Steady State Analysis of Single phase AC Circuit

2.1 Circuit with pure resistance only

A pure resistance is that in which there is ohamic voltage drop only. consider a circuit having a pure resistance R as shown in fig.1 below.

Let the instantaneous value of the alternating voltage applied be

$$v = V_m \sin \omega t$$

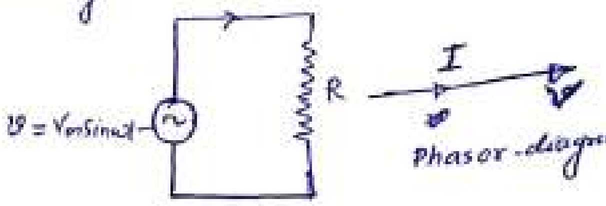

Fig.1

the instantaneous value of current

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$

* Phase difference between current and voltage waveform is zero
* instantaneous power $p(t) = v(t)\, i(t)$

$$p(t) = V_m \sin \omega t \; I_m \sin \omega t$$

$$\boxed{p(t) = \frac{V_m I_m}{2} [1 - \cos 2\omega t]}$$

2.2 Circuit with Pure inductance only

A pure inductive circuit possesses only inductance and no resistance or capacitance as shown in fig.2

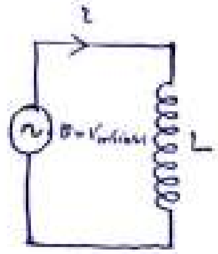

Fig.2

Let the applied voltage $v = V_m \sin \omega t$ —— ①

when an alternating voltage is applied to it, a back emf $(-L\frac{di}{dt})$ of self inductance is induced in it. As there is no ohmic resistance drop, the applied voltage has to oppose the self induced emf only. So the applied voltage is equal and opposite to back emf at all instants.

instantaneous value of self induced emf is v'.

$$v' = -L\frac{di}{dt} = -v$$

$$\boxed{v' = -v}$$

$$-L\frac{di}{dt} = -V_m \sin\omega t$$

$$di = \frac{V_m}{L}\sin\omega t \cdot dt$$

integrating both side

$$\int di = \frac{V_m}{L}\int \sin\omega t \cdot dt$$

$$i = \frac{V_m}{\omega L}(-\cos\omega t)$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$i = \frac{V_m}{X_L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)} \quad — (2)$$

$\boxed{X_L = \omega L}$ Ohm

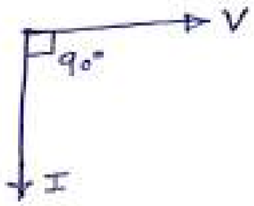

fig. 3(a) Phasor diagram

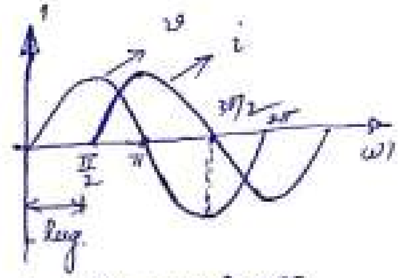

fig. 3(b) wave form

Observing eq (1) and eq (2) we find that current lags the applied voltage by 90° or $\frac{\pi}{2}$ radian. $\boxed{j = 1\angle 90^\circ}$

Circuit impedance $Z = \frac{V_m}{I_m} = \frac{V_m \angle 0^\circ}{\frac{V_m}{\omega L}\angle -\frac{\pi}{2}} = \omega L \angle \frac{\pi}{2} = j\omega L$

The quantity ωL is called inductive reactance and is usually devoted by symbol X_L and units is ohms

Average Power:-

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} v \cdot i \; d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \cdot I_m \sin\left(\omega t - \frac{\pi}{2}\right) d(\omega t)$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} - V_m I_m \sin\omega t \cos\omega t \cdot d(\omega t)$$

$$= -\frac{V_m I_m}{2\pi}\int_0^{2\pi} \frac{\sin 2\omega t}{2} \, . d(\omega t)$$

$$= 0$$

Note. (1) The average power consumption in an inductive circuit is zero, and is periodic with twice the supply frequency.

