Design of Experiments-I

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Definition

- Designing an experiment means deciding how the observations or measurement should be taken to answer a particular question in a valid, efficient and economical way.

- A well designed experiment helps the workers to properly partition the variation of the data into respective component in order to draw valid conclusion.

- The logical construction of the experiment in which the degree of uncertainty with which the inference is drawn may be well defined.

Basic Notations and Terminology

- ***Experiment***
  A device or a means of getting an answer to the problem under consideration. Broadly classified into two categories:

  - ***Absolute:*** *Designed to calculate certain measures of relationships. Eg. Correlation between age and height of a group of people, average marks of a class etc.*

  - ***Comparative:*** *Designed to compare the effect of two more characteristics. Eg. Comparison of effect of two or medicines, fertilizers etc.*

- ***Treatments***
  Various objects of comparison in a comparative experiment are termed as treatments.
  Eg: In an agricultural experiment, different fertilizers, different varieties of crops or different methods of cultivation are treatments.

Contd…

- ***Experimental Unit***
  Smallest division of the experimental material to which we apply the variable under study i.e. treatments.
  Eg: In agricultural experiments, the plot or land is the experimental unit and in medical experiments, the experimental unit may be a patient or a hospital.

- ***Blocks***
  In agricultural experiments, most of the times we divide the whole experimental unit (field) into relatively homogeneous sub-groups or strata, these strata, which are more uniform amongst themselves than the field as a whole are known as blocks.

- ***Yield***
  The measurement of the variable under study on different experimental units are termed as yield.

Contd…

- ***Experimental Error***

  Extraneous or random (or chance or non assignable error) variation in the yield due to the inherent variability in the experimental material to which treatment are applied is called an experimental error.

  The lack of uniformity in the methodology of conducting the experiment and the lack of representative ness of the sample to the population under study are called experimental error.

- ***Precision***

  The degree of uncertainty with which the inferences are drown from the results of experiments is called precision of the experiment. It is measured as the reciprocal of the variance of mean i.e.

$$Precision = \frac{1}{V(\bar{x})} = \frac{1}{\sigma^2/r} = r/\sigma^2$$

where r is treatment replication and σ^2 *is the variance*

Contd…

- ***Uniformity Trials***

  The fertility of the soil does not increase or decrease uniformity in any direction but is distributed over the entire field in an erratic manner. Uniformity trails enable us to have an idea about the fertility variation of the field. By uniformity trail, we mean a trail in which the field (experimental material) is divided into small units (plots) and same treatment is applied on each of the units and their yields are recorded.

Basic Principles of Design of Experiments

- ***Replication***
  Replication means the repetition of an experiment more than once. By replication the experimenter tries to average out as far as possible the effect of uncontrolled factors.

- ***Randomization***
  The allocation of treatments to the various experimental unit in a purely chance manner is called randomization. The purpose of randomization is to assure that the source of variation, not controlled in the experiment, operate randomly so that the average effect of any group of unit is zero.

- ***Local Control***
  The process of reducing the experimental error by dividing the relatively heterogeneous experimental area (field) into homogenous blocks is known as Local control.

Completely Randomized Design

- *The Completely Randomized Design(CRD) is the most simplest design based on principle of replication and randomization.*
- *All treatments are randomly allocated to the experimental units.*
- *It allows every experimental unit to have an equal probability of receive a treatment.*

Layout

- *Let us suppose that we have p treatments and i^{th} treatment is replicated n_i times $(i = 1,2, \dots, p)$.*
- *The layout is given as:*

Treatments	Replications				Total $(y_{i.})$
1	y_{11}	y_{12}	…	y_{1n_1}	$y_{1.}$
2	y_{21}	y_{22}	…	y_{2n_2}	$y_{2.}$
…	…	…	…	…	…
p	y_{p1}	y_{p2}	…	y_{pn_p}	$y_{p.}$

Statistical Analysis of CRD

- *The analysis of the CRD is analogous to one-way analysis of variance (ANOVA).*
- *Let us suppose that we have p treatments and i^{th} treatment is replicated n_i times $(i = 1,2, \dots p)$.*
- *The model is given as:*

$$\boldsymbol{y_{ij} = \mu + \alpha_i + \epsilon_{ij}}\ ; \quad i = 1,2, \dots p\ ; j = 1,2, \dots, n_i$$

where

- y_{ij} *is the* j^{th} *observation of* i^{th} *treatment.*
- μ *is general mean effect.*
- α_i *is the effect of* i^{th} *treatment.*
- ϵ_{ij} *is the error.*
- *Also* $n = \sum n_i$ *is the total number of observations.*
- ***Model Assumptions:*** $\sum_{i=1}^{p} \alpha_i = 0$; ϵ_{ij} *are i.i.d.* $N(0, \sigma^2)$.

