

**Centre of Excellence in Renewable Energy Education and Research,
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Module REC-202: Wind Energy Conversion Systems**

(Unit-2) Contents

Aerodynamic design principles; Aerodynamic theories; Axial momentum, blade element and combine theory.

Rotor characteristics: Solidity, Tip speed ratio, Tip loss correction, Maximum Power coefficient; Dynamic matching, Extension of linear momentum theory, Power extraction by a turbine.

Airfoils and General Concepts of Aerodynamics

Wind turbine blades use airfoils to develop mechanical power. The cross-sections of wind turbine blades have the shape of airfoils. The width and length of the blade are functions of the desired aerodynamic performance, the maximum desired rotor power, the assumed airfoil properties and strength considerations. Before the details of wind turbine power production are explained, aerodynamic concepts related to airfoils need to be reviewed.

Airfoil terminology

A number of terms are used to characterize an airfoil, as shown in Figure. The **mean camber line** is the locus of points halfway between the upper and lower surfaces of the airfoil. The most forward and rearward points of the mean camber line are on the **leading and trailing edges**, respectively. The straight line connecting the leading and trailing edges is the **chord line** of the airfoil, and the distance from the leading to the trailing edge measured along the chord line is designated as the **chord (c)**, of the airfoil. The **camber** is the distance between the mean camber line and the chord line, measured perpendicular to the chord line. The **thickness** is the distance between the upper and lower surfaces, also measured perpendicular to the chord line. Finally, the **angle of attack (α)**, is defined as the angle between the relative wind and the chord line. Not shown in the figure is the **span** of the airfoil, which is the length of the airfoil perpendicular to its cross-section. The geometric parameters that have an effect on the aerodynamic performance of

an airfoil include: the leading edge radius, mean camber line, maximum thickness and thickness distribution of the profile and the trailing edge angle.

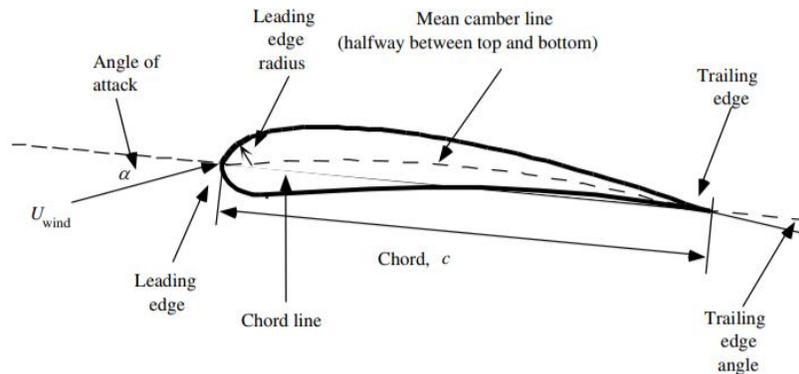


Fig: Airfoil Nomenclature

Aerodynamics

Air flow over a stationary airfoil produces two forces, a **lift force** perpendicular to the air flow and a **drag force** in the direction of air flow, as shown in Fig. 1. The existence of the lift force depends upon laminar flow over the airfoil, which means that the air flows smoothly over both sides of the airfoil. If turbulent flow exists rather than laminar flow, there will be little or no lift force. The air flowing over the top of the airfoil has to speed up because of a greater distance to travel and this increase in speed causes a slight decrease in pressure. This pressure difference across the airfoil yields the lift force, which is perpendicular to the direction of air flow.

The air moving over the airfoil also produces a drag force in the direction of the air flow. This is a loss term and is minimized as much as possible in high performance wind turbines.

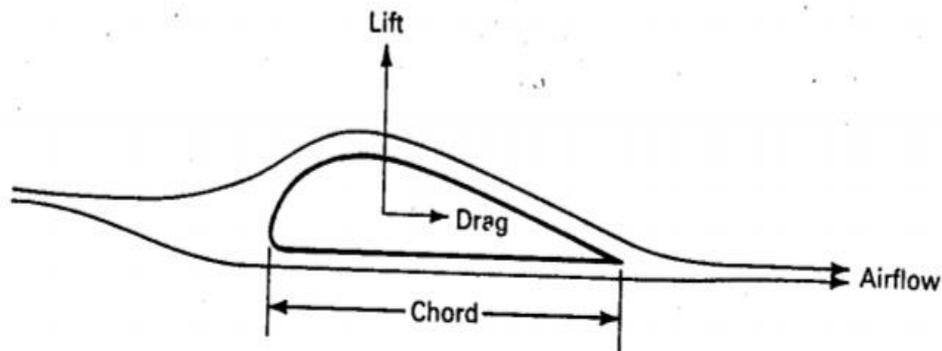


Fig: 1 Lift and Drag on a Stationary airfoil

Both the lift and the drag are proportional to the air density, the area of the airfoil, and the square of the wind speed.

Suppose that we allow the airfoil to move in the direction of the lift force. This motion or translation will combine with the motion of the air to produce a relative wind direction shown in Fig. 2. The airfoil has been reoriented to maintain a good lift to drag ratio. The lift is perpendicular to the relative wind but is not in the direction of airfoil translation.

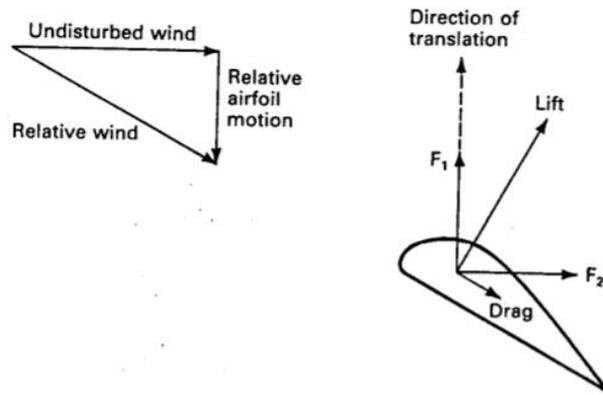


Fig 2: Lift and drag on a translating airfoil.

