Subject :- Dynamics of Machines
Course :- B.Tech
Year :- Third
Branch :- Mechanical Engineering
Faculty :- Er. Sandeep Kumar Gupta
Unit No :- V
Topic :- Mechanical Vibration

Vibration :- A body is said to be vibrate if it has a to and fro motion.

Vibration

- Natural Vibration
  - No friction
  - No External force
- Damped Vibration
  - friction
  - Gradually decreases
- forced Vibration
  - Repeated force acts continuously

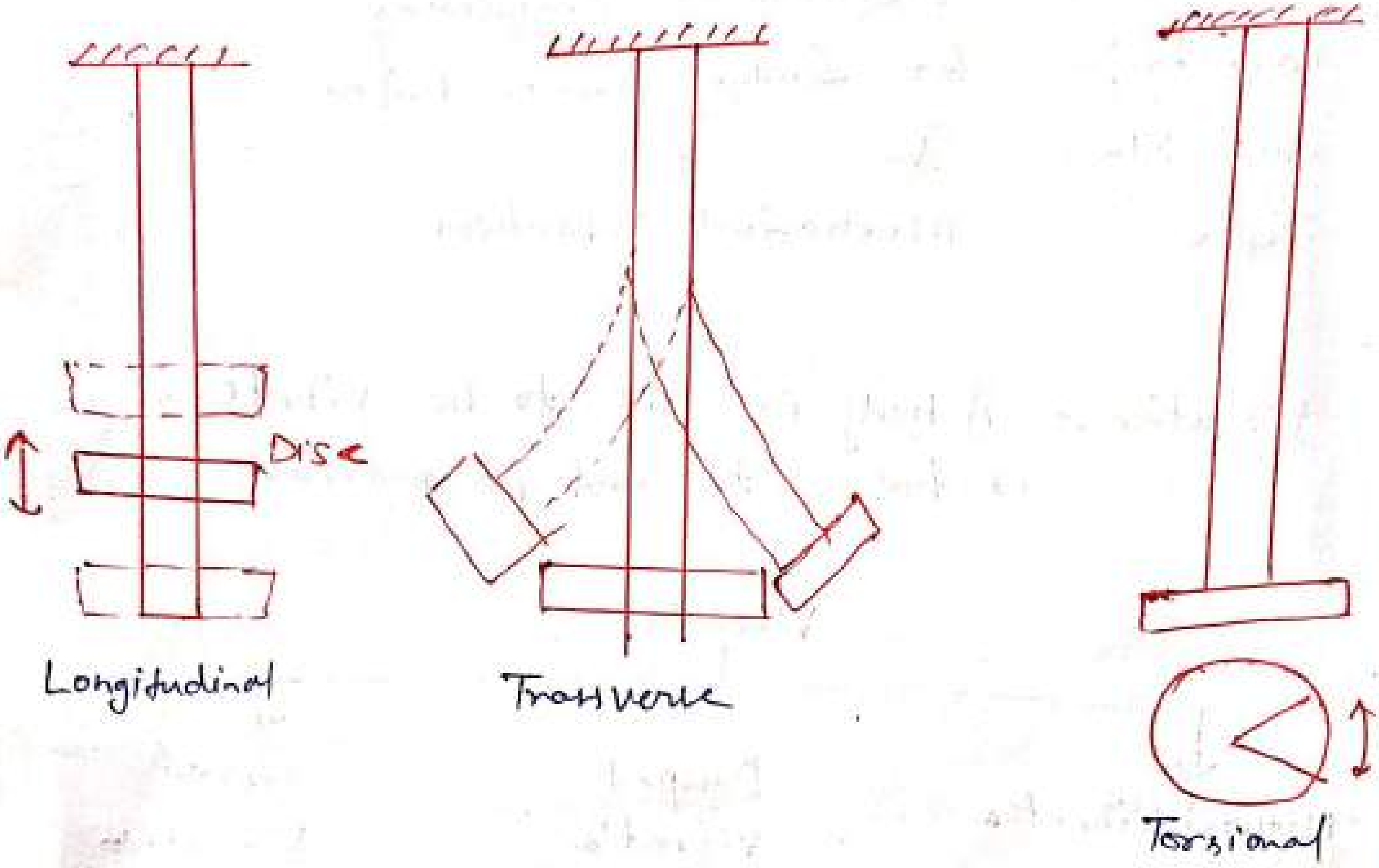
Types of Natural Vibrations : (According to the motion)

(i) **Longitudinal Vibrations :-**
* If the shaft is elongated and shortened so that the mass moves up and down resulting in tensile and compressive stresses in the shaft.

(ii) **Transverse Vibration :-** When the shaft is bent alternately and tensile and compressive stresses due to bending result, the vibrations called transverse.