ANOVA Table for CRD

Source of variation	Degree of Freedom (d.f.)	Sum of squares (SS)	Mean sum of squares (MSS)	F- ratio
Treatment	$p-1$	SST	$MST = \frac{SST}{p-1}$	$F_T = \frac{MST}{MSE}$
Error	$n-p$	SSE	$MSE = \frac{SSE}{n-p}$	
Total	$n-1$	TSS		

where

- $TSS = \sum_{i=1}^{p}\sum_{j=1}^{n_i} y_{ij}^2 - \frac{G^2}{n}$; $\quad SST = \sum_{i=1}^{p} \frac{y_{i.}{}^2}{n_i} - \frac{G^2}{n}$
- $SSE = TSS - SST$
- here $y_{i.}$ = total for the i^{th} treatment.
- $G = \sum_{i=1}^{p}\sum_{j=1}^{n_i} y_{ij}$ = Grand total

Hypotheses

- ***Null Hypothesis***

  $H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_p = 0$ *i.e., the effect of each treatments is same.*

  or $\mu_1 = \mu_2 = \cdots = \mu_p$ *i.e., the mean effect of each treatments is same.*

- ***Alternative Hypothesis***

  H_1: *The effect of at least two treatments are not same.*

- ***Test statistics is***

$$F_T = \frac{MST}{MSE} \sim F_{p-1,n-p}$$

- *If* $F_T > F_{(p-1,n-p)}(\alpha)$, *then* H_0 *is rejected at* $100\alpha\%$ *level of significance and we conclude that treatments differ significantly, otherwise not.*

Advantages of CRD

- *The layout of design is easy.*
- *Very useful to conduct small experiments.*
- *The no. of replications need not to be same for each treatment.*
- *The CRD provides maximum d.f. for estimation of error variance, which increases the sensitivity or the precision of the experiments when the number of treatments are small.*
- *The statistical analysis remains simple even if some or all observations for any treatment are rejected or lost or missing for some purely accidental reasons.*
- *CRD results in the maximum use of the experimental units since all the experimental material can be used.*

Disadvantages of CRD

- *If experimental materials are not homogenous, the design suffers from the disadvantage of being inherently less informative than other more sophisticated designs.*
- *Not suited for a large number of treatments.*

Uses of CRD

- *Under conditions where the experimental material is homogenous e.g. in physics, chemistry and biological experiment for some green house studies.*
- *CRD may be used in a chemical or baking experiment where the experimental units are the part of the thoroughly mixed chemical or powder.*

Example

- *A company is considering three different covers for boxes of a brand of cereal. Box type A has picture of a sports hero eating the cereal, type B has a picture of a child eating the cereal, and type C has a picture of a bowl of the cereal. The company wants to determine which cereal box type provides for the most sales. Eighteen test markets were selected by the company and each box type was randomly assigned to six markets. The number of boxes of this cereal sold per 10000 population in a specified period is recorded for each test market. The data are as follows:*

Type A	52.4	47.8	52.4	51.3	50	52.1
Type B	50.1	45.2	46	46.5	47.4	46.2
Type C	49.2	48.3	49	47.2	48.6	48.2

- *Test whether there is any significant difference in the mean of three means.*

Solution

- H_0: No difference in means of three treatments i.e. $\mu_i = \mu \; \forall i = 1,2,3.$

v/s H_1: At least two means are not same i.e., $\mu_i \neq \mu$ for some i

- We have, $p = 3, n_i(q\ say) = 6 \; \forall \; i = 1,2,3$ and $n = 18$

therefore,

- $TSS = \sum_{i=1}^{p} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{G^2}{n}$

$$= (52.4^2 + 47.8^2 + \cdots + 48.6^2 + 48.2^2) - \frac{877.9^2}{18} = 85.40$$

- $SST = \sum_{i=1}^{p} \frac{y_{i\cdot}^{\;2}}{n_i} - \frac{G^2}{n}$

$$= \frac{1}{6}(306.0^2 + 281.4^2 + 290.5^2) - \frac{877.9^2}{18} = 51.57$$

- $SSE = TSS - SST = 85.40 - 51.57 = 33.83$

ANOVA Table

Source of variation	d.f.	Sum of squares	Mean sum of squares	F- ratio
Treatment	2	51.57	25.79	11.41
Error	15	33.83	2.26	
Total	17	85.40		

