Electric Quadrupole moment of Deuteron

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Basic data on deuteron Ground state

1- Binding energy = 2.2245 MeV
2- Total angula momentum = $1\hbar$
3- Parity = even
4- Isospin T = 0
5- Magnetic dipole moment = $0.8574\ \mu_N$
6- Electric quadrupole moment = $2.82 \times 10^{-27} cm^2$

$$= 0.282\ fm^2$$

we can also observe that magnetic moment of deuteron is not equal to the sum of magnetic moment of proton and neutron

$$\mu_p = 2.7927\ \mu_N$$

$$\mu_n = -1.9130\ \mu_N$$

$$\mu_p + \mu_n - \mu_d = 0.02237\ \mu_N$$

This small definite difference between μ_d and $\mu_p + \mu_n$ indicates that the ground state of the deuteron is not a pure ^3S state but has an admixture of ^3D state to a small extent.

The mixing of ^3S and ^3D state is possible only if the nucleon-nucleon potential has a tensor component.

small but finite +ve value ($0.282\ fm^2$) for deuteron quadrupole moment suggest that charge distribution is not spherically symmetric i.e. $L = 0$ alone is not correct description so it is mixed with $L = 2$ state.

If the orbital angular momenta get mixed up
$\Rightarrow$ the potential is not central potential.

Hence the nuclear potential should have a non central term also.

Nuclear non central character, analogy can be drawn from mag dipoles.

The force between two mag dipoles is different for different orientations of mag dipoles, depending on angles.

Mag. energy for this kind of system

$$\left[\frac{3(\vec{m}_1 \cdot \vec{r})(\vec{m}_2 \cdot r)}{r^2} - \vec{m}_1 \cdot \vec{m}_2\right]$$

Hence non central part of nucleon-nucleon potential can be written

$$\left[\frac{3(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \sigma_1 \cdot \sigma_2\right] V_T(r)$$

The electric quadrupole moment operator is defined by

$$Q_{op} = r^2(3\cos^2\theta - 1) = \sqrt{\frac{16\pi}{5}}\, r^2\, Y_{20}(\theta,\phi)$$

In deuteron, contribution to quadrupole moment comes only from proton. If r is the relative separation between neutron and proton, then distance of proton from COM is $\frac{r}{2}$.

So

$$Q_{op} = \sqrt{\frac{\pi}{5}}\, r^2\, Y_{20}(\theta,\phi) \quad \text{replace } r \text{ by } \frac{r}{2}$$

$$Q_{obs} = \sqrt{\frac{\pi}{5}}\, \langle \psi_M | r^2 Y_{20} | \psi_M \rangle$$

where $\psi_M = \sum_{l=0,2} |l, s=1, J=1, M=1\rangle \frac{u_l(r)}{r}$

So

$$Q = \sqrt{\frac{\pi}{5}} \Big[\langle 0,1,1,1 | Y_{20} | 2,1,1,1\rangle \langle r^2\rangle_{SD}$$
$$+ \langle 2,1,1,1 | Y_{20} | 0,1,1,1\rangle \langle r^2\rangle_{DS}$$
$$+ \langle 2,1,1,1 | Y_{20} | 2,1,1,1\rangle \langle r^2\rangle_{DD} \Big]$$

where

$$\langle r^2\rangle_{SD} = \langle r^2\rangle_{DS} = \int_0^\infty u_0(r)\, r^2\, u_2(r)\, dr$$

$$\langle r^2\rangle_{DD} = \int_0^\infty u_2(r)\, r^2\, u_2(r)\, dr$$

$$\langle 0,1,1,1 | Y_{20} | 2,1,1,1\rangle = \langle 2,1,1,1 | Y_{20} | 0,1,1,1\rangle$$

But

$$\langle 0,1,1,1 | Y_{20} | 2,1,1,1 \rangle = \frac{1}{2}\sqrt{\frac{1}{10\pi}}$$

$$\langle 2,1,1,1 | Y_{20} | 2,1,1,1 \rangle = -\frac{1}{2}\sqrt{\frac{1}{20\pi}}$$

So

$$Q_{obs} = \sqrt{\frac{\pi}{5}}\left\{\sqrt{\frac{1}{10\pi}}\langle r^2\rangle_{SD} - \frac{1}{2}\sqrt{\frac{1}{20\pi}}\langle r^2\rangle_{DD}\right\}$$

$$= \sqrt{\frac{1}{50}}\langle r^2\rangle_{SD} - \frac{1}{20}\langle r^2\rangle_{DD}$$

$$Q_{obs} = \sqrt{\frac{1}{50}}\langle r^2\rangle_{SD}, \text{ as } \langle r^2\rangle_{DD} \text{ is negligible}$$

$$Q_{obs} = \sqrt{\frac{1}{50}}\int_0^\infty u_0(r)\, u_2(r)\, r^2\, dr$$

Let us assume that

$$u_0(r) = c\, e^{-\alpha r} \qquad \text{when } c = \sqrt{2\alpha}$$

$$u_0(r) = \sqrt{2\alpha}\, e^{-\alpha r} \qquad \alpha^2 = \frac{ME_{B.E}}{\hbar^2}$$

$$u_2(r) = \sqrt{2\alpha}\, n\, e^{-\alpha r}$$

and $\int |u_2(r)|^2 dr = n^2 = P_D$

$$2\alpha n^2 \int_0^\infty e^{-2\alpha r}\, dr = 2\alpha n^2 (-) \left.\frac{e^{-2\alpha r}}{-2\alpha}\right|_0^\infty$$

$$= n^2 = P_D$$

when P_D is the ^3D state probability.

$$Q_{obs} = \sqrt{\frac{I}{50}}\sqrt{2\alpha}\sqrt{2\alpha}\, n \int_0^\infty r^2 e^{-2\alpha r} dr$$

$$= \sqrt{\frac{I}{50}}\, 2\alpha n \left[\frac{1}{4\alpha^3}\right]$$

$$= \sqrt{\frac{I}{50}}\,\frac{n}{2\alpha^2}$$

but $\alpha = \sqrt{\frac{M E_{B.E}}{\hbar^2}} \simeq 0.232\ fm^{-1}$

Substituting Q_{obs}, α values in the above equation, we get

$$n = 0.214$$

Hence the probability of ^3D state P_D comes out to be around 0.046.

Reference Book

Nuclear Physics by Devnathan