The lift and drag forces can be split into components parallel and perpendicular to the direction of the undisturbed wind, and these components combined to form the net force F_1 in the direction of translation and the net force F_2 in the direction of the undisturbed wind. The force F_1 is available to do useful work. The force F_2 must be used in the design of the airfoil supports to assure structural integrity.

A practical way of using F_1 is to connect two such airfoils or blades to a central hub and allow them to rotate around a horizontal axis, as shown in Fig. 3. The force F_1 causes a torque which drives some load connected to the propeller. The tower must be strong enough to withstand the force F_2 .

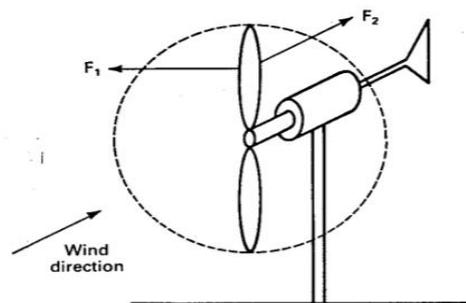


Fig 3: Aerodynamic forces on a turbine blade

These forces and the overall performance of a wind turbine depend on the construction and orientation of the blades. One important parameter of a blade is the **pitch angle**, which is the angle between the chord line of the blade and the plane of rotation, as shown in Fig. 4. The **chord line** is the straight line connecting the leading and trailing edges of an airfoil. The **plane of rotation** is the plane in which the blade tips lie as they rotate. The blade tips actually trace out a circle which lies on the plane of rotation. Full power output would normally be obtained when the wind direction is perpendicular to the plane of rotation. The pitch angle is a static angle, depending only on the orientation of the blade.

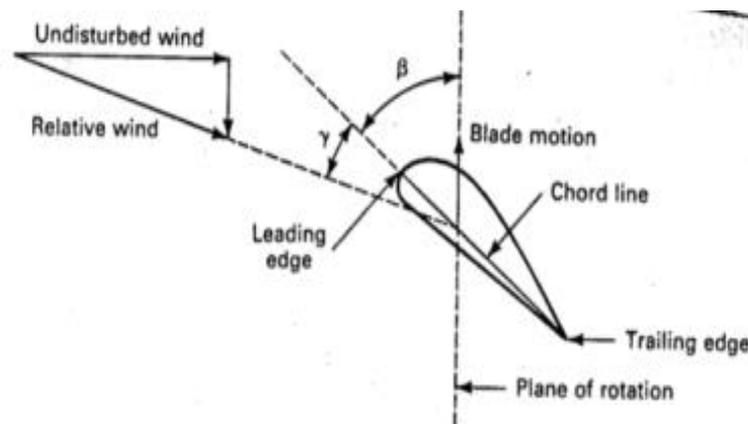


Fig 4: Definition of pitch angle β and angle of attack α .

Another important blade parameter is the **angle of attack**, which is the angle $\alpha(\gamma)$ between the chord line of the blade and the relative wind or the effective direction of air flow. It is a dynamic angle, depending on both the speed of the blade and the speed of the wind. The blade speed at a distance r from the hub and an angular velocity ω_m is $r\omega_m$.

A blade with twist will have a variation in angle of attack from hub to tip because of the variation of $r\omega_m$ with distance from the hub. The lift and drag have optimum values for a single angle of attack so a blade without twist is less efficient than a blade with the proper twist to maintain a nearly constant angle of attack from hub to tip. Even the blades of them old Dutch windmills were twisted to improve the efficiency. Most modern blades are twisted, but some are not for cost reasons. A straight blade is easier and cheaper to build and the cost reduction may more than offset the loss in performance.

When the blade is twisted, the pitch angle will change from hub to tip. In this situation, the pitch angle measured three fourths of the distance out from the hub is selected as the reference.

Airfoil section

Airfoil is profiled to have very small drag. A typical airfoil having 200 mm chord and 15% thickness has drag equivalent to a wire of 1mm diameter. It is characterized by coefficient of lift, C_L and coefficient of drag, C_D that are function of angle of attack. Refer Figure 5. C_L and C_D are defined as,

$$C_L = \frac{F_L}{2\rho AV^2}; \quad C_D = \frac{F_D}{2\rho AV^2}$$

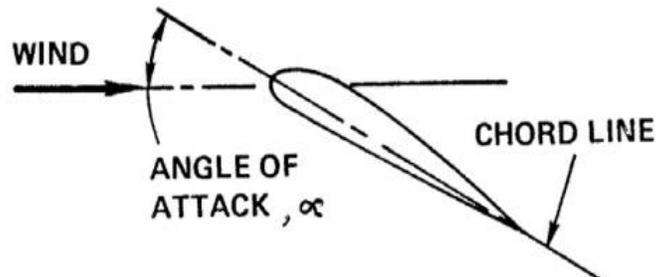


Fig 5: Definition of chord and angle of attack for an airfoil section

Different types of airfoil sections are shown in Figure 6.

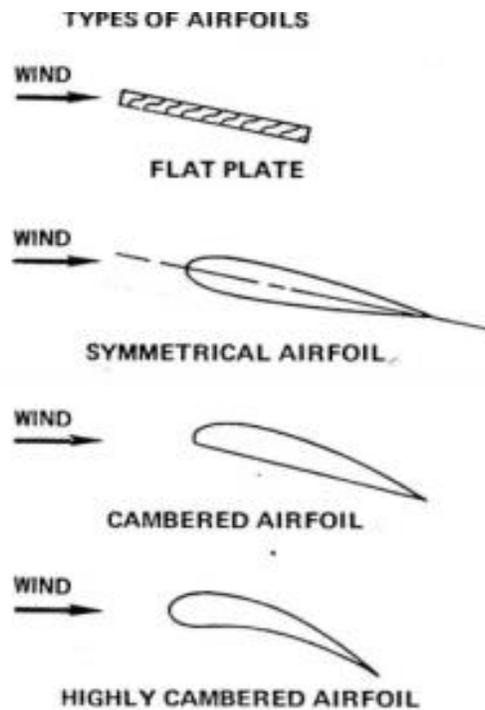


Fig 6: Types of airfoil.

