t = P \cos \alpha \sin \gamma + \mu P \cos \gamma$$

$$\therefore \quad (P_1)_t = P(\cos \alpha \sin \gamma + \mu \cos \gamma) \qquad (20.26)$$

Similarly, $(P_1)_a = P\cos \alpha \cos \gamma - \mu(\sin \gamma)$

$$\therefore \quad (P_1)_a = P(\cos \alpha \cos \gamma - \mu \sin \gamma \qquad (20.27)$$

and $\quad (P_1)_r = P\sin \alpha \qquad (20.28)$

In practice, the tangential component $(P_1)_t$ on the worm is determined from the torque that is transmitted from the worm to the worm wheel. Therefore,

$$(P_1)_t = \frac{2 M_t}{d_1} \qquad (20.29)$$

From Eqs (20.26) and (20.27),

$$(P_1)_a = (P_1)_t \times \frac{(\cos \alpha \cos \gamma - \mu \sin \gamma)}{(\cos \alpha \sin \gamma + \mu \cos \gamma)} \qquad (20.30)$$

From Eqs (20.26) and (20.28),

$$(P_1)_r = (P_1)_t \times \frac{\sin \alpha}{(\cos \alpha \sin \gamma + \mu \cos \gamma)} \qquad (20.31)$$

Equations (20.29), (20.30) and (20.31) are used to determine the magnitude of components of the resultant tooth force.

**NUMERICAL**

**Example 20.2** A pair of worm and worm wheel is designated as

3/60/10/6

The worm is transmitting 5 kW power at 1440 rpm to the worm wheel. The coefficient of friction is 0.1 and the normal pressure angle is 20°. Determine the components of the gear tooth force acting on the worm and the worm wheel.

*Solution*

$z_1 = 3 \quad z_2 = 60$ teeth $\quad q = 10 \quad m = 6$ mm

$d_1 = qm = 10(6) = 60$ mm

$$\tan\gamma = \frac{z_1}{q} = \frac{3}{10} = 0.3 \text{ or } \quad \gamma = 16.7°$$

$$M_t = \frac{60 \times 10^6 (\text{kW})}{2\pi n_1} = \frac{60 \times 10^6 (5)}{2\pi(1440)}$$

$$= 33157.28 \text{ N} - \text{mm}$$

From Eq. (20.29),

$$(P_1)_t = \frac{2M_t}{d_1} = \frac{2(33157.28)}{60} = 1105.24 \text{ N} \quad \text{(a)}$$

From Eqs (20.30),

$$(P_1)_a = (P_1)_t \times \frac{(\cos\alpha \cos\gamma - \mu \sin\gamma)}{(\cos\alpha \sin\gamma + \mu \cos\gamma)}$$

$$= 1105.24 \times \frac{[\cos(20)\cos(16.7) - 0.1\sin(16.7)]}{[(\cos(20)\sin(16.7) + 0.1\cos(16.7)]}$$

$$= 2632.55\text{N} \quad \text{(b)}$$

From Eq. (20.31),

$$(P_1)_r = (P_1)_t \times \frac{\sin\alpha}{(\cos\alpha \sin\gamma + \mu \cos\gamma)}$$

$$= 1105.24 \times \frac{\sin(20)}{[\cos(20)\sin(16.7) + 0.1\cos(16.7)]}$$

$$= 1033{\cdot}35 \text{ N} \quad \text{(c)}$$

The force components acting on the worm wheel are as follows (Eqs. 20.22 to 20.24),

$(P_2)_t = (P_1)_a = 2632.55$ N

$(P_2)_a = (P_1)_t = 1105.24$ N

$(P_2)_r = (P_1)_r = 1033.35$ N

**THANK YOU**

**Question Session**

**?**