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2.3 Circuit with Pure Capacitance only:

Applied voltage $v = V_m \sin \omega t$ —— (1)

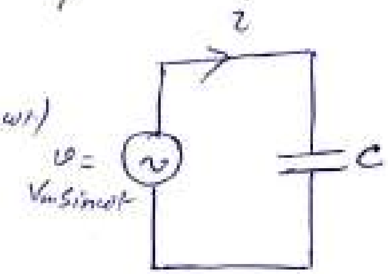


Fig. 4 (a)

Current $i(t) = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin \omega t)$

$$i(t) = \underbrace{V_m \omega C}_{I_m} \cos \omega t$$

$$i(t) = I_m \cos \omega t$$

$$i(t) = I_m \sin(\omega t + 90^\circ) \quad \text{—— (2)}$$

from eq (1) and eq (2) it is observed that current lead the applied voltage by 90°.

Here $I_m = \omega C V_m = \frac{V_m}{1/\omega C}$, $\boxed{X_C = \frac{1}{\omega C}}$

The quantity $1/\omega C$ is called inductive capacitance and is usually denoted by X_C.

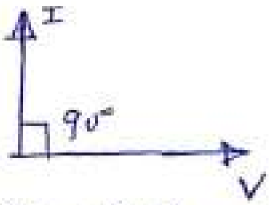


Fig. 4 (b) Phasor diagram

(a) Circuit impedance:-

Impedance $Z = \frac{V}{I} = \frac{V_m \angle 0^\circ}{I_m \angle 90^\circ}$

$$Z = \frac{V_m}{I_m} \angle -90^\circ$$

$$Z = X_C \angle -90^\circ \quad \left[\text{Since } \frac{V_m}{I_m} = X_C\right]$$

$$Z = -jX_C = -\frac{j}{\omega C}\ \Omega$$

$$\boxed{Z = -\frac{j}{\omega C}\ \Omega}$$

(b) Average Power:-

Instantaneous power $P = v \cdot i = V_m \sin \omega t \cdot I_m \sin(\omega t + \frac{\pi}{2}) = V_m I_m \sin \omega t \cos$

$$P = \frac{V_m I_m}{2} \sin 2\omega t$$

$$P_{av} = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\omega t \, d(\omega t) = 0$$

This shows that power consumed in purely capacitive circui is zero.

AC Through series RL Circuit

$V_R \rightarrow$ voltage drop across resistance

$V_L \rightarrow$ voltage across reactance

$V_R = IR \longrightarrow$ in phase with current vector I

$V_L = IX_L = I\omega L \rightarrow 90°$ ahead of current vector I

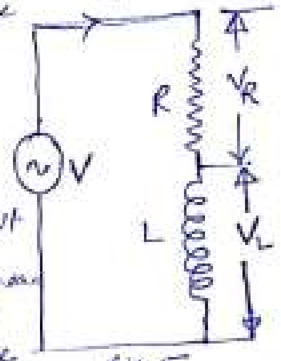


Fig. 5

$v = V_m \sin\omega t$ — instantaneous value of voltage

V = rms or effective value of applied voltage

I = rms or effective value of current

ⓐ Phasor diagram :-

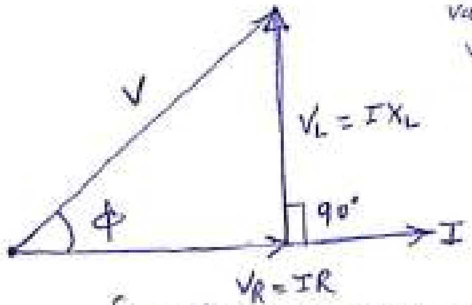


Fig. 5 ⓐ voltage Triangle

ⓑ Circuit impedance -

Applying KVL in fig. 5

$$V = V_R \angle 0° + V_L \angle 90°$$

$$IZ = I \cdot R + j\, I X_L$$

$$\boxed{Z = R + j X_L}$$

Magnitude of circuit impedance

$$|Z| = \sqrt{R^2 + X_L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$Z = |Z| \angle \phi$$