- *The tabulated value* $(F_{(2,15)}(0.05)) = 3.86$.
- *Hence F-ratio > tabulated* $(11.41 > 3.86)$, *so, we reject null hypothesis.*
- *Thus, there is a significant difference between the means of three treatments at the 5% level of significance.*

Randomized Block Design (RBD)

- *If a large number of treatments are to be compared, then a large number of experimental units are required, therefore, the variation among the responses/experimental units will increase or experimental material will not be homogeneous, then CRD may not be appropriate to use.*
- *In such situation, RBD enables us to take care of the variability among the experimental units by dividing the experimental area into smaller homogenous blocks.*
- *RBD deals with blocking is done in one direction.*
- *All the three principles of designs i.e. replication, randomization and local control are used in RBD.*

Layout

- *Let us suppose that we have p treatments and q blocks.*
- *The layout is as follows:*

Blocks / Treatments	1	2	…	q	Total $(y_{i.})$
1	y_{11}	y_{12}	…	y_{1q}	$y_{1.}$
2	y_{21}	y_{22}	…	y_{2q}	$y_{2.}$
…	…	…	…	…	…
p	y_{p1}	y_{p2}	…	y_{pq}	$y_{p.}$
Total $(y_{.j})$	$y_{.1}$	$y_{.2}$	…	$y_{.q}$	$y_{..}$

Statistical Analysis of RBD

- *The analysis of the RBD is analogous to two-way ANOVA.*
- *Let us suppose we have p treatments and q blocks that are to be compared..*
- *The model is given as:*

$$\boldsymbol{y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij}}\ ; \quad i = 1,2, \dots p\ ; j = 1,2, \dots, q$$

where,

- y_{ij} *is the* j^{th} *block observation of* i^{th} *treatment.*
- μ *is general mean effect.*
- α_i *is the effect of* i^{th} *treatment.*
- β_j *is the effect of* j^{th} *block.*
- ϵ_{ij} *is the error and it is independently normally distributed with mean zero and variance* σ^2.
- *Also* $n = pq$ *is the total number of observations.*
- ***Model Assumptions:*** $\sum_{i=1}^{p} \alpha_i = \sum_{j=1}^{q} \beta_j = 0$; ϵ_{ij} *are i.i.d.* $N(0, \sigma^2)$.

ANOVA Table for RBD

Source of variation	Degree of Freedom (d.f.)	Sum of squares (SS)	Mean sum of squares (MS)	F- ratio
Treatment	$p-1$	SST	$MST = \frac{SST}{p-1}$	$F_T = \frac{MST}{MSE}$
Blocks	$q-1$	SSB	$MSB = \frac{SSB}{q-1}$	$F_T = \frac{MSB}{MSE}$
Error	$(p-1)(q-1)$	SSE	$MSE = \frac{SSE}{(p-1)(q-1)}$	
Total	$n-1$	TSS		

where

- $TSS = \sum_{i=1}^{p}\sum_{j=1}^{q} y_{ij}^2 - \frac{G^2}{n}$;
- $SSB = \frac{1}{p}\sum_{j=1}^{q} y_{.j}^2 - \frac{G^2}{n}$;
- $y_{i.}$ = total for the i^{th} treatment
- $y_{.j}$ = total for the j^{th} block.
- $G = \sum_{i=1}^{p}\sum_{i=1}^{q} y_{ij} = y_{..}$ =Grand total

$$SST = \frac{1}{q}\sum_{i=1}^{p} y_{i.}^{\ 2} - \frac{G^2}{n}$$

$$SSE = TSS - SST - SSB$$

Hypotheses

- ***Null Hypotheses***

  $H_{0T}: \alpha_1 = \alpha_2 = \cdots = \alpha_p = 0$ *i.e., the effect of each treatment is same.*

  $H_{0B}: \beta_1 = \beta_2 = \cdots = \beta_q = 0$ *i.e., the effect of each block is same.*

