Input-Output Analysis

Jai Lakshmi sharma

Input-Output Analysis

Input-output analysis is a technique invented by Professor Wassily W. Leontief in 1951. It is used to analyse inter-industry relationship in order to understand the inter-dependencies and complexities of the economy and thus the conditions for maintaining equilibrium between supply and demand.

As the inputs of one industry the outputs of another industry and vice versa. An input is obtained but an output is produced. Thus input represents the expenditure of the firm, and output its receipts. The sum of the money values of inputs is the total cost of a firm and the sum of the money values of the output is its total revenue.

There are flows of goods in “whirlpools and cross currents” between different industries. The supply side consists of large inter-industry flows of intermediate products and the demand side of the final goods. In essence, the input-output analysis implies that in equilibrium, the money value of aggregate output of the whole economy must equal the sum of the money values of inter-industry inputs and the sum of the money values of inter-industry outputs. For example,Coal is an input for steel industry and steel is an input for coal industry, though both are the outputs of their respective industries. A major part of economic activity consists in producing intermediate goods (inputs) for further use in producing final goods (outputs).

There are two types of relationships which indicate and determine the manner in which an economy behaves and assumes a certain pattern of flows of resources.The internal stability or balance of each sector of the economy, and the external stability of each sector or intersectoral relationships. Professor Leontief calls them the "fundamental relationships of balance and structure." When expressed mathematically they are known as the "balance equations' and the "structural equations".

The Static Input-Output Model:

The whole economy is divided into two sectors—“inter-industry sectors” and “final-demand sectors,” both being capable of sub-sectoral division.The total output of any inter-industry sector is generally capable of being used as inputs by other inter-industry sectors, by itself and by final demand sectors. Prices, consumer demands and factor supplies are given. There are no external economies and diseconomies of production.

Assumptions:

(i)No two products are produced jointly. Each industry produces only one homogeneous product.

(ii) Each producing sector satisfies the properties of linear homogeneous production function i.e. Production of each sector is subject to constant returns to scale.

(iii) The combinations of inputs are employed in rigidly fixed proportions. The inputs remain in constant proportion to the level of output. It implies that there is no substitution between different materials and no technological progress. There are fixed input coefficients of production.

Example

- For understanding, a four-sector economy is taken in which there are three industry sectors, $X_{1,}$ X_{2} , X_3 and one final demand sector.
- Horizontal explanation-Products of the these three industries are being used as an intermediate product(input) and final consumption by government or household sector.
- Vertical explanation-total inputs(from all sectors)utilised by each sector for its production.

Input-output table

	Input requirements of producing sectors			
Total output of the sectors	X_1	X_2	X_3	Final demand
X_1	X_{11}	X_{12}	X_{13}	F_1
X_2	X_{21}	X_{22}	X_{23}	F_2
X_3	X_{31}	X_{32}	X_{33}	F_3
Primary input or labor	L_1	L_2	L_3	

Rows which are consumption centres can be written as

- $X_1 = X_{11} + X_{12} + X_{13} + F_1$
- $X_2 = X_{21} + X_{22} + X_{23} + F_2$
- $X_3 = X_{31} + X_{32} + X_{33} + F_3$

$L = L1 + L2 + L3$

So $X_i = \Sigma X_{ij} + \Sigma F_i$

and $L = \Sigma L_i$

Where all i and j varies from 1 to 3.

Columns which are production functions can be written as:

- $X_1 = X_{11} + X_{21} + X_{31} + L_1$
- $X_2 = X_{12} + X_{22} + X_{32} + L_2$
- $X_3 = X_{13} + X_{23} + X_{33} + L_3$

Technological coefficient matrix

	Input requirements of producing sectors			
Total output of the sectors	X_1	X_2	X_3	Final demand
X_1	$a_{11}X_1$	$a_{12}X_2$	$a_{13}X_3$	F_1
X_2	$a_{21}X_1$	$a_{22}X_2$	$a_{23}X_3$	F_2
X_3	$a_{31}X_1$	$a_{32}X_2$	$a_{33}X_3$	F_3
Primary input or labor	L_1	L_2	L_3	

Technological coefficient matrix

From the assumption of fixed input requirements, the input used for the i^{th} commodity for a fixed amount , in order to produce j^{th} commodity can be denoted by aij

$$\mathbf{a_{ij} = X_{ij}/X_j}$$

- $X_1 = a_{11}X_1 + a_{12}X_2 + a_{13}X_3 + F_1$
- $X_2 = a_{21}X_1 + a_{22}X_2 + a_{23}X_3 + F_1$
- $X_3 = a_{31}X_1 + a_{32}X_1 + a_{33}X_1 + F_1$

$$L = l_1X_1 + l_1X_1 + l_1X_1$$

So $X_i = \Sigma a_{ij}X_j + F_j$ for i=1,2 and 3

and $L = \Sigma\, l_iX_i$

- $\begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} = \begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix} \begin{bmatrix} X1 \\ X2 \\ X3 \end{bmatrix} + \begin{bmatrix} F1 \\ F2 \\ F3 \end{bmatrix}$

