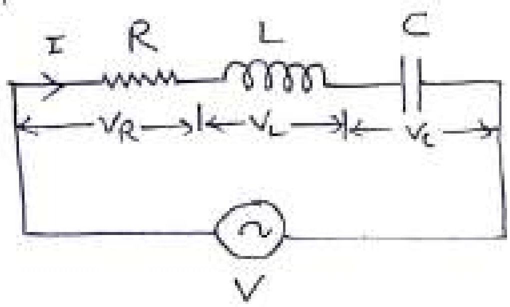
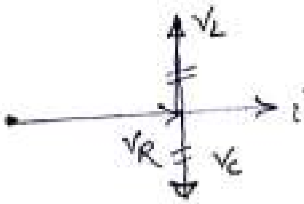
2·10 Series RLC Resonance:-

For the occurance of resonance in any system two energies are required, in the electric circuit inductor is having energy in the form of magnetic field (Kinetic energy) and capacitor is having energy in the form of electric field (potential energy), when these two energies are present at a particular frequency wide variations are present in the system.

* The circuit is said to be in resonance when source current is in phase with source voltage
* The frequency at which $X_L = X_C$ is called as resonant frequency
* The resonant frequency indicates the rate at which energy transformation is done between inductor and capacitor.





'Phasor diagram'

for resonance - $V_L = V_C$

$$I X_L = I X_C$$

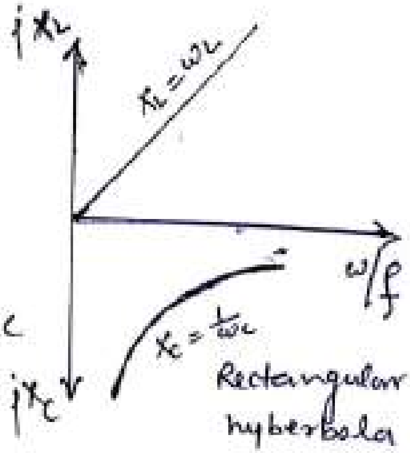
$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/sec}$$

$$2\pi f_0 = \frac{1}{\sqrt{LC}}$$

Resonant frequency $\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$. Hz

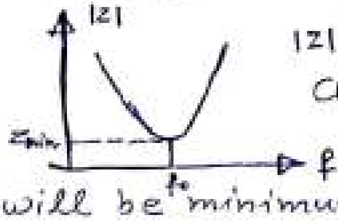


variation of X_L & X_C with frequency.

(a) Properties of series RLC resonant circuit

(I) $Z = R + j(X_L - X_C)$

at resonance $X_L = X_C$



|Z| Vs f Curve

Hence $Z_{min} = R$ (impedance will be minimum)

(II) Current will be maximum

$$I_{max} = \frac{V_s}{Z_{min}} = \frac{V_s}{R}$$

(III) $\cos\theta = 1 \rightarrow$ power factor will be unity

(IV) Active component of voltage $V_R = V_s$

(V) Net reactive voltage = 0 $(V_L - V_C = 0)$

(VI) Voltage across inductor and voltage across capacitor are greater than source voltage. Hence this phenomena is called as voltage magnification.

(VII) Series resonant circuit is also called as acceptor circuit.

(b) Applications -

Series RLC resonance phenomena is used in -

(I) Oscillators

(II) Band pass ~~and band reject~~ filters

(III) Tuning circuits

(IV) induction heating

(c) Quality factor :- Defined only correspond to resonance condition.

Quality factor is the 2π times of maximum energy stored in the circuit to power dissipation per cycle

$$Q = 2\pi \times \frac{\text{Max. energy stored in the ckt}}{\text{power dissipation per cycle}}$$