One-Dimensional Momentum Theory and the Betz Limit -

It is used to determine the power from an ideal turbine rotor, the thrust of the wind on the ideal rotor and the effect of the rotor operation on the local wind field.

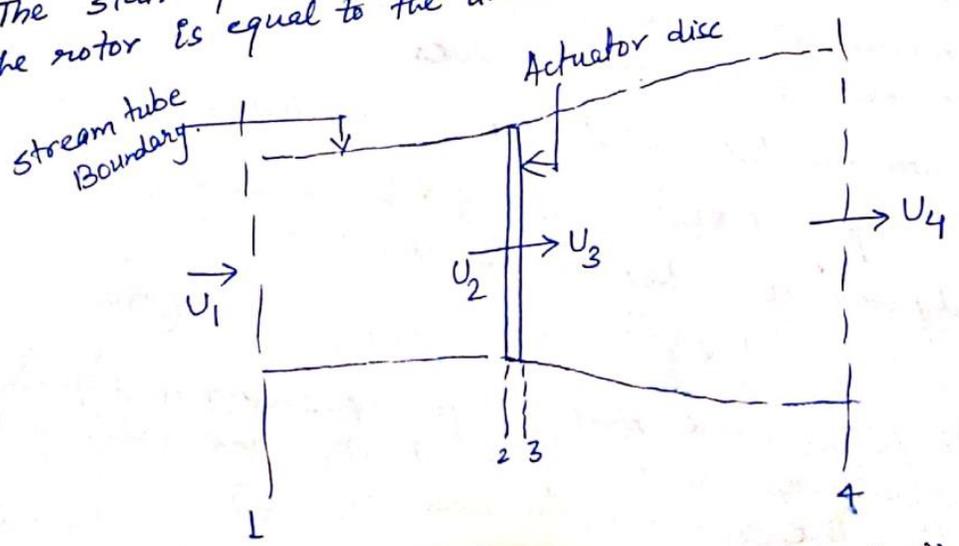
This simple model is based on a linear momentum theory to predict the performance of propellers.

The analysis assumes a control volume, in which the control volume boundaries are the surface of a stream tube and two cross-sections of the stream tube, The flow only flow is across the ends of the stream tube.

The turbine is represented by a uniform "actuator disk" which creates a discontinuity of pressure in the stream tube of air flowing through it.

The Analysis uses the following assumptions :-

- * Homogeneous, incompressible, steady state fluid flow.
- * No frictional drag.
- * An infinite No. of blades
- * Uniform thrust over the disk or rotor area.
- * The static pressure far upstream and far downstream of the rotor is equal to the undistributed ambient static pressure



Actuator disc of a wind Turbine; U = mean air velocity; 1, 2, 3 and 4 indicates location.

Applying the conservation of linear momentum to the control volume enclosing the whole system, The net force is equal and opposite to the thrust F_T , which is the force of the wind on the wind turbine.

From the conservation of linear momentum for a 1-D, incompressible, steady flow, the thrust is equal and opposite to the change in momentum of air stream:

$$F_T = U_1 (\rho A)_1 - U_4 (\rho A)_4 \quad \text{--- (1)}$$

where ρ is the air density, A is the cross sectional area, U is the air velocity.

For steady state flow, $(\rho A)_1 = (\rho A)_4 = \dot{m}$, where \dot{m} is the mass flow rate.

Therefore eqⁿ (1) can be written as

$$F_T = \dot{m} (U_1 - U_4) \quad \text{--- (2)}$$

The thrust is positive so the velocity behind the rotor, U_4 is less than the free stream velocity, U_1 . No work is done on either side of the rotor.

The Bernoulli eqⁿ can be used in the two control volume on either side of the actuator disk.

In the stream tube upstream of the disk:—

$$P_1 + \frac{1}{2} \rho U_1^2 = P_2 + \frac{1}{2} \rho U_2^2 \quad \text{--- (3)}$$

similarly for downstream of the disk:—

$$P_3 + \frac{1}{2} \rho U_3^2 = P_4 + \frac{1}{2} \rho U_4^2 \quad \text{--- (4)}$$

where it is assumed that the far upstream and far downstream pressure are equal ($P_1 = P_4$) and velocity across the disk remains the same. ($U_2 = U_3$)

The thrust can also be expressed as the net sum of the forces on each side of the actuator disc.

$$F_T = A_2 (P_2 - P_3) \quad \text{--- (5)}$$

from eqⁿ (3), (4) & (5)

$$F_T = \frac{1}{2} \rho A_2 (U_1^2 - U_4^2) \quad \text{--- (6)}$$

from eqⁿ (2) & (6) [equating thrust force]

$$\rho A_2 U_2 (U_1 - U_4) = \frac{1}{2} \rho A_2 (U_1^2 - U_4^2)$$

$$\boxed{U_2 = \frac{1}{2} (U_1 + U_4)} \quad \text{--- (7)}$$

Thus, the wind velocity at the rotor plane, using the Axial momentum theory, the average of upstream and downstream wind speeds.

Axial Induction factor (a) — It is the fractional decrease in wind velocity between the free stream and the rotor plane.

$$a = \frac{U_1 - U_2}{U_1}$$

$$U_2 = U_1 (1 - a)$$

$$U_4 = U_1 (1 - 2a)$$

$$\left. \begin{array}{l} U_2 = U_1 (1 - a) \\ U_4 = U_1 (1 - 2a) \end{array} \right\} \text{--- (8) } \left\{ \text{from eqⁿ (7)} \right.$$

The quantity, $U_1 a$ is often referred to as the induced velocity at the rotor, in which case velocity of the wind at the rotor is a combination of the free stream velocity and the induced wind velocity.