ⓒ Circuit Current

$$I = \frac{V}{Z} = \frac{V\angle 0°}{|Z|\angle\phi}$$

$$I = \frac{|V|}{|Z|} \angle -\phi$$

instantaneous value of current $\boxed{i = I_m \sin(\omega t - \phi)}$

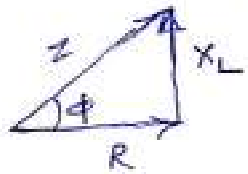
where $I_m = \dfrac{|V|}{|Z|}$

ⓓ Phase difference b/w Applied voltage and current ;-

In R-L circuit current lags the applied voltage by angle ϕ.

$\phi = \tan^{-1}\frac{X_L}{R}$

ⓔ Impedance Triangle -



$$\cos\phi = \frac{R}{Z}$$

$\cos\phi$ is called the power factor of the circuit.

The power factor is lagging in an inductive circuit.

2.5: AC Through Series RC Circuit

$v = V_m \sin\omega t \rightarrow$ instantaneous applied voltage

V = rms or effective value of applied voltage

$V_R = IR \rightarrow$ in phase with current I

$V_C = I \cdot \frac{1}{\omega C} = I X_C$, 90° lagging with current I

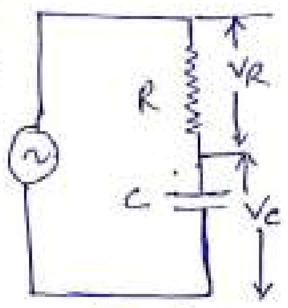


fig. 6

ⓐ Phasor diagram –

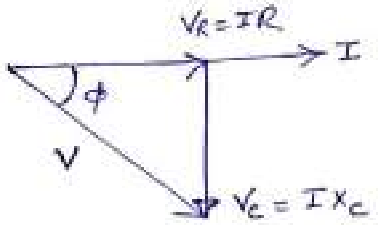


fig. 6 ⓐ Voltage triangle

$$V = \sqrt{V_R^2 + V_C^2}$$

Circuit current I leads the applied voltage by an angle ϕ.

ⓑ Circuit Impedance :-

By KVL –

$$V = V_R \angle 0° + V_C \angle -90°$$

$$IZ = IR + \frac{I_C}{j\omega C} \quad \left(\frac{1}{j} = 1\angle -90°\right)$$

$$\boxed{Z = R - jX_C}$$

$$|Z| = \sqrt{R^2 + X_C^2}$$

$$\phi = -\tan^{-1}\left[\frac{\text{Imaginary Part}}{\text{real part}}\right] = -\tan^{-1}\left(\frac{-X_C}{R_C}\right) = -\tan^{-1}\left(\frac{X_C}{R}\right)$$

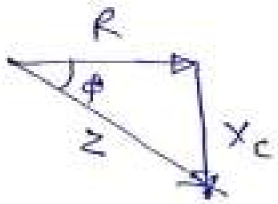


fig. 6 ⓑ impedance Triangle

ⓒ Power factor :-

$$\text{Power factor } \cos\phi = \frac{V_R}{V} = \frac{R}{Z} \quad (\text{Leading})$$