(d) expression of Quality factor for series RLC resonant circuit

let- $i(t) = I_m \sin\omega t$ ——— (1)

$$V_c = \frac{1}{C}\int i \cdot dt = \frac{1}{C}\int I_m \sin\omega t \, dt$$

$$V_c = -\frac{I_m}{\omega C}\cos\omega t \quad \text{——— (2)}$$

Total energy stored in series RLC circuit

$$W_e = \frac{1}{2}Li^2 + \frac{1}{2}CV_c^2$$

$$W_e = \frac{1}{2}L(I_m\sin\omega t)^2 + \frac{1}{2}C\left(-\frac{I_m}{\omega C}\cos\omega t\right)^2$$

$$W_e = \frac{1}{2}LI_m^2\sin^2\omega t + \frac{1}{2}C\frac{I_m^2}{\omega^2C^2}\cos^2\omega t$$

$$W_e = \frac{1}{2}LI_m^2\sin^2\omega t + \frac{1}{2}\frac{I_m^2}{\omega^2C}\cos^2\omega t$$

$$W_e = \frac{1}{2}LI_m^2\sin^2\omega t + \frac{1}{2}LI_m^2\cos^2\omega t \qquad \left(\omega^2 = \frac{1}{LC},\ L = \frac{1}{\omega^2C}\right)$$

$$W_e = \frac{1}{2}LI_m^2(\sin^2\omega t + \cos^2\omega t)$$

$$W_e = \frac{1}{2}LI_m^2$$

Quality factor $Q = 2\pi \cdot \dfrac{\frac{1}{2}LI_m^2}{\left(\frac{I_m}{\sqrt{2}}\right)^2 \cdot R \cdot \frac{1}{f}} = \dfrac{2\pi fL}{R}$

$$\boxed{Q = \frac{2\pi fL}{R} = \frac{\omega L}{R}}$$

As $\omega = \dfrac{1}{\sqrt{LC}}$

$$\boxed{Q = \frac{1}{R}\sqrt{\frac{L}{C}}}$$

some other expression of Quality factor-

$$Q = \frac{X_L}{R} = \frac{IX_L}{IR} = \frac{V_L}{V_R} \qquad \boxed{Q = \frac{V_L}{V_R} = \frac{V_c}{V_R}}$$

$V_s \rightarrow$ supply voltage

$V_s = V_R$ at resonance

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At lower half frequency ω_1, $X_c > X_L$

Hence $X_c - X_L = R$

$$\frac{1}{\omega_1 C} - \omega_1 L = R$$

$$1 - \omega_1^2 LC = \omega_1 RC$$

$$\omega_1^2 LC + \omega_1 RC - 1 = 0$$

$$\omega_1^2 + \frac{R}{L}\omega_1 - \frac{1}{LC} = 0 \qquad \left[\frac{-b \pm \sqrt{b^2-4ac}}{2a}\right]$$

$$\omega_1 = -\frac{R}{2L} \pm \left[\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}\right]^{1/2}$$

$$\omega_1 = -\alpha \pm [\alpha^2 + \omega_0^2]^{1/2} \quad \text{—— (4)}$$

Here $\alpha = \frac{R}{2L}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

At upper half frequency ω_2, $(X_L > X_c)$

ω_2 $X_L - X_c = R$

$$\omega_2 L - \frac{1}{\omega_2 C} = R$$

$$\omega_2^2 - \frac{R}{L}\omega_2 - \frac{1}{LC} = 0$$

$$\omega_2 = \frac{R}{2L} \pm \left[\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}\right]^{1/2}$$

$$\omega_2 = \alpha \pm [\alpha^2 + \omega_0^2]^{1/2} \quad \text{—— (5)}$$

Bandwidth $= \omega_2 - \omega_1$

$\Delta\omega = B.W. = 2\alpha$.

$\Delta\omega = B.W. = 2 \cdot \frac{R}{2L}$

$$\boxed{B.W = \frac{R}{L}}$$

lower cut off frequency

$\omega_1 = \omega_0 - \frac{\Delta\omega}{2}$

$$\boxed{\omega_1 = \omega_0 - \frac{R}{2L}}$$

upper cut off frequency

$\omega_2 = \omega_0 + \frac{\Delta\omega}{2}$

$$\boxed{\omega_2 = \omega_0 + \frac{R}{2L}}$$

2.11 Parallel RLC Resonance

A parallel combination of R, L and C branches connected to a source will produce a parallel resonance when the resultant current through the combination is in phase with applied voltage.