The power out P , is equal to the thrust times the ^{velocity} velocity at the disc.

$$\begin{aligned} \text{Power} &= \text{Thrust force} \times \text{velocity at rotor} \\ &= \frac{1}{2} \rho A_2 (U_1^2 - U_4^2) \times U_2 \\ &= \frac{1}{2} \rho A_2 U_2 (U_1 - U_4) (U_1 + U_4) \end{aligned}$$

Substitute the value of U_2 & U_4 from eqⁿ 8 in above eqⁿ.

$$\begin{aligned} \text{Power} &= \frac{1}{2} \rho A_2 [U_1(1-a)] [\{U_1 - U_1(1-2a)\}] [\{U_1 + U_1(1-2a)\}] \\ &= \frac{1}{2} \rho A_2 U_1^3 [1-a] [1 - 1 + 2a] [1 + 1 - 2a] \\ &= \frac{1}{2} \rho A_2 U_1^3 (1-a) (2a) \times 2(1-a) \\ &= \frac{1}{2} \rho A_1 U_1^3 4a(1-a)^2 \end{aligned}$$

$$\boxed{P = \frac{1}{2} \rho A U^3 4a(1-a)^2} \quad \text{--- (9)}$$

where the control volume at the rotor A_2 is replaced with A , the rotor area, and the free stream velocity U_1 is replaced by U .

Wind turbine rotor performance is usually characterized by its power coefficient, C_p :

$$C_p = \frac{\text{Rotor Power}}{\text{Power in the wind}} = \frac{P}{\frac{1}{2} \rho A U^3} \quad \text{--- (10)}$$

The non-dimensional power coefficient represents the fraction of the power in the wind that is extracted by the rotor.
from eqⁿ (9) & (10)

$$C_p = 4a(1-a)^2$$

The max C_p is determined by taking the derivative of the power coefficient with respect to 'a' and equating to zero.

$$\frac{dC_p}{da} = 0 \Rightarrow \frac{d}{da} [4a(1-a)^2] = 0$$

$$4a \cdot 2(1-a)(-1) + (1-a)^2 \times 4 = 0$$

$$\& 2a = (1-a)$$

$$3a = 1 \Rightarrow \boxed{a = \frac{1}{3}}$$

$$\boxed{C_{p, \max} = \frac{16}{27} = 0.5926}$$

Betz coefficient -
This results indicates that, if an ideal rotor were designed and operated such that the wind speed at the rotor were $\frac{2}{3}$ of the free stream wind speed, then it would be operating at the point of Max. Power production, this is the max. power possible.

from equation (6) and (8)

$$\begin{aligned} F_T &= \frac{1}{2} \rho A (U_1^2 - U_2^2) \\ &= \frac{1}{2} \rho A U_1^2 [1 - (1-a)^2] \\ &= \frac{1}{2} \rho A U_1^2 [1 - 1 + 2a - a^2] \\ &= \frac{1}{2} \rho A U_1^2 [2a - a^2] \end{aligned}$$

$$\left\{ U_2 = U_1 (1-a) \right.$$

$$\boxed{F_T = \frac{1}{2} \rho A U_1^2 [4a(1-a)]} \quad \text{--- (11)}$$

Similarly to the power, the thrust on the wind turbine can be characterized by a non-dimensional thrust coefficient

$$C_T = \frac{F_T}{\frac{1}{2} \rho U^2 A} = \frac{\text{Thrust force}}{\text{Dynamic force}} \quad \text{--- (12)}$$

from eqn (1) & (2) $C_T = 4a(1-a)$

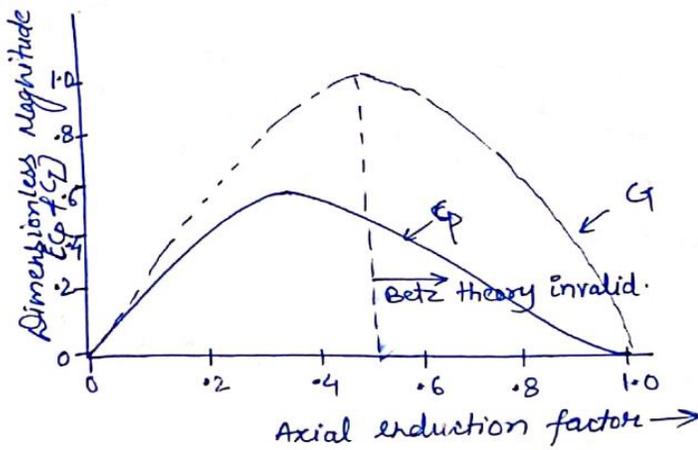
∴ The thrust coefficient for an ideal wind turbine is equal to $4a(1-a)$.

C_T has max of 1, when $a = 0.5$

i.e. $U_4 = U_1(1-2a) = 0$ [Downstream velocity is zero]
i.e. Not a feasible case.