- ***Alternative Hypotheses***

  H_{1T}: *The effect of at least two treatments are not same.*

  H_{1B}: *The effect of at least two blocks are not same.*

- ***Test statistics are***

  $$F_T = \frac{MST}{MSE} \sim F_{\{p-1,(p-1)(q-1)\}} \text{ and } F_B = \frac{MSB}{MSE} \sim F_{\{q-1,(p-1)(q-1)\}}$$

- *If* $F_T > F_{\{p-1,(p-1)(q-1)\}}(\alpha)$, *then* H_0 *is rejected at* $100\alpha\%$ *level of significance and we conclude that the treatments effect differ significantly, otherwise not.*

- *If* $F_B > F_{\{q-1,(p-1)(q-1)\}}(\alpha)$, *then* H_0 *is rejected at* $100\alpha\%$ *level of significance and we conclude that the blocks effect differ significantly, otherwise not.*

Advantages of RBD

- ***Accuracy***

  This design has been shown to be more efficient or accurate than C.R.D for most types of experimental work. The elimination of between S.S, usually results in a decrease of error mean S.S.

- ***Flexibility***

  In R.B.D. no restrictions are placed on the number of treatments or the number of replicates. In general, at least two replicates are required to carry out the test of significance (Factorial design is an exception). In addition, control (check) or some other treatments may be included more than once without complications in the analysis.

- ***Ease of analysis***

  Statistical analysis is simple and rapid. Moreover the error of any treatment can be isolated and any number of treatments may be omitted from the analysis without complicating it.

Disadvantages of RBD

- *As the number of treatments increases then blocks size increases, therefore, we have lesser control over error.*
- *RBD may give misleading results if blocks are not homogeneous.*
- *If the data on more than two plots is missing, the statistical analysis becomes quite tedious and complicated.*

Example

- *A seed company performs an experiment to compare four varieties of rice. Five fields are available for the study and each field is subdivided into four plots of equal size. Each variety is randomly assigned to a plot, and the yield in bushels is recorded as follows:*

		Field				
		1	**2**	**3**	**4**	**5**
Variety of rice	**1**	45	37	41	48	32
	2	47	41	38	46	37
	3	53	47	50	56	45
	4	38	32	40	43	29

- *Test whether there is any significant difference in the yield mean according to variety of rice and fields..*

Solution

- $\boldsymbol{H_{0A}}$: *There is no difference among the yield means of varieties i.e.*

  $\mu_{1A} = \mu_{2A} = \mu_{3A} = \mu_{4A}$

v/s $\boldsymbol{H_{1A}}$: *At least two means are not same.*

- $\boldsymbol{H_{0B}}$: *There is no difference among the yield means of fields i.e.*

  $\mu_{1B} = \mu_{2B} = \mu_{3B} = \mu_{4B} = \mu_{5B}$

v/s $\boldsymbol{H_{1B}}$: *At least two means are not same.*

- *We have,* $p = 4\ q = 5$, *and* $n = pq = 20$, *therefore,*
- $TSS = \sum_{i=1}^{p} \sum_{j=1}^{q} y_{ij}^2 - \frac{G^2}{n}$

$$= (45^2 + 47^2 + \cdots + 45^2 + 29^2) - \frac{845^2}{20} = 957.75$$

Contd…

- $SST = \frac{1}{q}\sum_{i=1}^{p} y_{i}.^{2} - \frac{G^2}{n}$

$$= \frac{1}{5}(203^2 + 209^2 + 251^2 + 182^2) - \frac{845^2}{20} = 501.75$$

- $SSB = \frac{1}{p}\sum_{i=1}^{q} y_{.j}^{2} - \frac{G^2}{n}$

$$= \frac{1}{4}(183^2 + \cdots + 143^2) - \frac{845^2}{20} = 398.00$$

- $SSE = TSS - SSA - SSB$

$$= 957.75 - 501.75 - 398 = 58.00$$

ANOVA Table

Source of variation	d.f.	Sum of squares	Mean sum of squares	F- ratio
Variety	3	501.75	167.25	34.63
Field	4	398.00	99.50	20.60
Error	12	58.00	4.83	
Total	19	957.75		

