Torsional Vibrations :- (for Numerical Purpose only) (8)

Torsional stiffness

$$\boxed{S\theta = \frac{GJ}{l}}$$

Length ← L

Area ← A

Modulus ← G of Rigidity

$$\boxed{\omega_n = \sqrt{\frac{S\theta}{I}} \text{ rad/sec}}$$

If the shaft is also having Moment of Inertia = I_{shaft}

$$\boxed{\omega_n = \sqrt{\frac{S\theta}{I + \frac{I_{shaft}}{3}}}}$$

Longitudinal stiffness

* Force Required to produce unit deflection

$$E = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

$$F = \frac{AE\Delta L}{L}$$

$$\frac{F}{\Delta L} = \frac{AE}{L}$$

$$\boxed{S = \frac{AE}{L}}$$

Transverse stiffness

* Torque Required to produce unit angle of twist

$$\theta = \frac{TL}{GJ}$$

$$\frac{T}{\theta} = \frac{GJ}{L}$$

$$\boxed{S\theta = \frac{GJ}{L}}$$

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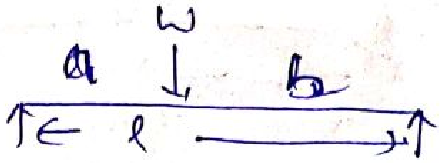
Natural frequency of free Transverse Vibrations :-

I

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

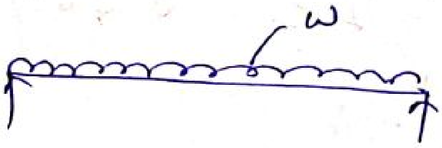


$$\boxed{f_n = \frac{0.4985}{\sqrt{\delta_s}}\ Hz}$$

$$\boxed{\delta = \frac{Wa^2b^2}{3EIl}}$$

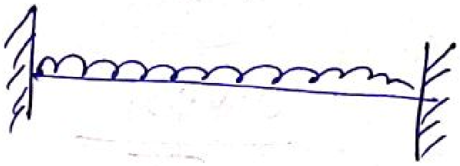
II

$$\boxed{f_n = \frac{0.5615}{\sqrt{\delta}}\ Hz}$$



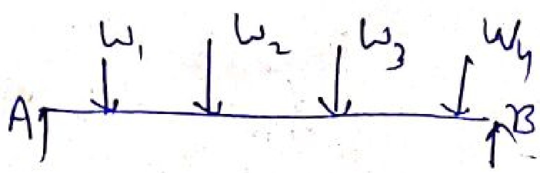
III

$$\boxed{f_n = \frac{0.571}{\sqrt{\delta_s}}\ Hz}$$



Natural frequency of a free Transverse Vibrations for a shaft subjected to a Number of Point Loads

Dunkerley's method :-



$$\frac{1}{(f_n)^2} = \frac{1}{(f_1)^2} + \frac{1}{(f_2)^2} + \frac{1}{(f_3)^2} - - -$$

$$\boxed{f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 - - -}}}$$

$$\frac{1}{(f_n)^2}$$

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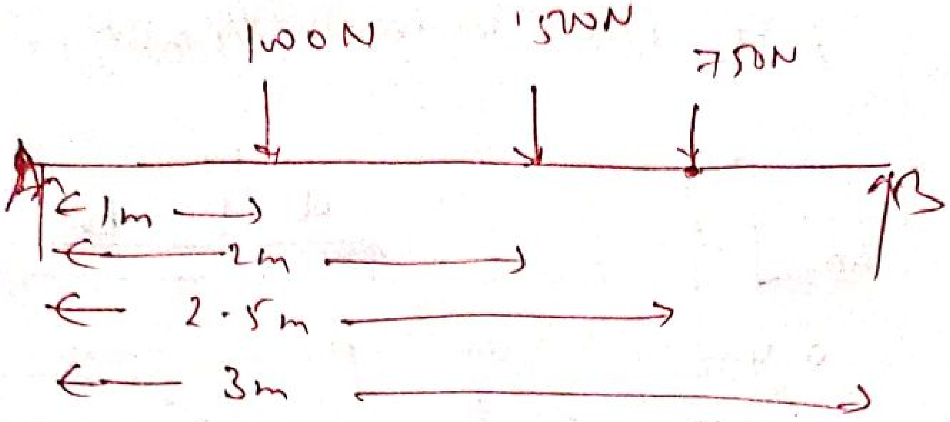
Q:



Given $E = 200\ GN/m^2$

$I = 0.307 \times 10^{-6}\ m^4$

find $f_n = ?$

$$\frac{1}{(f_n)^2} = \frac{1}{(f_{n_1})^2} + \frac{1}{(f_{n_2})^2} + \frac{1}{(f_{n_3})^2}$$

$$f_{n_1} = \frac{0.4985}{\sqrt{\delta_1}}$$

where $\delta = \frac{W a^2 b^2}{3 E I l}$

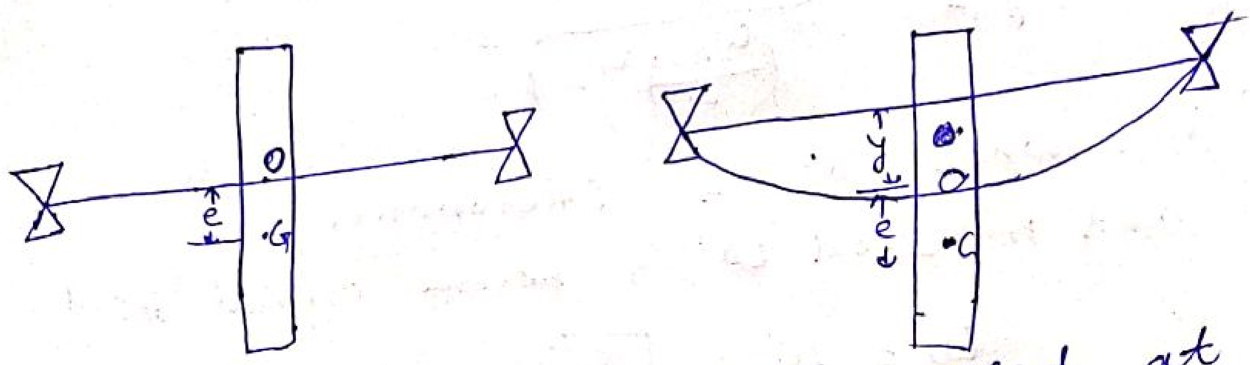
$$\delta = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3}$$

$$= 10.86 \times 10^{-3}\ m$$

Similarly solve

$$\boxed{f_n = 3.5\ Hz}$$ Answer

Critical or whirling speed of a shaft:-



* Critical or whirling speed is the speed at which the shaft tends to vibrate violently in the transverse direction.

* The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed of shaft.

$$f_c = m(y+e)\omega^2$$

where $\omega \rightarrow$ Angular velocity of shaft.

The shaft behaves like a spring. Therefore the force resisting the deflection $= s.y$

$$\therefore \quad sy = m(y+e)\omega^2$$

$$sy = \cancel{m(y+e)}\ my\omega^2 + me\omega^2$$

$$y(s - m\omega^2) = me\omega^2$$

$$y = \frac{me\omega^2}{s - m\omega^2}$$

$$y = \frac{e}{\frac{s}{m\omega^2} - 1}$$

$$y = \frac{e}{\left(\frac{\omega_n}{\omega}\right)^2 - 1}$$

Thus when $\omega = \omega_n$ (Resonance)

and the speed ω is become critical speed

$$\omega_c = \omega_n = \sqrt{s/m} = \sqrt{\frac{g}{\delta}}\ Hz$$

if N_c is the critical speed in rps then

$$2\pi N_c = \sqrt{\frac{g}{\delta}}$$

$$\boxed{N_c = \frac{0.4985}{\sqrt{\delta}}\ r.p.s} \qquad (23\text{-}7)$$

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