At maximum power output $a = \frac{1}{3}$, $C_T = \frac{8}{9}$

A graph of the power and thrust coefficients for an ideal Betz turbine —



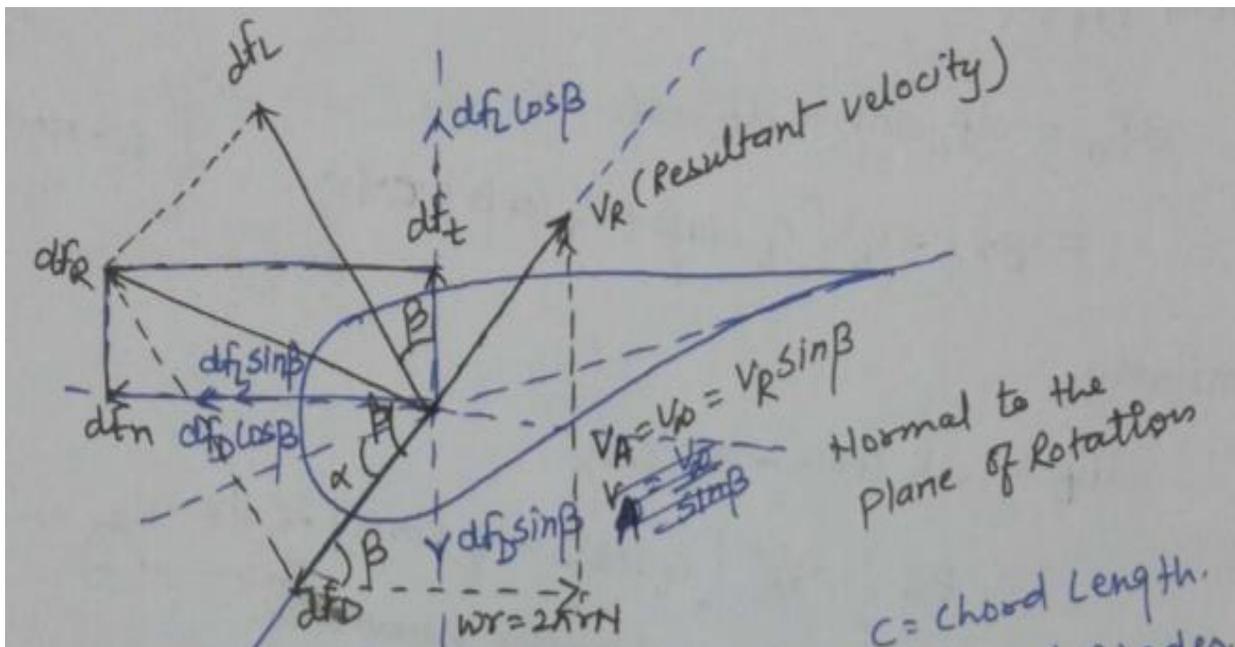
This idealized model is not valid for axial induction factors greater than 0.5.

The Betz limit, $C_{p, \max} = \frac{16}{27}$, is the max. theoretically possible rotor power coefficient.

Blade Element Momentum (BEM) and Combine Theory

Actuator disc theory (Momentum Theory) provides simple formula to calculate the power extracted and thrust acting on the wind turbine rotor.

- It provides a theoretical limit on the power that can be extracted from the wind.
- However, it is unable to predict the performance of wind turbine rotor blades as a function of the rotor design parameter such as:
 - i. Rotor Radius
 - ii. Number of Blades
 - iii. Blade Chord
 - iv. Blade Twist
 - v. Airfoil Section Shape



- The angle of attack of the airfoil section on the rotor is the angle between the airfoil chord line and the resultant velocity airfoil section experiences.
- With rotation, the resultant velocity, V_R is made up of the vector sum of the wind speed and the rotation speed of the blade section.

$$V_R = \sqrt{[U_\infty(1-a)]^2 + [\omega r(1+a')]^2}$$

Note: Both the wind speed and rotational velocity are modified by the axial and angular induction factor.

BEM Nomenclature:

α = Angle of attack

β = Resultant angle

ω = Angular speed of the rotar (rad/sec.)

ρ = Density of air

R = Radius of rotor

c = Chord length

B = No. of blades

V_A = Wind velocity

V_R = Resultant velocity

dF_n = Elemental normal force, normal to the plane of rotation

dF_t = Elemental tangential force, in the plane of rotation

dF_D = Elemental drag force

dF_L = Elemental lift force

dF_R = Resultant force

dA = Elemental Area = $c \cdot dr$

$d\tau$ = Differential torque

dP = Differential power

C_D = Coefficient of drag

C_L = Coefficient of lift

- The thrust force acting on any section of the rotor blade section acts normal to the plane of rotation of the blade.
- The torque on any section of the rotor blade equals the net aerodynamic force in any plane of rotation times its distance to the axis of rotation.
- The normal and tangential force on a section of the blade can be expressed in terms of the differential lift and drag forces.

Lift Forces:

$$dF_L = C_L \frac{\rho A V_R^2}{2} = C_L \frac{\rho V_R^2 \cdot c \cdot dr}{2}$$

Drag Forces:

$$dF_D = C_D \frac{\rho A V_R^2}{2} = C_D \frac{\rho V_R^2 \cdot c \cdot dr}{2}$$

The lift and drag are function of the airfoil section and angle of attack, α .

From fig:

$$dF_n = dF_L \sin \beta + dF_D \cos \beta$$

$$dF_n = B \left[C_L \frac{\rho V_R^2 \cdot c \, dr}{2} \cdot \sin \beta + C_D \frac{\rho V_R^2 \cdot c \, dr}{2} \cdot \cos \beta \right] \quad \{B = \text{No. of blades}\}$$

$$= B \frac{\rho V_R^2}{2} [C_L \cdot \sin \beta + C_D \cdot \cos \beta] \cdot c \, dr \dots \dots \dots \{1\}$$

Similarly,

$$dF_t = dF_L \cos \beta - dF_D \sin \beta$$

$$= B \frac{\rho V_R^2}{2} [C_L \cdot \cos \beta - C_D \cdot \sin \beta] \cdot c \, dr \dots \dots \dots \{2\}$$

By considering new dimensional parameter,

$$C_n = C_L \cdot \sin \beta + C_D \cdot \cos \beta$$

$$C_t = C_L \cdot \cos \beta - C_D \cdot \sin \beta \dots \dots \dots \{3\}$$

From equation 1, 2 & 3

$$dF_n = C_n \cdot B \frac{\rho V_R^2}{2} \cdot c \, dr \dots \dots \dots \{4\}$$

$$dF_t = C_t \cdot B \frac{\rho V_R^2}{2} \cdot c \, dr \dots \dots \dots \{5\}$$

