Investment Return $R_p = \frac{V_1 - V_0 + D_1}{V_0}$

assumption 1) any int or dividend income recd in between is reinvested.

2) Dist. occurs at end of interval

Portfolio risk : extent to which possible future portfolio values diverge from expected or predicted value.
especially future P values are less than expected.

Expected Portfolio returns :

eg. Outcome	Possible return	Subjective Probability
1	50%	0.1
2	30	0.2
3	10	0.4
4	-10	0.2
5	-30	0.1
		1.00

expected return is weighted avg of possible outcomes

$E(R_p) = P_1 R_1 + P_2 R_2 + \dots + P_n R_n$

$= \sum_{j=1}^{n} P_j R_j$

$= 10\%$

If risk is defined as chance of achieving returns lower than expected It it would be logical to measure risk by dispersion of possible returns below the expected value.

But if distribution of future return is reasonably symmetric about expected value.

measures of total variability of return will be twice as large as measures of portfolio's variability below expected return

Thus if total variability is used as risk surrogate, the risk ranking for a gp. of portfolios will be same as when variability below expected return is used.

∴ Total variability is used as surrogate for risk.

most commonly used variance and S.d.

Diversification

empirical facts

1) s.d. of return for individual stocks in portfolio is considerably larger than portfolio

2) avg return of individual stock is less than portfolio return.

this is so

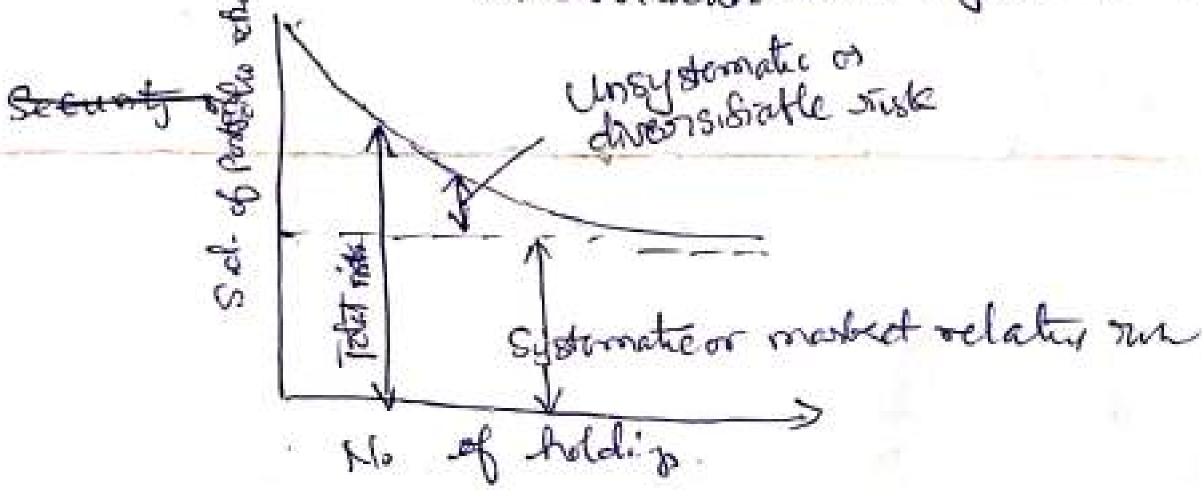
∵ not all of an individual stock's risk is relevant.

Much of the total risk (which equals s.d. of return) is diversifiable.

Diversification — combining stocks which are less than perfectly correlated to reduce portfolio risk.

Wagner & Lau — NYSE — upto 20 randomly picked stocks eliminate 40% of risk.

but some risk cannot be eliminated through diversification. this is systematic risk



~~Rate of~~ Security return can be divided into 2 parts

one perfectly correlated with and proportionate to market return — systematic return (β)

and second, independent of market — unsystematic return

Security return. $R = \beta R_m + \epsilon'$ — epsilon (unsystematic)

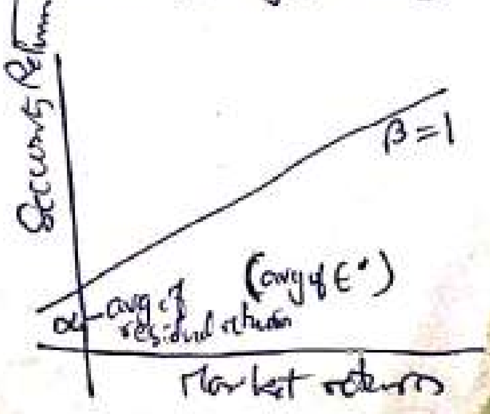
(R_m: market return)

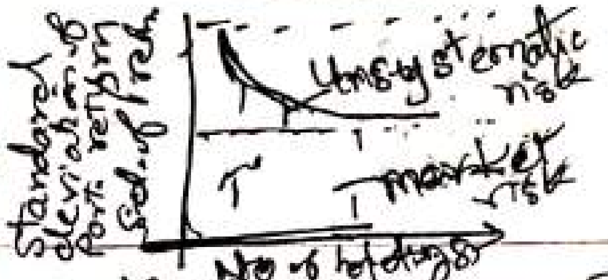
β = proportionality factor

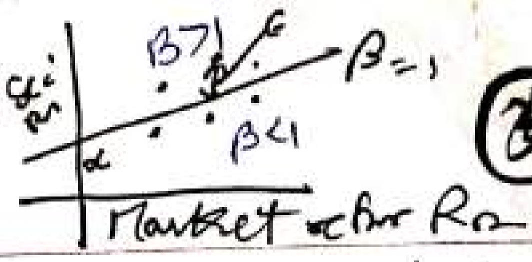
on how sensitive, security return is to changes in market $R = \beta R_m + \epsilon'$