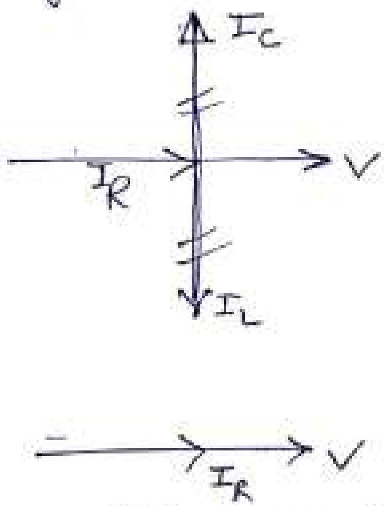


Fig (b) Phasor diagram

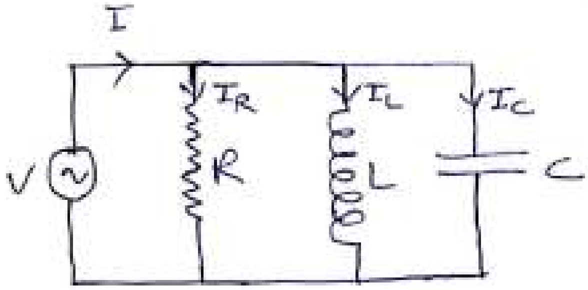


Fig. (a)

$$\boxed{Y = G + j(B_L \sim B_c)}$$

Applying KCL:

$$I = I_R\angle 0 + I_L\angle -90^\circ + I_C\angle +90^\circ$$

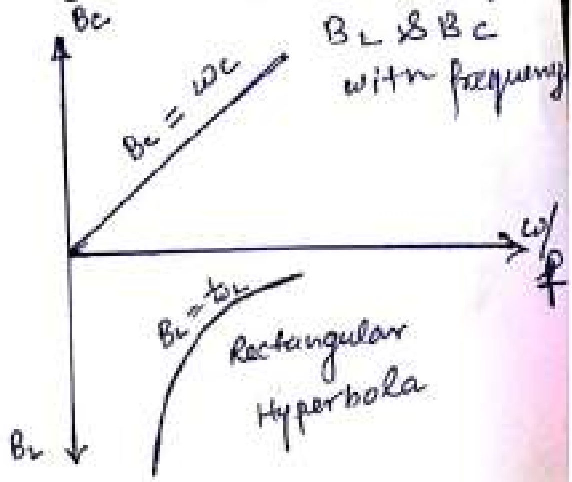
$$I_L = I_C \quad [\text{for resonance}]$$

$$\frac{V}{X_L} = \frac{V}{X_C}$$

$$\frac{1}{X_L} = \frac{1}{X_C}$$

$$\boxed{B_L = B_C}$$

Variation of B_L & B_C with frequency

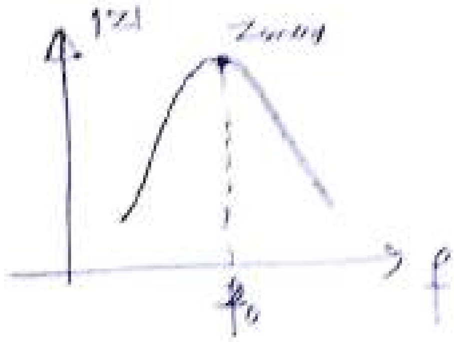


Resonant frequency $\omega_0 = \dfrac{1}{\sqrt{LC}}$

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}}$$

(a) Properties of Parallel RLC resonating ckt

(I) $Y = G + j(B_L \sim B_C)$



At Resonance $B_L = B_C$

Hence $\boxed{Y_{min} = G}$

$$Z_{max} = \frac{1}{Y_{min}}$$

impedance will be maximum.

(II) $I_{min} = \frac{V}{Z_{max}}$ → current will be minimum

(III) active component of the current $I = I_R$

(IV) Net susceptance $B_{net} = 0$

(V) current flowing through inductor and capacitor are greater than total current. This phenomen is called current magnification.

