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M.Sc (Sem II) Physics

Unit IV

Representation of group of finite groups

In physics, we are intrested to study group transformations which act on suitable Hilbert spaces of physical systems, each vector of the Hilbert space characterizing a state of the system. Here matrix representing an operator in a Hilbert space is introduced in group's elements. So the study of such matrices are used under the representation theory of group Here it is considered only finite groups, although most of the results either hold good as they are modified to the case of infinite groups.

Representation of Group:-

Let $G = \{E, A, B, C \ldots\}$ be a finite group of order g. Let $T = \{T(E), T(A), T(B), T(C) \ldots\}$ be a collection of non-singular square matrices, all of the same order, which have the properties

$$T(A).T(B) = T(AB)$$

since $AB = C$ in the group G, then

$$T(A).T(B) = T(C)$$

The collection T of the matrices is called the representation of Group. The order of the matrices of T is called the dimension of representation.

Let L_n be an n-dimensional vector space on which the operation of G. Let $\{\phi_i\}$ be an orthogonal basis in L_n. The operation of an element

$A \in G$ on a basis vector is given as

$$A\phi_i = A\sum_{j=1}^{n} \phi_j T_{ji}(A)$$

Where $T(A)$ is the matrix representing A with the basis $\{\phi_i\}$. An element of the matrix $T(A)$ is given as

$$T_{ji}(A) = (\phi_j, A\phi_i)$$

Similarly we can write

$$AB\phi_i = A\sum_{j=1}^{n} \phi_j T_{ji}(B) = \sum_{k,j=1}^{n} \phi_k T_{kj}(A) T_{ji}(B)$$

Thus $AB\phi_i = \sum_{k=1}^{n} \phi_k T_{ki}(AB)$

From above results it is clear that

$$\sum_{k,j=1}^{n} \phi_k T_{kj}(A) T_{ji}(B) = \sum_{k=1}^{n} \phi_k T_{ki}(AB)$$

$\forall\, 1 \le i,k \le n$

or $T(A)T(B) = T(AB)$

If all the matrices of T are distinct, there is clearly a one-to-one correspondence between the elements of G and the matrices of T. In this case, the groups G and T are isomorphic to each other and the representation generated by the matrices of T is called a faithful representation of G. On the other hand, if the matrices of T are not distinct, there exists only a homomorphism from G to T and such a representation is called a unfaithful representation of G.

The simple representation of a group is obtained when we associate unity [a constant number is a spacial case of a matrix - it is a square matrix of order one] . Let us considere the group C_{4V} with correspondance representation one.

Element : $E \; C_4 \; C_4^2 \; C_4^3, m_x, m_y, \sigma_u, \sigma_v$
Representation : 1 1 1 1 1 1 1 1

The set (1, 1, 1, 1, 1, 1) shows form of representation of any group. The product of two elements for an example $C_4 m_x = \sigma_u$. corresponds to $1 \times 1 = 1$ in the representation This is known as the identity representation. which is unfaithful representation of any group. Every group has at least one faithful representation.