Investment Return \( R_p = \frac{V_f - V_o + D}{V_o} \)

- Any interest earned is reinvested in between.
- Distributions occur at the end of the interval.

Portfolio risk: Extent to which possible future portfolio values diverge from expected or predicted value, especially future values are less than expected.

<table>
<thead>
<tr>
<th>Expected Portfolio Return</th>
<th>Outcome</th>
<th>Possible Return</th>
<th>Subjective Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>50%</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>30</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>10</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>-10</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-30</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ E(R_p) = \sum_{j=1}^{n} p_j R_j \]

Expected return is weighted avg of possible outcomes.

\[ E(R_p) = 10\% \]
If risk is defined as chance of achieving returns lower than expected, it would be logical to measure risk by dispersion of possible returns below the expected value.

But if distribution of future return is reasonably symmetric about expected value, measures of total variability will be twice as large as measures of portfolio's variability below expected return.

Thus, if total variability is used as risk surrogate, the risk ranking for a group of portfolios will be same as when variability below expected return is used.

Total variability is used as surrogate for risk.

most commonly used variance and S.D.
Diversification - empirical facts

1) S.d. of return for individual stock in portfolio is considerably larger than portfolio average return of individual stock is less than portfolio return.

This is so because not all of an individual stock's risk is relevant. Much of the total risk (which equals s.d. of return) is diversifiable.

Diversification - combining stocks which are less than perfectly correlated to reduce portfolio risk.

Wagner Law - up to 20 randomly picked stocks eliminate 40% of risk

but some risk cannot be eliminated through diversification. This is systematic risk.

*Adapted from Security, return can be divided into 2 parts:

- perfectly correlated with and proportional to market return - systematic return ($\beta$)

- second, independent of market - unsystematic return

security return, $R = \beta R_m + \epsilon'$

$\beta$ = proportionality factor

$\epsilon'$ = unsystematic return

$R = \alpha + \beta R_m + \epsilon$ where $\epsilon' = \alpha + \epsilon$, then avg $\epsilon = 0$
Variance of return is weighted sum of squared deviations from expected return:

\[ \sigma_p^2 = p_1 [R_1 - E(R_1)]^2 + p_2 [R_2 - E(R_2)]^2 + \ldots + p_n [R_n - E(R_n)]^2 \]

\[ \sigma_p^2 = 0.1(50-10)^2 + 0.2(30-10)^2 + 0.1(40-10)^2 + 0.2(-10-10)^2 + 0.1(30-10)^2 \]
\[ = 480 \text{% squared} \]

\[ \sigma_p = 22 \% \]

Larger the variance larger the uncertainty.

Systematic risk of a security is \( \beta_p \).

Unsystematic risk = \( \sigma_e \) [S.D. of residual return] = \( \sigma_p - \beta_p \sigma_m \)

Portfolio systematic risk = \( \beta_p \sigma_p \)

Portfolio \( \beta = \beta_1 \beta_1 + \beta_2 \beta_2 + \ldots + \beta_n \beta_n \)

\[ \beta_p = \sum \beta_i \beta_i \] [Market model]

Measure of total risk = \( \sigma \), measure of relative index is \( \beta \)
Thus systematic risk of a portfolio is simply market-value weighted average of the systematic risk of individual securities. It follows that \( \beta \) of a portfolio consisting of all stocks is 1.

If a stock's \( \beta \) exceeds 1, it is above avg.

If \( \beta < 1 \), it is below avg.

(The unsystematic risk of a portfolio is also a fn. of unsystematic security risks. But \( \to 0 \) with increasing diversification.)

1) 40% to 50% of total security risk can be eliminated through diversification.

2) Remaining security risk = security \( \beta \) times market risk.

3) Portfolio systematic risk is a weighted avg. of Security systematic risks.

:. Realized rates of return over substantial periods of time are expected to be related to systematic as opposed to total risk of securities.

2) Security systematic risk = security Beta times \( \beta \), Beta is a useful relative risk measure. \( \beta \) gives systematic risk of a security (or portfolio) relative to risk of market index.
measure of total risk = \sigma

measure of relative index = \beta

degree of systematic risk

risk and return

Capital Asset Pricing Model (CAPM)

Basic Postulate - Assets with same systematic risk should have the same expected rate of return.

ie. price of assets in capital market should adjust until equivalent risk assets have identical expected returns. This principle is called Law of one price.

ie. if \beta = 0, investor should expect rate of return on risk less assets such as T-B.

ie. no risk - only risk less rate of return.
CAPM is often stated in Risk Premium form.

Risk Premiums or excess returns are often obtained by subtracting risk-free rate from rate of return.

Expected Portfolio Risk Premium

\[ E(r_p) = E(r_m) - R_f \]

\[ E(r_m) = E(r_m) - R_f \]

Put in eq (2)

\[ E(r_p) = \beta P E(r_m) \]

Expected Risk Premium for investors of portfolio

\[ \beta P E(r_m) \]

For Safe investments, \( \beta = 0 \)

For risky investments, \( \beta > 0 \) — investors would expect a rate of expected return, or expected return beyond risk-free rate.
mixture
\[ X \text{ in risky} \]
\[ \beta_p = X \times 1 + (1-X) \times 0 \]
\[ = X \]
\[ \beta_p = \text{fraction of money invested in risky portfolio} \]
\[ \text{Portfolio } \beta \]
\[ 0 < \beta_p < 1 \]

If investor borrows risk-free rate and invests money in portfolio so that \( X \) is larger than 1 \( \Rightarrow (1-X) \) is negative
\[ \text{Portfolio } \beta > 1 \]

(E) expected return on composite portfolio is weighted avg of expected returns on 2 portfolios
\[ E(R_p) = (1-X)R_e + X \times E(R_m) \]
from eq 1
\[ x = \beta_p \]
\[ E(R_p) = (1-\beta_p)R_e + \beta_p \times E(R_m) \]
\[ [E(R_p) = R_e + \beta_p [E(R_m) - R_e] \]

Capital asset pricing model.
\[ \text{ie: expected return on a portfolio should exceed the riskless rate of return by an asset that is in its portfolio beta} \]
Bonds

\[ P = \sum_{i=1}^{n} \frac{C_i}{(1+y)^i} + \frac{M.V.}{(1+y)^n} \]

Perpetuity

\[ P = \frac{C}{(1+y)^1} + \frac{C}{(1+y)^2} + \ldots + \frac{C+i}{(1+y)^i} \]

\[ P = \text{Price of Bond} \]
\[ y = \text{Yield to Maturity} \]
\[ C = \text{Coupon Rate} \]
\[ M.V. = \text{Maturity Value} \]
\[ t = \text{Time to Maturity} \]

\[ E(r_p) = \beta_p E(r_m) \]

1. There is a linear relationship between average risk premium rates return on market and average risk premium return on a portfolio and its slope is \( \beta \).
2. Linear relationship should pass through origin.

(Contd. from prev. page)