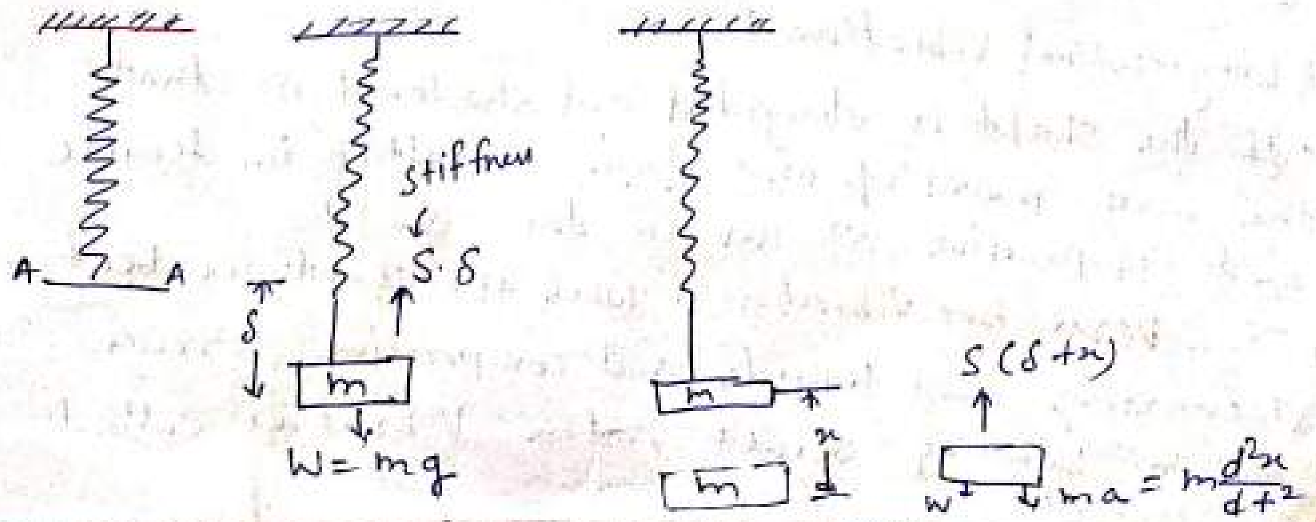
(iii) Torsional Vibrations:-

When the shaft is twisted and untwisted alternately and torsional shear stresses are induced, the vibration are know as torsional vibration.



Methods of finding Natural frequency of Longitudinal Vibrations:-

I Equilibrium Method:-



Restoring force $= W - S(\delta + x)$

$= S\delta - S\delta - Sx$

$= -Sx$ —— ①

Accelerating force $=$ mass $\times$ Acceleration

$= m \times \frac{d^2x}{dt^2}$ — ②

equating eqn ① & ②

$$m \times \frac{d^2x}{dt^2} = -S \cdot x$$

$$m \times \frac{d^2x}{dt^2} + S \cdot x = 0$$

$$\boxed{\frac{d^2x}{dt^2} + \frac{S}{m} \times x = 0}$$ —— ③ → S.H.M (like)

$$\boxed{\frac{d^2x}{dt^2} + \omega_n^2 \times x = 0}$$ —— ④ ~~where $\omega = \sqrt{S/m}$~~

→ fundamental S.H.M

On comparing ③ & ④ we have

$$\omega^2 = \frac{S}{m}$$

$$\boxed{\omega_n = \sqrt{S/m}}$$

$$2\pi f_n = \sqrt{S/m}$$

$$\boxed{f_n = \frac{1}{2\pi} \times \sqrt{S/m}}$$ $$\boxed{T_p = 2\pi \sqrt{m/S}}$$

5

Since $s \cdot \delta = m \cdot g$

$$\frac{s}{m} = \frac{\delta}{g}$$

$$s \cdot \delta = m \cdot g$$

$$\frac{s}{m} = \frac{g}{\delta}$$

$$\therefore \quad f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}}$$

$$\boxed{f_n = \frac{0.4985}{\sqrt{\delta}} \text{ Hz}}$$

II Energy Method:

$$\frac{d}{dt}(K.E + P.E) = 0$$

$$K.E. = \frac{1}{2} m \times \dot{x}^2 \qquad \text{where } \dot{x} = \frac{dx}{dt} = v$$

$$P.E = \left(\frac{0 + s.x}{2}\right) \times x = \frac{1}{2} s.x^2$$

$$\therefore \frac{d}{dt}\left(\frac{1}{2} m\dot{x}^2 + \frac{1}{2} s.x^2\right) = 0$$

$$\frac{1}{2} m\, 2\dot{x}\ddot{x} + \frac{1}{2} s. 2x\dot{x} = 0 \qquad \text{where } \ddot{x} = \frac{d^2x}{dt^2}$$

$$m\ddot{x} + sx = 0 \quad \text{or} \quad m\frac{d^2x}{dt^2} + sx = 0$$

$$\boxed{\omega_n = \sqrt{s/m}}$$

$$\boxed{f_n = \frac{0.4985}{\sqrt{\delta}}\ Hz}$$

III Rayleigh's Method:-

In this method the max K.E = Max P.E

max K.E ↓ at mean position

Max P.E ↓ at extreme position

Assume the motion

$$x = X \sin \omega t$$

where x = displacement of body

X = Maximum displacement.