The differential torque, $d\tau = r \cdot dF_t$ and the differential power, $dP = \omega \cdot d\tau$,

$$d\tau = r \cdot dF_t = C_t \cdot B \frac{\rho V_R^2}{2} \cdot c \cdot r \cdot dr$$

$$dP = C_t \cdot B \frac{\rho V_R^2}{2} \cdot c \cdot \omega r \cdot dr \dots \dots \dots \{6\}$$

The differential thrust dT (Momentum Theory) is equivalent to the differential normal force dF_n (Blade Element Theory),

$$dT = dF_n$$

$$2\rho V_\infty^2 \cdot a(1-a) \times 2\pi r \, dr = C_n \cdot B \frac{\rho V_R^2}{2} \cdot c \, dr \dots \dots \dots \{ \text{From eq. 6 and Momentum theory} \}$$

By solving above equation and substituting the value of, $V_R = \frac{V_\infty(1-a)}{\sin \beta}$

$$\frac{a}{1-a} = \frac{BC_n \cdot c}{8\pi r \cdot \sin^2 \beta} \dots \dots \dots \{7\}$$

Defining a new parameter, σ_r (Profile Geometry) $\sigma_r = \frac{B \cdot c}{2\pi r}$

$$\frac{a}{1-a} = \frac{\sigma_r \cdot C_n}{4 \sin^2 \beta}$$

$$4 \sin^2 \beta \cdot a = \sigma_r \cdot C_n - a \cdot \sigma_r \cdot C_n$$

$$a \cdot [4 \sin^2 \beta + \sigma_r \cdot C_n] = \sigma_r \cdot C_n$$

$$a = \frac{\sigma_r \cdot C_n}{4\sin^2\beta + \sigma_r \cdot C_n} =$$

$$\mathbf{a} = \frac{\mathbf{1}}{\frac{4\sin^2\beta}{\sigma_r \cdot C_n} + \mathbf{1}} \dots\dots\dots \{8\}$$

Obtains a relation for the axial induction factor.

Similarly, the torque (power) equation for momentum and BEM theory,

$$\rho V_\infty^2 \cdot 2a'(1 - a') \times 2\pi r \cdot \omega \cdot dr = C_t \cdot B \frac{\rho V_R^2}{2} \cdot c \cdot \omega r \cdot dr \dots\dots\dots \{9\}$$

By solving above equation and substituting the value of, $V_R = \frac{V_\infty(1-a')}{\sin\beta}$

$$\mathbf{a}' = \frac{\mathbf{1}}{\frac{4\sin\beta \cdot \cos\beta}{\sigma_r \cdot C_t} - \mathbf{1}} \dots\dots\dots \{10\}$$

Obtains a relation for the angular induction factor.

Efficiency, power and torque characteristics

Any wind turbine or windmill rotor can be characterized by plotting experimentally derived curves of power against rotational speed at various wind speeds; Fig. 1(A). Similarly the torque produced by a wind rotor produces a set of curves such as in Fig. 2(B).

The maximum efficiency coincides with the maximum power output in a given Wind speed. Efficiency is usually presented as a non-dimensional ratio of shaft-power divided by wind-power passing through a disc or shape having the same area as the vertical profile of the windmill rotor; this ratio is known as the "**Power Coefficient**" or C_p and is numerically expressed as:

$$C_p = \frac{P}{2\rho AV_i^3}$$

The speed is also conventionally expressed non-dimensionally as the "**tip-speed ratio**" (λ). This is the ratio of the speed of the windmill rotor tip, at radius R when rotating at ω radians/second, to the speed of the wind, V, and is numerically:

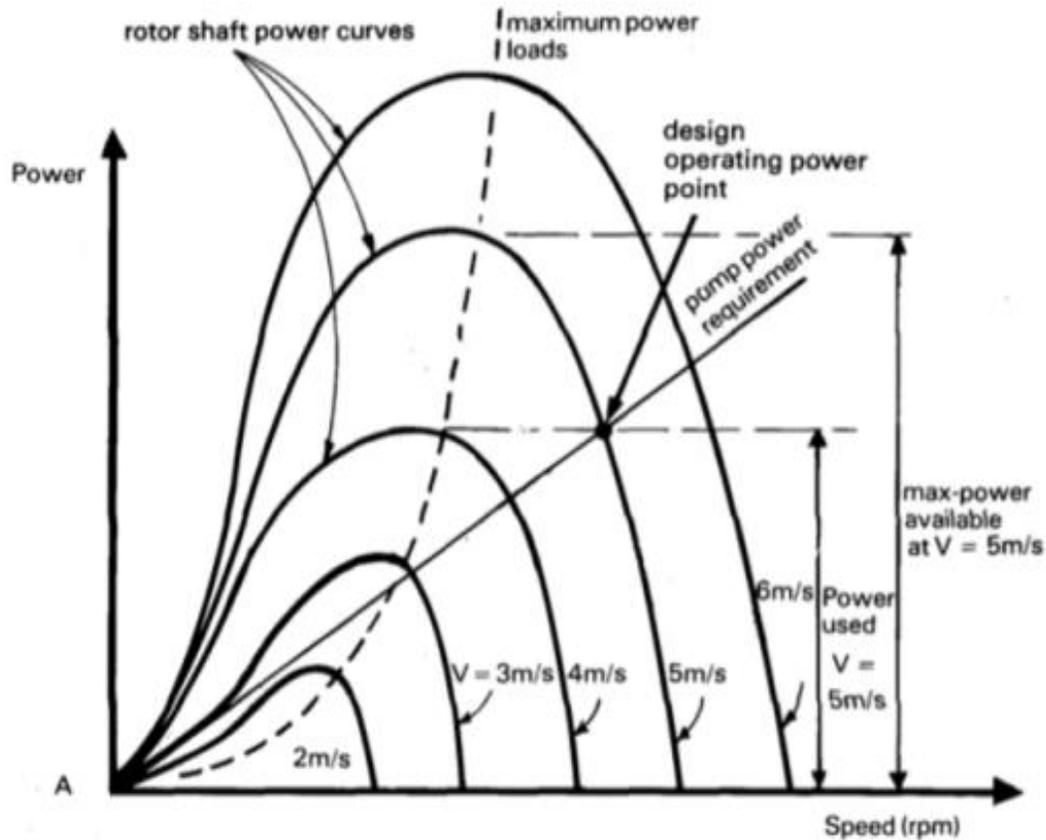
$$\lambda = \frac{\omega R}{V}$$

When the windmill rotor is stationary, its tip-speed ratio is also zero, and the rotor is stalled. This occurs when the torque produced by the wind is below the level needed to overcome the resistance of the load. A tip-speed ratio of 1 means the blade tips are moving at the same speed as the wind (so the wind angle "seen" by the blades will be 45°) and when it is 2, the tips are moving at twice the speed of the wind, and so on.