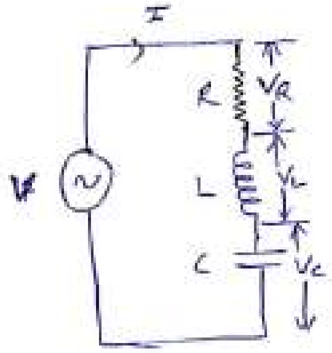
ⓓ instantaneous value of current –

$$I = I_m \sin(\omega t + \phi)$$

2.6 AC Through Series RLC Circuit

~~By KVL~~

(a) Circuit Impedance :-



By KVL

$$V = V_R \angle 0 + V_L \angle 90^\circ + V_C \angle -90^\circ$$

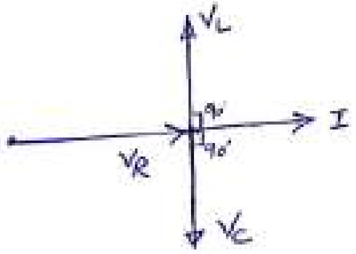
$$IZ = IR + jIX_L - jIX_C$$

$$\boxed{Z = R + j(X_L - X_C)}$$

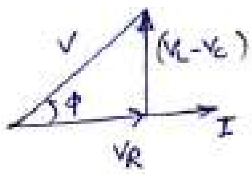
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

(b) Phasor diagram :-



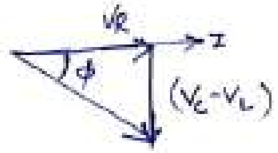
→ Case-1 when $V_L > V_C$



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

ϕ → lagging

→ Case-2 - when $V_C > V_L$



$$V = \sqrt{V_R^2 + (V_C - V_L)^2}$$

ϕ → leading

→ Case-3. when $V_L = V_C$

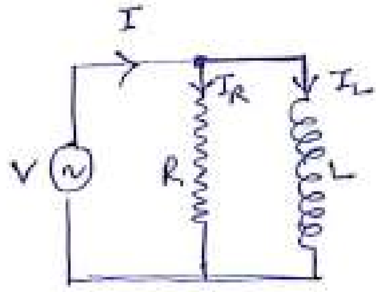
$$\boxed{V = V_R}$$

$$\phi = 0^\circ$$

2.7 R-L Parallel Circuit:-

By KCL -

$\vec{I} = \vec{I_R} + \vec{I_L}$



~~I = Vp~~

$\bar{I} = I_R \angle 0° + I_L \angle -90°$

$\frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L}$

Note:- In parallel ckt voltage is taken as reference.

$\frac{1}{Z} = \frac{1}{R} - j\frac{1}{X_L}$

$$\boxed{Y = G - jB_L}$$

$Y \rightarrow$ admittance

$B_L \rightarrow$ Inductive susceptance $= \frac{1}{X_L}$

(a) Phasor diagram

(a)

$I = \sqrt{I_R^2 + I_L^2}$

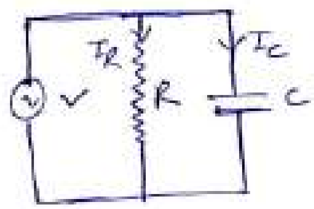
$\phi = \tan^{-1}\left(\frac{-I_L}{I_R}\right)$

$Y = \sqrt{G^2 + B_L^2}$

$\phi = \tan^{-1}\left(\frac{-B_L}{G}\right)$

2.8 R-C- Parallel Circuit

(a) By KCL



$I = I_R \angle 0° + I_C \angle +90°$

$\frac{V}{Z} = \frac{V}{R} + j\frac{V}{X_C}$

$\frac{1}{Z} = \frac{1}{R} + j\frac{1}{X_C}$

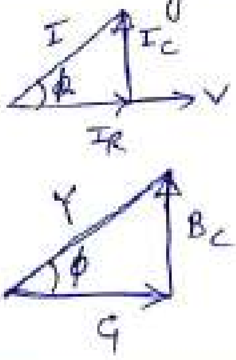
$$\boxed{Y = G + jB_C}$$

Y = admittance $= \frac{1}{Z}$

G = conductance $= \frac{1}{R}$

B_C = Capacitive susceptance $= \frac{1}{X_C}$

(b) Phasor diagram



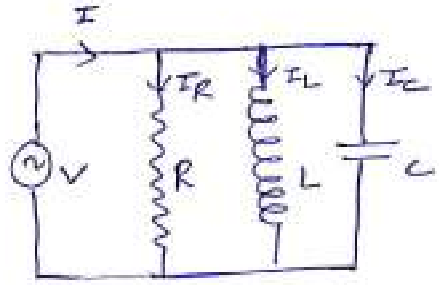
2.9. Parallel RLC Circuit:

By KCL.

$$I = I_R \angle 0^\circ + I_L \angle -90^\circ + I_C \angle +90^\circ$$

$$\frac{V}{Z} = \frac{V}{R} - j\frac{V}{X_L} + j\frac{V}{X_C}$$

$$\boxed{Y = G + j(B_C - B_L)}$$



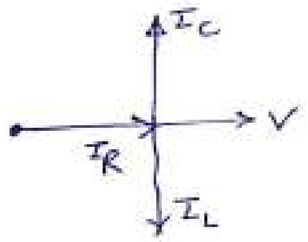
(a)

$$G = \frac{1}{R}$$

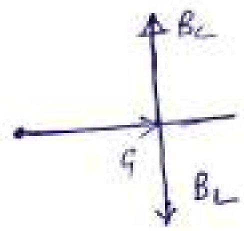
$$B_C = \frac{1}{X_C}$$

$$B_L = \frac{1}{X_L}$$

Phasor diagram:-



(b)



(c)

Problems on Series RLC Circuit

Q.1 A two element series circuit is connected across an AC source $V = 300\cos(314t + 20^\circ)$ volt. The current is drawn $15\cos(314 - 10^\circ)$ Amp. Determine circuit impedance magnitude and phase angle. What is average power drawn.

$\sin(\theta + 90^\circ) = \cos\theta$

Sol: Applied voltage $V = 300\cos(314t + 20^\circ)$

$V = 300\sin(314t + 20^\circ + 90^\circ)$

$V = 300\sin(314t + 110^\circ)$

$V = 300/\sqrt{2} \angle 110^\circ$

Circuit Current $i = 15\cos(314t - 10^\circ)$

$i = 15\sin(314t - 10^\circ + 90^\circ)$

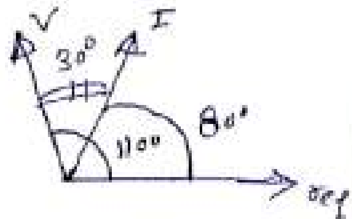
$i = 15\sin(314t + 80^\circ)$

$i = \frac{15}{\sqrt{2}} \angle 80^\circ$

Circuit impedance $Z = \frac{V}{I}$

$$Z = \frac{300/\sqrt{2} \angle 110^\circ}{15/\sqrt{2} \angle 80^\circ}$$

$Z = 20 \angle 30^\circ$



The angle b/w V & I $= 30^\circ$

Current lag the voltage by 30°

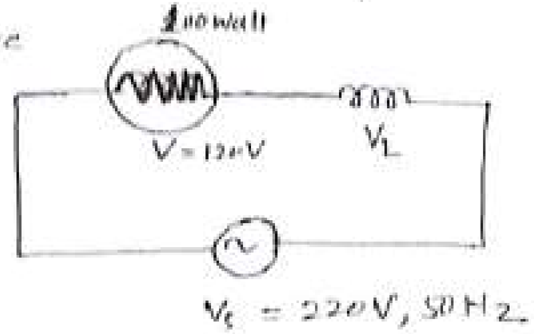
$P_{av} = V_{rms} \cdot I_{rms} \cos\phi$

$= \frac{300}{\sqrt{2}} \times \frac{15}{\sqrt{2}} \cos 30^\circ$

$= 1949.85$ Watt

Q.1 A 120V, 100W lamp is to be connected to a 220V, 50Hz AC supply. What value of pure inductance should be connected in series in order to run the lamp on rated voltage.

Hint :- Lamp is resistive
Solve with the help
of voltage triangle.
Ans – $L = 0.7046H$



Q.2 A coil connected to 100V DC supply draws 10 Amp and same coil connected 100V, ac voltage of frequency 50Hz draws 5 Amp. Calculate the parameter (~~either~~ R, L & C) of coil and power-factor.