$R = \alpha + \beta R_m + \epsilon$

[where $\epsilon' = \alpha + \epsilon$ where avg $\epsilon = 0$]







34

variance of return is weighted sum of sqd deviations from expected return

$$\sigma_p^2 = P_1[R_1 - E(R_p)]^2 + P_2[R_2 - E(R_p)]^2 + \dots + P_n[R_n - E(R_p)]^2$$

$$\sigma_p^2 = 0.1(50-10)^2 + 0.2[30-10]^2 + 0.4(0-10)^2 + 0.2(-10-10)^2 + 0.1(-30-10)^2$$

$= 480\%$ sqd.

[β = slope of line
β gives expected increase in security return agst 1% inc in market return]

$\sigma_p = 22\%$

larger the variance larger the uncertainty

systematic risk for a security ~~is~~ $= \underline{\beta\sigma_m}$

unsystematic risk $= \sigma_\epsilon$ [s.d. of residual return factor]

Portfolio systematic risk $= \beta_p \sigma_\epsilon$

Portfolio $\beta = x_1\beta_1 + x_2\beta_2 + \dots + x_n\beta_n$

$\beta_p = \sum_{i=1}^{n} x_i \beta_i$ [market model.

measure of total risk $= \sigma$, measure of relative indexing systematic risk $= \beta$

Thus systematic risk of a portfolio is simply market value weighed average of the systematic risk of individual security. It follows that β of a portfolio consisting of all stocks is 1

If a stocks β exceeds 1 it is above avg if $\beta < 1$, it is below avg.

(The unsystematic risk of a portfolio is also a fn. of unsystematic security risks. But $\rightarrow 0$ with increasing diversification).

1) 40% to 50% of total security risk can be eliminated through diversification

2) remaining security risk ~~is β~~ = security β times market risk.

3) portfolio systematic risk is a weighed avg of security systematic risks.

∴ 1) Realized rates of return over substantial periods of time are expected to be related to systematic as opposed to total risk of securities.

2) ∵ Security systematic risk = security Beta times σ_m, beta is a useful relative risk measure

β gives systematic risk of a security (or portfolio) relative to risk of market index.

measure of total risk — σ

measure of relative index of systematic risk $= \beta$

risk and return.

Capital Asset Pricing Model. CAPM

Basic Postulate — Assets with same Systematic risk should have the same expected rate of return.

ie. price of assets in capital markets should adjust until ~~exp~~ equivalent risk assets have identical expected returns. This principle is called Law of one Price.

ie if $\beta = 0$. investor should expect rate of return on risk less assets such as T.B.
ie. no risk. — only risk less rate of return.

CAPM is often stated in Risk Premium form.

Risk Premium or excess return are obtained by subtracting risk free rate from rate of return.

Expected Portfolio risk premium and Expected Market risk premium
$E(r_p)$ $E(r_m)$

$$E(r_p) = E(R_p) - R_f$$

$$E(r_m) = E(R_m) - R_f$$

Putting in eq (2)

$$E(r_p) = \beta_p E(r_m)$$

expected risk premium for investors of portfolio $= \beta E(R_m)$

Or. Expected risk premium should be equal to quantity of risk (as measured by β) and market price of risk (as measured by expected market risk premium)

[for safe investments $\beta = 0$
for risky " $\beta > 0$ — investors would expect
a rate of excess return proportional to ... invest

mixture
X in risky
(1-X) in riskless.

$$\beta_p = X \times 1 + (1-X) \times 0 \qquad (1)$$
$$= X$$

i.e. β_p = fraction of money invested in risky portfolio.

Portfolio β. β_p $0 < \beta_p < 1$

If investor borrows at risk free rate and invests in risky portfolio so that X is larger than 1 and (1-X) is negative portfolio $\beta > 1$.

(ii) expected return on composite portfolio is weighted avg of expected returns on 2 portfolios

i.e. $E(R_p) = (1-X) \times R_f + X \times E(R_m) \qquad (2)$

Risk free rate

from eq (1)
$x = \beta_p$.

$\therefore E(R_p) = (1-\beta_p) \times R_f + \beta_p \times E(R_m)$

$$\boxed{E(R_p) = R_f + \beta_p \left[E(R_m) - R_f \right]}$$

Capital asset Pricing Model.

i.e. expected return on a portfolio should exceed the riskless rate of return by an amount that is ∝ to the portfolio Beta.

proportional

Bonds

$$P = \sum_{i=1}^{n} \frac{CF}{(1+r)^i} + \frac{M.V.}{(1+r)^n}$$

Perpetuity

$$P = \frac{C}{(1+y)^1} + \frac{C}{(1+y)^2} + \cdots + \frac{C+M}{(1+y)^n}$$

yield to maturity

P = Price of Bond.
C = Coupon ints
M = M.V. (maturity value)
n = time to maturity.

Expected portfolio risk prem $\quad E(r_p) = \beta_p E(r_m)$ — expected market risk premium

implies

(i) There is linear relationship between avg risk premium ~~ratio~~ return on market and avg risk premium return on a portfolio and its slope is β.

(2) linear relationship should pass through origin

(Contd. from prev. page)