(VI) Parallel resonant circuit is also called as anti resonant circuit or rejector circuit.

(b) Quality factor:- (Parallel RLC circuit) (At resonance condition

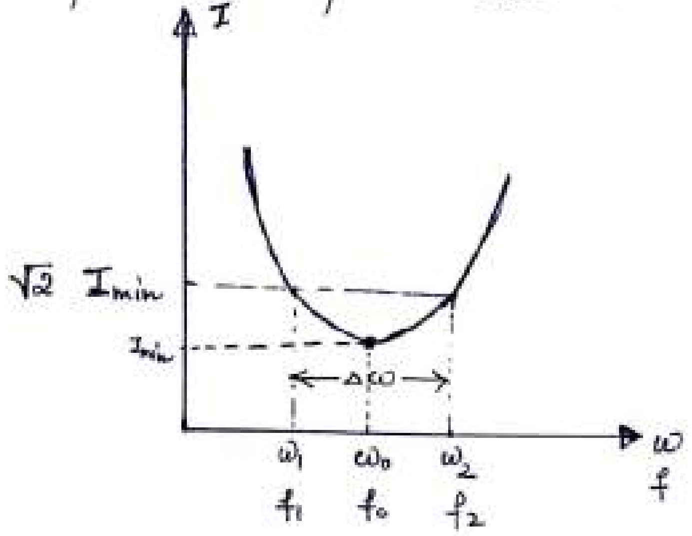
$$Q = \frac{\text{Reactive component of current}}{\text{active component of current}}$$

$$Q = \frac{I_L}{I_R} = \frac{V/X_L}{V/R} = \frac{R}{X_L} = \frac{R}{\omega L}$$

$$\boxed{Q = \frac{R}{\omega L}}$$

$$\boxed{Q = R\sqrt{\frac{C}{L}}}$$ by putting $\omega = \frac{1}{\sqrt{LC}}$

(c) Expression of Bandwidth for Parallel RLC resonating circuit



Bandwidth is a range of frequencies on either side of the resonant frequency when current rises from minimum value to $\sqrt{2}$ times of minimum value of current.

It is given by –

$$\Delta\omega = B.W = \omega_2 - \omega_1 = f_2 - f_1$$

~~At~~ Admittance of Parallel RLC circuit is given by

$$Y = G + j(B_L \sim B_C) = G + j\,B_{net}$$

At ω_0, $Y = G = \frac{1}{R}$

at ω_1 & ω_2 $Y = \frac{1}{\sqrt{2}R}$ (as $I = \sqrt{2} I_{min}$

$$\therefore \sqrt{G^2 + B_{net}^2} = \frac{1}{\sqrt{2}R}$$

Hence $\boxed{B_{net} = \frac{1}{R}}$ —— (1)

At ω_1, $B_{net} = \frac{1}{R}$

$$B_L \sim B_C = \frac{1}{R}$$

$$\frac{1}{X_L} \sim \frac{1}{X_C} = \frac{1}{R}$$

$$\frac{1}{\omega_1 L} \sim \frac{1}{1/\omega_1 C} = \frac{1}{R}$$

$$\frac{1}{\omega_1 L} - \omega_1 C = \frac{1}{R} \quad \left(\text{as } \frac{1}{\omega_1 L} > \frac{1}{\omega_1 C}\right.$$
at low frequency)

$$1 - \omega_1^2 LC = \frac{\omega_1 L}{R}$$

$$\omega_1^2 LC + \frac{\omega_1 L}{R} - 1 = 0$$

$$\omega_1^2 + \frac{\omega_1}{RC} - \frac{1}{LC} = 0 \quad \text{——— (2)}$$

$$\omega_1 = \frac{-\frac{1}{RC} \pm \sqrt{\frac{1}{R^2C^2} + \frac{4}{LC}}}{2 \cdot 1}$$