$$\frac{dx}{dt} = \omega \times X \cos \omega t$$

Since at mean position $t = 0$

$$\therefore v_{max} = \frac{dx}{dt} = \omega \cdot X$$

$$\therefore (K.E)_{max} = \frac{1}{2} m \cdot (\omega \cdot X)^2 \quad -①$$

$$(P.E)_{max} = \left(\frac{0 + s.X}{2}\right) X = \frac{1}{2} \times s.X^2 \quad -②$$

$$\therefore \frac{1}{2} m \omega^2 \cdot x^2 = \frac{1}{2} s \cdot x^2$$

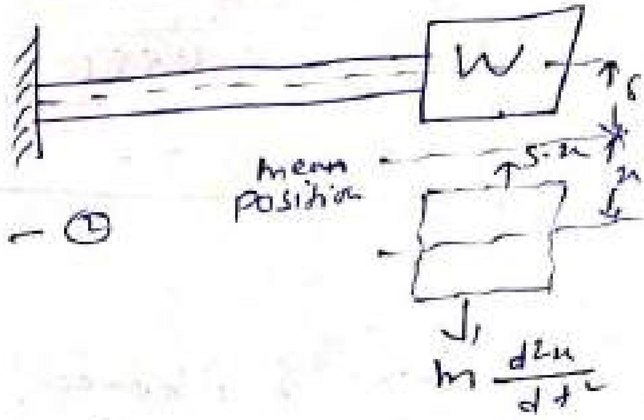
$$\omega^2 = s/m, \quad \omega = \sqrt{s/m}$$

$$\therefore \boxed{f_n = \frac{0.4985}{\sqrt{\delta}}}$$

Natural frequecy of a free Transverse Vibrations:-

Restoring force $= -S.x$ — ①

accelerating force $= m \times \frac{d^2x}{dt^2}$ — ②



$$m \times \frac{d^2x}{dt^2} = -S.x$$

Rest is same as privious

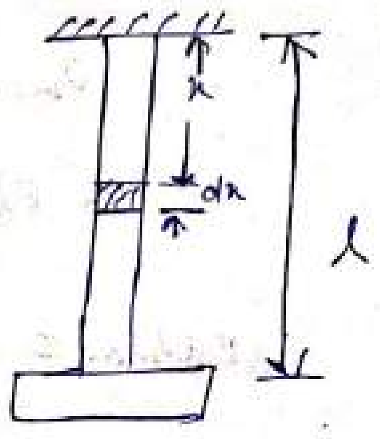
$$\boxed{\frac{d^2x}{dt^2} + \frac{S}{m} \times x = 0}$$

$$\boxed{\omega_n = \sqrt{S/m}}$$

$$\boxed{f_n = \frac{0.4985}{\sqrt{S}} \ Hz}$$

Effect of Inertia of the constraints in longitudinal and Transvers Vibrations:-

Longitudinal Vibrations:-



Let $m' =$ mass of the spring wire per unit length

$v =$ velocity of the free end of the spring at the instant under consideration

$l =$ total length of the spring wire

(7)

K.E of element = $\frac{1}{2} \times$ mass of element $\times$ (velocity)^2

$$= \frac{1}{2} (m' dx) \times \left(\frac{x}{l} v\right)^2$$

$\therefore$ K.E of spring $= \int_0^l \frac{1}{2} m' v^2 \left(\frac{x}{l}\right)^2 dx$

$$= \frac{1}{2} \frac{m' v^2}{l^2} \int_0^l x^2 dx$$

$$= \frac{1}{2} \times \frac{m' v^2}{l^2} \times \frac{l^3}{3}$$

$$= \frac{1}{3} \frac{1}{2} (m' l) v^2$$

$$= \frac{1}{3} \times \frac{1}{2} (M) v^2$$

$$= \frac{1}{2} \times \left(\frac{M}{3}\right) \times v^2 = \frac{1}{2} (M_e) v^2$$

$\therefore$ ~~equivalent~~ Equivalent mass at free end $= m + \frac{m_c}{3}$

$$\therefore \boxed{f_n = \frac{1}{2\pi} \sqrt{\frac{s}{m + \frac{m_c}{3}}}}$$

Transverse vibration:

$$\boxed{f_n = \frac{1}{2\pi} \sqrt{\frac{s}{m + \frac{33 m_c}{140}}}}$$