Hint :- (i) in case of dc, $X_L = \omega L = 0$ (as $\omega = 2\pi f, \because f = 0$)

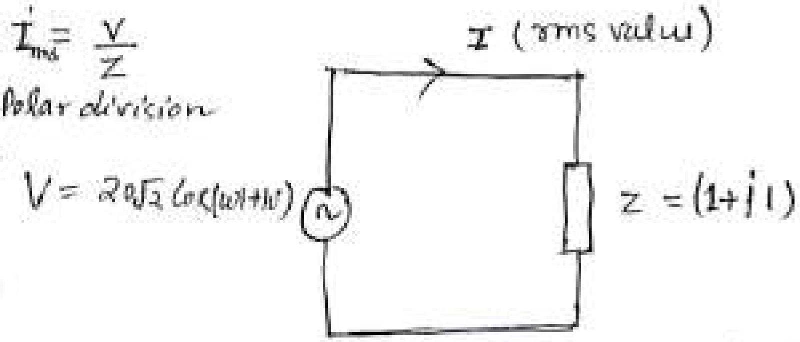
(ii) $z = \frac{V}{I}$

Ans – $L = 0.05H$, $\cos\phi = 0.5$ lagging

Q.3 A Load having impedance of $(1+j1)\Omega$ is connected to an AC voltage represented as $V = 20\sqrt{2}\cos(\omega t + 10°)$ Volt. Find the current in load expressed in the form of $i = I_m \sin(\omega t + \phi)$ A.

Ans – $i = 20\sin(\omega t + 55°)$

Hint : (i) $I_{rms} = \frac{V}{Z}$
(ii) Polar division



Q.4 An ac emf is given by $100\sin(314t - \frac{\pi}{4})$ volts in applied to a circuit and the current is $20\sin(314t - \frac{\pi}{2})$ A. Find (i) frequency (ii) circuit element (~~either~~ R, L or C)

Ans – $f = 50Hz$, $L = 0.2758H$, $R = 50\Omega$

Q.1 A 120V, 100 Watt lamp is to be connected to a 220V, 50Hz AC supply. What value of pure inductance should be connected in series in order to run the lamp on rated voltage.

Solution :-

$V_R = 120V$

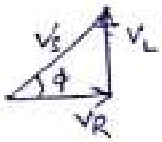
$V_L = ?$

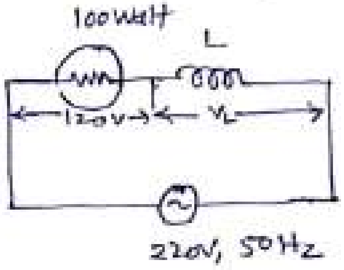
$V_s = V_R + jV_L$

$V_s = \sqrt{V_R^2 + V_L^2}$

$220V = \sqrt{120^2 + V_L^2}$

$V_L = 184.39$ Volt





⇒ Current through the lamp and inductance is same.

Current Through lamp $I = \frac{P}{V} = \frac{100}{120}$ A

$V_L = I X_L = I \times 2\pi f L$

$L = \frac{V_L}{I \times 2\pi f}$

$L = \frac{184.39}{\frac{100}{120} \times 2 \times \pi \times 50}$

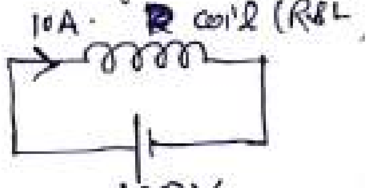
$$\boxed{L = 0.7046 \text{ H}}$$

Q.2 A coil connected to 100V DC supply draws 10 A and the same coil connected 100V, AC voltage of frequency 50 Hz draws 5A. Calculate the parameters of the coil and power factor.