X=AX+F
&
L= $\Sigma\, l_i X_i$
So
X=AX+F
X-AX=F
$[I - A]$X=F
X= $[I - A]^{-1}$F
This way we can get the value of X_1, X_2 and X_3.
Let us take one example

Input-output table

	Purchasing sectors		(in rupees crore)	Total output or total revenue
Sectors	Inputs to agriculture	Inputs to industries	Final demand	
Agriculture	50	150	100	300
industries	100	250	150	500
Value added(labour)	150	100	0	250
Total input or total cost	300	500	250	1050

- The first row total shows that agricultural output is valued at Rs. 300 crores per year.Of this total, Rs. 100 crores go directly to final consumption (demand), that is, household and government, as shown in the third column of the first row. The remaining output from agriculture goes as inputs: 50 to itself and 150 to industry. Similarly, the second row shows the distribution of total output of the industrial sector valued at Rs. 500 crores per year. Columns 1, 2 and 3 show that 100 units of manufactured goods go as inputs to agriculture, 250 to industry itself and 150 for final con-sumption to the household sector.
- The first column describes the input or cost structure of the agricultural industry. Agricultural output valued at Rs. 300 crores is produced with the use of agricultural goods worth Rs. 50, manufactured goods worth Rs. 100 and labour services valued at Rs. 150. To put it differently, it costs Rs. 300 crores to get revenue of Rs. 300 crores from the agricultural sector. Similarly, the second column explains the input structure of the industrial sector (i.e., 150 + 250 + 100 = 500).
- Thus "a column gives one point on the production function of the corresponding industry." The 'final demand' column shows what is available for consumption and government expenditure. The third row corresponding to this column has been shown as zero. This means that the household sector is simply a spending (consuming) sector that does not sell anything to itself. In other words, labour is not directly consumed.

Technological coefficient matrix

	Purchasing sectors	
Sectors	Inputs to agriculture	Inputs to industries
Agriculture	50/300=0.17	150/500=0.30
industries	100/300=0.33	250/500=0.50

$$Here\ A=\begin{bmatrix} 0.17 & 0.30 \\ 0.33 & 0.50 \end{bmatrix}$$

$$I=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[I-A]\mathrm{X}=\mathrm{F}$$

So $[I-A]=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$- $\begin{bmatrix} 0.17 & 0.30 \\ 0.33 & 0.50 \end{bmatrix}$=$\begin{bmatrix} 1-0.17 & 0-0.30 \\ 0-0.33 & 1-0.50 \end{bmatrix}$

$$=\begin{bmatrix} 0.83 & -0.30 \\ -0.33 & 0.50 \end{bmatrix}$$

$[I - A] = \begin{bmatrix} 0.83 & -0.30 \\ -0.33 & 0.50 \end{bmatrix}$ *and F*= $\begin{bmatrix} 100 \\ 150 \end{bmatrix}$

$[I - A]^{-1} = \begin{bmatrix} & \end{bmatrix}$

The value of inverse=Adjoint/value of determinant

Value of $[I - A] =$ *0.80*0.50-(-0.30*-0.33)*

=0.400

Adjoint $[I - A] = \begin{bmatrix} 0.50 & 0.30 \\ 0.33 & 0.83 \end{bmatrix}$ *hence*

$$[I-A]^{-1} = \frac{1}{0.40} * \begin{bmatrix} 0.50 & 0.30 \\ 0.33 & 0.83 \end{bmatrix}$$

$$[I-A]^{-1} * F = \frac{1}{0.40} * \begin{bmatrix} 0.50 & 0.30 \\ 0.33 & 0.83 \end{bmatrix} \begin{bmatrix} 100 \\ 150 \end{bmatrix}$$

$$[I-A]^{-1} * F = \begin{bmatrix} (0.50 * 100 + 0.30 * 150)/0.4 \\ 0.33 * 100 + 0.83 * 150)/0.4 \end{bmatrix}$$

$$[I-A]^{-1} * F = \begin{bmatrix} 237.5 \\ 393.75 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Closed model and open model

In open model when value of final demand is given, we can find out absolute level of production. But in a closed model , where value of final demand is not given absolute values cannot be found.