The C_p versus curves for three different types of rotor, with configurations A, B, C, D, E₁, E₂ and F as indicated, are shown in Fig. 2. The second set of curves show the torque coefficients, which are a non-dimensional measure of the torque produced by a given size of rotor in a given wind speed (torque is the twisting force on the drive shaft). The torque coefficient, C_t , is defined as:

$$C_t = \frac{T}{2\rho AV_i^2 R}$$

Where T is the actual torque at wind speed V_i for a rotor of that configuration and radius R.



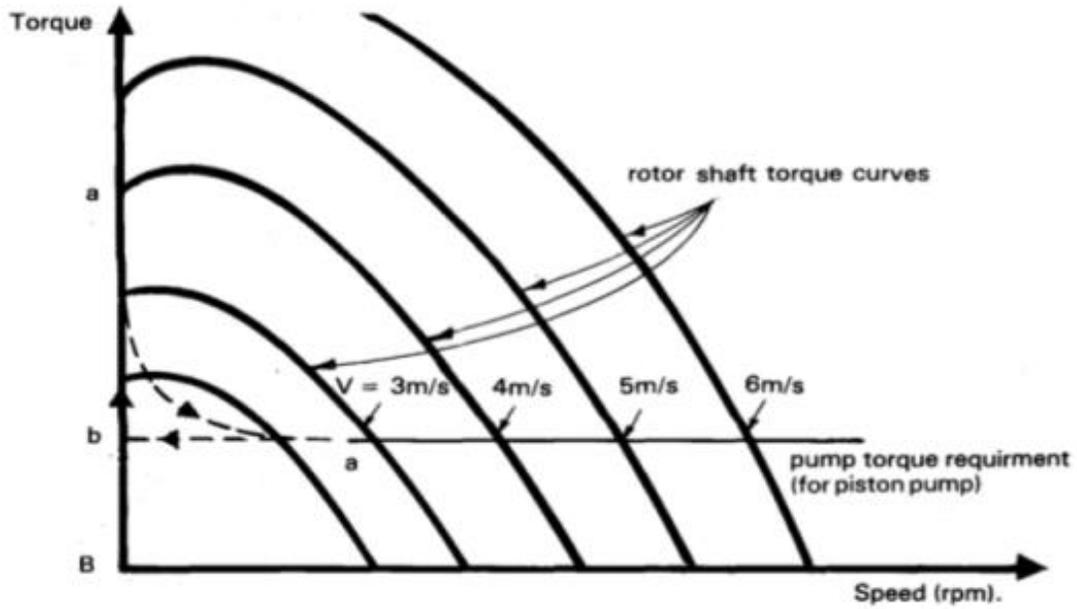


Fig: 1 The power (A) and The Torque (B) of a wind rotor as a function of rotational speed for different wind speeds.