- *The tabulated values are* $F_{(3,12)}(0.05) = 3.49$ *and* $F_{(4,12)}(0.05) = 3.26$.
- *The F-ratio > tabulated value* $(34.63 > 3.49)$, *so, we reject null hypothesis* H_{0A} *and conclude that there is a significant difference among the means of four varieties of rice at the 5% level of significance.*
- *The F-ratio > tabulated value* $(20.60 > 3.26)$, *so, we reject null hypothesis* H_{0B} *and conclude that there is a significant difference among the means of five fields at the 5% level of significance.*

Latin Square Design (LSD)

- *The randomized block design is a design that reduces the residual error in an experiment by removing the variability due to a known and controllable nuisance variable.*
- *The Latin square design is used to eliminate two nuisance sources of variability; that is, it systematically allows blocking in two directions.*
- *The rows and columns in a Latin square design represent two restrictions/directions.*
- *Each of the resulting squares contains one letter corresponding to a treatment, and each letter occurs once and only once in each row and each column.*
- *Treatments are assigned at random within rows and columns, with each treatment once per row and once per column.*
- *There are equal numbers of rows, columns, and treatments.*
- *There are some other designs that utilizes the blocking principle.*

Layout

- *The layout of Latin square for $m = 4$ treatments is given below:*

Columns / *Rows*	1	2	3	4
1	A	B	C	D
2	B	C	D	A
3	C	D	A	B
4	D	A	B	C

Statistical Analysis of LSD

- *The model is given as:*

$$\boldsymbol{y_{ijk} = \mu + \alpha_i + \beta_j + + \gamma_k + \epsilon_{ijk}}\,; \quad i, j, k = 1,2, \ldots m$$

where

- y_{ijk} *is the* k^{th} *treatment observation for* $(i, j)^{th}$ *cell.*
- μ *is general mean effect.*
- α_i *is the effect of* i^{th} *row*
- β_j *is the effect of* j^{th} *column.*
- γ_k *is the effect of* k^{th} *treatment.*
- ϵ_{ijk} *is the error .*
- *Also* $n = m^2$ *is the total number of observations.*
- ***Model Assumptions:*** $\sum_{i=1}^{m} \alpha_i = \sum_{j=1}^{m} \beta_j = \sum_{k=1}^{m} \gamma_k = 0$ *and* ϵ_{ij} *are i.i.d.*$N(0, \sigma^2)$.

ANOVA Table for LSD

Source of variation	***d.f.***	***Sum of squares***	***Mean sum of squares***	***F- ratio***
Row	$m-1$	SSR	$MSR = \frac{SSR}{m-1}$	$F_R = \frac{MSR}{MSE}$
Column	$m-1$	SSC	$MSC = \frac{SSC}{m-1}$	$F_C = \frac{MSC}{MSE}$
Treatment	$m-1$	SST	$MST = \frac{SST}{m-1}$	$F_T = \frac{MST}{MSE}$
Error	$(m-1)(m-2)$	SSE	$MSE = \frac{SSE}{(m-1)(m-2)}$	
Total	$n-1$	TSS		

where

- $TSS = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} y_{ijk}^2 - \frac{G^2}{n}$
- $SSC = \frac{1}{m}\sum_{i=1}^{m} y_{i..}^2 - \frac{G^2}{n}$

Contd…

- $SSB = \frac{1}{m}\sum_{i=1}^{m} y_{.j.}^2 - \frac{G^2}{n}$;
- $SST = \frac{1}{m}\sum_{i=1}^{m} y_{..k}^2 - \frac{G^2}{n}$
- $SSE = TSS - SST - SSC - SSB$
- $y_{i..}$ = *total for the* i^{th} *row.*
- $y_{.j.}$ = *total for the* j^{th} *column.*
- $y_{..k}$ = *total for the* k^{th} *treatment.*
- $G = \sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m} y_{ijk} = y_{...}$ =*Grand total*

Hypotheses

- ***Null Hypotheses***

  $H_{0R}: \alpha_1 = \alpha_2 = \cdots = \alpha_m = 0$ i.e., the effect of each row is same.

  $H_{0C}: \beta_1 = \beta_2 = \cdots = \beta_m = 0$ i.e., the effect of each column is same.

  $H_{0T}: \gamma_1 = \gamma_2 = \cdots = \gamma_m = 0$ i.e., the effect of each treatment is same.

- ***Alternative Hypotheses***

  H_{1R}: The effect of at least two rows are not same.

  H_{1C}: The effect of at least two columns are not same.

  H_{1T}: The effect of at least two treatments are not same.