Sol. Coil means a resistance and inductance both.

let impedance of coil $Z = R + jX_L$

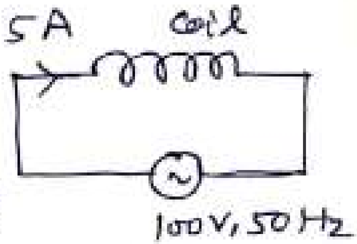
$X_L = 2\pi f L$



① in case of dc $f = 0$, Hence $X_L = 0$

Hence resistance of coil $R = \frac{V}{I} = \frac{100}{10}$

$R = 10\Omega$

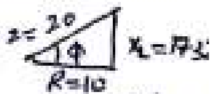


② in Case of AC supply impedance of coil

$|Z| = \frac{V}{I} = \frac{100}{5} = 20\Omega$

$\sqrt{R^2 + X_L^2} = 20 \Rightarrow X_L = 17.32\Omega$

$\Rightarrow L = X_L / 2\pi f = 0.05H$

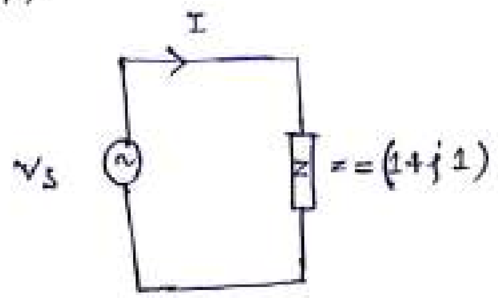


$\cos\phi = R/Z$

$\cos\phi = 0.5$

Q.3 A Load having impedance of $(1+j1)\ \Omega$ is connected to an ac voltage represented as $V = 20\sqrt{2}\cos(\omega t + 10^\circ)$ volt. Find the current in load expressed in the form of

$$i = I_m \sin(\omega t + \phi)$$



$$Z = 1 + j1 = |Z| \angle \phi$$

$$|Z| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\phi = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$$

$$Z = \sqrt{2} \angle 45^\circ$$

Voltage across the load $V = 20\sqrt{2}\cos(\omega t + 10^\circ)$

$$= 20\sqrt{2}\sin(\omega t + 100^\circ)$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \angle 100^\circ$$

$$V_{rms} = \frac{20\sqrt{2}}{\sqrt{2}} \angle 100^\circ = 20 \angle 100^\circ$$

Current Through load $I = \frac{V}{Z} = \frac{20\angle 100^\circ}{\sqrt{2}\angle 45^\circ}$

$$I = 14.144 \angle 55^\circ \ (\cancel{rms}$$

$$I_{rms} = 14.144$$

$$I_m = 14.144 \times \sqrt{2} = 20 \text{ and } \phi = 55^\circ$$

$$\boxed{i = 20 \sin(\omega t + 55^\circ)}$$

Q4 A emf is given by $100\sin(314t - \frac{\pi}{4})$ volt is applied to a circuit and the current is $20\sin(314t - \frac{\pi}{2})$ A. find (i) frequency (ii) circuit elements.

Sol.

$$\omega t = 314t$$

$$2\pi f = 314$$

$$\boxed{f = 50 Hz}$$

$$E = \frac{100}{\sqrt{2}} \angle -\frac{\pi}{4} \qquad I = \frac{20}{\sqrt{2}} \angle -\frac{\pi}{2}$$

45°

$$Z = \frac{E}{I} = \frac{\frac{100}{\sqrt{2}} \angle -\frac{\pi}{4}}{\frac{20}{\sqrt{2}} \angle -\frac{\pi}{2}} = \cancel{100 \angle -60}$$

$$= 5 \angle (90^\circ - 45^\circ)$$

$$= 5 \angle 45^\circ)$$

$$Z = R + jX_L = 5\cos 45^\circ + j5\sin 45^\circ$$

$$Z = \frac{5}{\sqrt{2}}(1 + j)$$

$$R = 5/\sqrt{2}\ \Omega, \quad X_L = 5/\sqrt{2}\ \Omega \Rightarrow L = \frac{X_L}{2\pi}$$