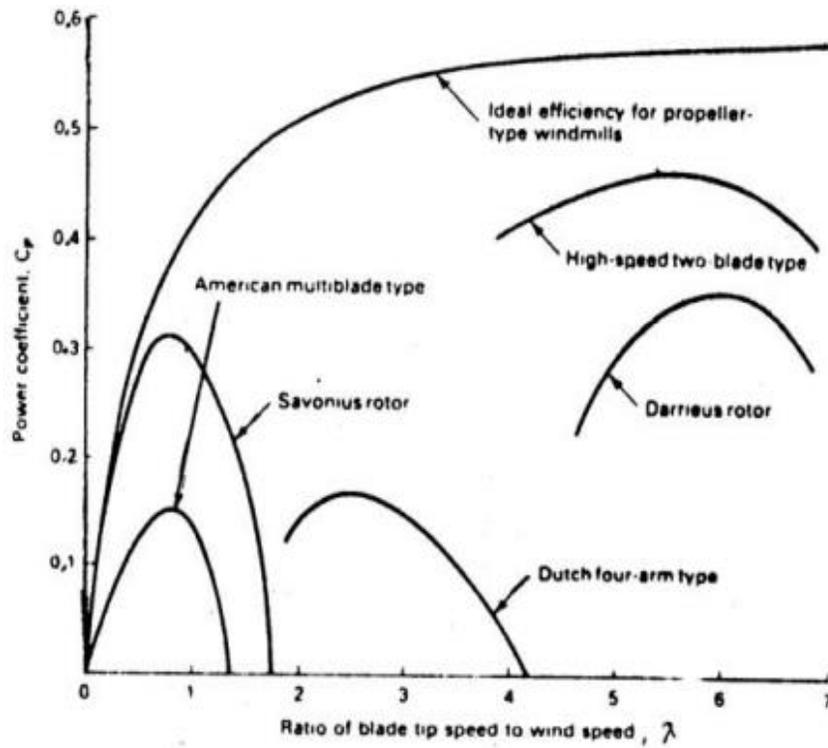


Fig: C_p - λ curves for practical wind machines

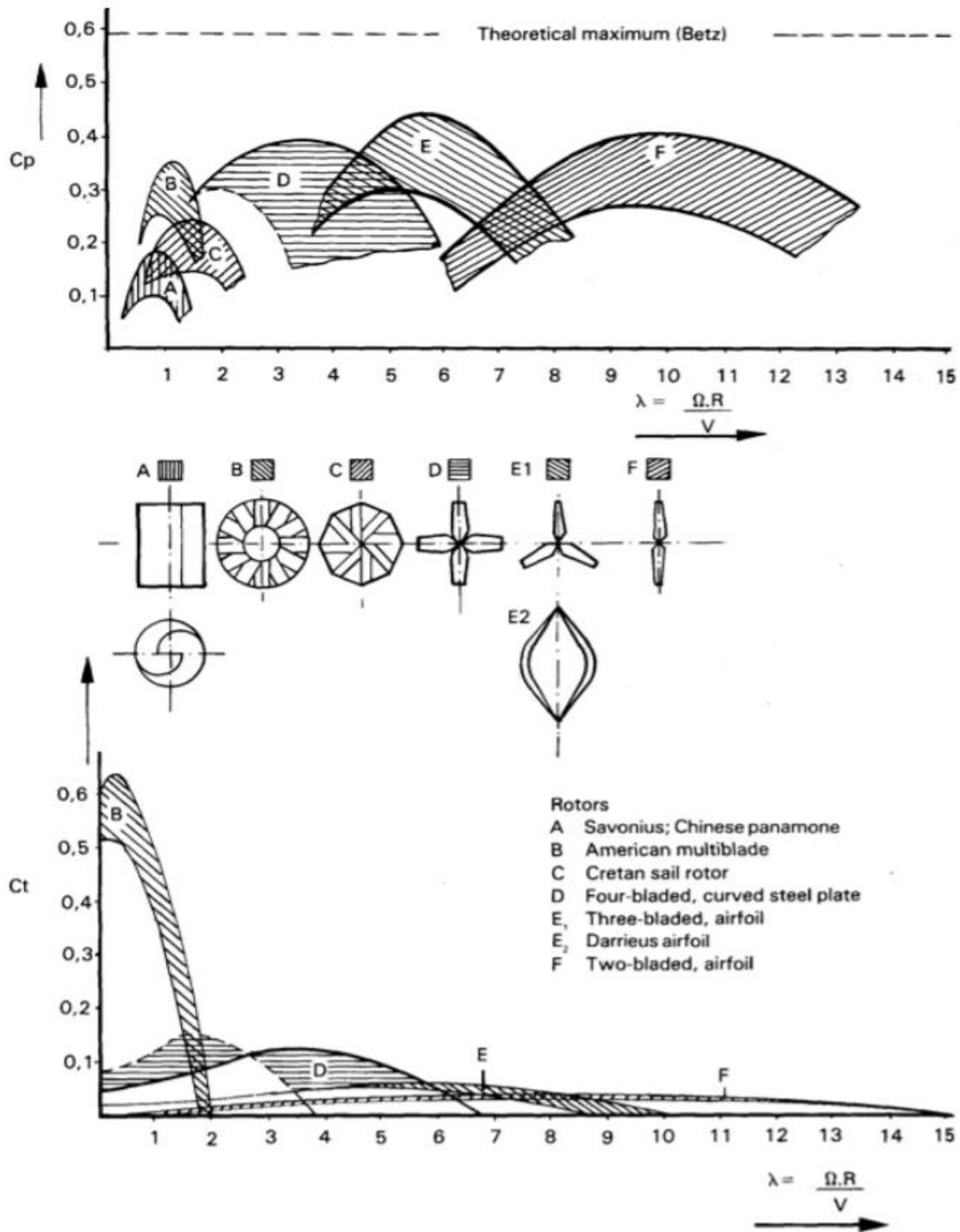


Fig: 2 The power coefficients (C_p) [above] and the torque coefficients (C_t) [below] of various types of wind turbine rotor plotted against tip-speed ratio (λ)

Rotor Solidity

"Solidity" (σ) is a fairly graphic term for the proportion of a windmill rotor's swept area that is filled with solid blades. It is generally defined as the ratio of the sum of the width, or "chords" of all the blades to the circumference of the rotor; i.e. 24 blades with a chord length (leading edge to trailing edge) of 0.3m on a 6m diameter rotor would have a tip solidity of:

$$\sigma = \frac{24 \times 0.3}{6\pi}$$

Multi-bladed rotors, as used on wind pumps, (eg. rotor "B" in Fig. 2) are said to have high "solidity", because a large proportion of the rotor swept area is "solid" with blades. Such machines have to run at relatively low speeds and will therefore have their blades set at quite a coarse angle to the plain of rotation, like a screw with a coarse thread. This gives it a low tip-speed ratio at its maximum efficiency, of around 1.25, and a slightly lower maximum coefficient of performance than the faster types of rotor such as "D", "E" and "F" in the figure. However, the multi-bladed rotor has a very much higher torque coefficient at zero tip-speed ratios (between 0.5 and 0.6) than any of the other types. Its high starting torque (which is higher than its running torque) combined with its slow speed of rotation in a given wind make it well-suited to driving reciprocating borehole pumps.

In contrast, the two or three-bladed, low-solidity, rotors "E₁" and "F" in Fig. 2, are the most efficient, (with the highest values for C_p), but their tips must travel at six to ten times the speed of the wind to achieve their best efficiency. To do so they will be set at a slight angle to the plain of rotation, like a screw with a fine thread and will therefore spin much faster for a given wind speed and rotor diameter than a high solidity rotor. They also have very little starting torque, almost none at all, which means they can only start against loads which require little torque to start them, like electricity generators (or centrifugal pumps) rather than positive displacement pumps.

All this may sound academic, but it is fundamental to the design of wind rotors; it means that multi-bladed "high-solidity" rotors run at slow speeds and are somewhat less efficient than few-bladed "low solidity" rotors, but they have typically five to twenty times the starting torque.

Note:

1. Power extraction by a turbine and maximum power coefficient derivation (Betz Limit) already done in Unit 1.