Hypotheses

- ***Test statistics are***

$$F_R = \frac{MSR}{MSE} \sim F_{\{m-1,(m-1)(m-2)\}}$$

$$F_C = \frac{MSC}{MSE} \sim F_{\{m-1,(m-1)(m-2)\}}$$

$$F_T = \frac{MST}{MSE} \sim F_{\{m-1,(m-1)(m-2)\}}$$

- If $F_R > F_{\{m-1,(m-1)(m-2)\}}(\alpha)$, then H_0 is rejected at 100α% level of significance and we conclude that rows effect differ significantly, otherwise not.
- If $F_C > F_{\{m-1,(m-1)(m-2)\}}(\alpha)$, then H_0 is rejected at 100α% level of significance and we conclude that columns effect differ significantly, otherwise not.
- If $F_T > F_{\{m-1,(m-1)(m-2)\}}(\alpha)$, then H_0 is rejected at 100α% level of significance and we conclude that treatments effect differ significantly, otherwise not.

Example

- *Five levels of fertilizers were tried in a* 5×5 *Latin square to see its effect on the yield of wheat. The yield are given below in kg. per plot along with the layout:*

B 37.0	C 35.9	E 30.9	A 28.2	D 35.8
D 37.3	E 38.3	A 26.9	B 36.6	C 37.6
C 34.8	A 27.4	B 34.2	D 37.4	E 34.4
E 31.3	B 38.4	D 38.0	C 39.4	A 30.3
A 24.2	D 38.0	C 36.8	E 30.8	B 34.5

- *Test whether there is any significant difference in the means due to 3 factors i.e, row, column & treatment (fertilizer).*

Solution

- **Null Hypotheses**

  $H_{0R}: \alpha_1 = \alpha_2 = \cdots = \alpha_5 = 0$ i.e., the effect of each row is same.

  $H_{0C}: \beta_1 = \beta_2 = \cdots = \beta_5 = 0$ i.e., the effect of each column is same.

  $H_{0T}: \gamma_1 = \gamma_2 = \cdots = \gamma_5 = 0$ i.e., the effect of each treatment is same.

- **Alternative Hypotheses**

  H_{1R}: The effect of at least two rows are not same.

  H_{1C}: The effect of at least two columns are not same.

  H_{1T}: The effect of at least two treatments are not same.

- *We have,* $m = 5$, *and* $n = m^2 = 25$, *therefore,*
- $TSS = \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{k=1}^{m} y_{ijk}^2 - \frac{G^2}{n}$

  $= (37.0^2 + 37.3^2 + \cdots + 30.3^2 + 34.5^2) - \frac{854.4^2}{25} = 429.67$

Contd…

- $SSR = \frac{1}{m}\sum_{i=1}^{m} y_{i..}^2 - \frac{G^2}{n}$

  $= \frac{1}{5}(167.8^2 + \cdots + 164^2) - \frac{854.4^2}{25} = 27.27$

- $SSC = \frac{1}{m}\sum_{i=1}^{m} y_{.j.}^2 - \frac{G^2}{n}$

  $= \frac{1}{5}(178.0^2 + \cdots + 172.6^2) - \frac{854.4^2}{25} = 22.41$

- $SST = \frac{1}{m}\sum_{i=1}^{m} y_{..k}^2 - \frac{G^2}{n}$

  $\frac{1}{5}(137.0^2 + \cdots + 165.7^2) - \frac{854.4^2}{25} = 340.13$

- $SSE = TSS - SST - SSR - SSC$

  $= 429.67 - 340.13 - 27.27 - 22.41 = 39.86$

ANOVA Table

Source of variation	d.f.	Sum of squares	Mean sum of squares	F- ratio
Row	4	27.27	6.82	2.74
Column	4	22.41	5.60	2.25
Treatment	4	340.12	85.03	34.15
Error	12	39.87	2.49	
Total	24	429.67		

- *The tabulated values is* $F_{(4,12)}(0.05) = 3.26$.
- *The F-ratios (2,74 & 2.25) < tabulated value (3.26), so, we do not reject null hypotheses* H_{0R} *and* H_{0C} *and conclude that there is no significant difference between the effects of rows and columns, respectively at 5% level of significance.*
- *The F-ratio (34.15) > tabulated value (3.26), so, we reject null hypothesis* H_{0T} *and conclude that there is a significant difference between the effects of fertilizers at the 5% level of significance.*

Thank you