$$\omega_1 = \frac{-1}{2RC} \pm \sqrt{\frac{1}{4R^2C^2} + \frac{1}{LC}}$$

$$\omega_1 = \frac{-1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

$$\omega_1 = -\alpha \pm \sqrt{\alpha^2 + \omega_0^2} \quad \text{(3)}$$

Here $\alpha = \frac{1}{2RC}$

similarly at ω_2. $\frac{1}{\omega_2 L} - \omega_2 C = -\frac{1}{R}$ $\left(\text{as } \omega_2 C > \frac{1}{\omega_2 L}\right)$

$$\omega_2^2 - \frac{\omega_2}{RC} - \frac{1}{LC} = 0 \quad \text{——— (4)}$$

$$\omega_2 = \alpha \pm \sqrt{\alpha^2 + \omega_0^2} \quad \text{——— (5)}$$

eq (5) − eq (3)

$$\omega_2 - \omega_1 = 2\alpha$$

$$\Delta\omega = 2 \cdot \frac{1}{2RC} = \frac{1}{RC}$$

$$\boxed{\Delta\omega = \frac{1}{RC}}$$

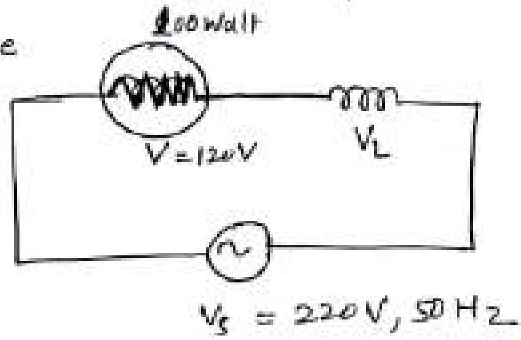
lower cutoff frequency
$\omega_1 = \omega_0 - \frac{\Delta\omega}{2}$
upper cut off frequency
$\omega_2 = \omega_0 + \frac{\Delta\omega}{2}$

Some Problems on RLC Circuit

Q.1 A 120V, 100W lamp is to be connected to a 220V, 50Hz AC supply. What value of pure inductance should be connected in series in order to run the lamp on rated voltage.

Hint :- Lamp is resistive
solve with the help
of voltage triangle.

Ans – $L = 0.7046\,H$



Q.2 A coil connected to 100V DC supply draws 10 Amp and same coil connected 100V, ac voltage of frequency 50Hz draws 5 Amp. Calculate the parameter (~~either~~, R, L ~~&~~ C) of coil and power factor.

Hint :- (i) in case of dc, $X_L = \omega L = 0$ (as $\omega = 2\pi f$, $\therefore f = 0$)

(ii) $Z = \frac{V}{I}$

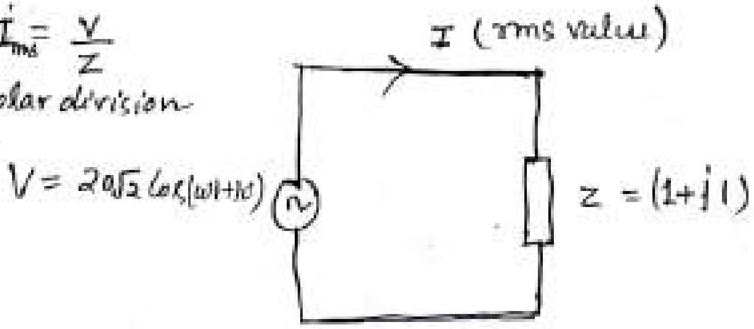
Ans – $L = 0.05\,H$, $\cos\phi = 0.5$ lagging

Q.3 A Load having impedance of $(1+j1)\,\Omega$ is connected to an AC voltage represented as $V = 20\sqrt{2}\cos(\omega t + 10^\circ)$ Volt. Find the current in load expressed in the form of $i = I_m \sin(\omega t + \phi)\,A$.

Ans – $i = 20\sin(\omega t + 55^\circ)$

Hint ! (i) $I_{ms} = \frac{V}{Z}$

(ii) Polar division



Q.4 An ~~ef~~ emf is given by $100\sin(314t - \frac{\pi}{4})$ volts in applied to a circuit and the current is $20\sin(314t - \frac{\pi}{2})\,A$. Find (i) frequency (ii) circuit element (~~eithe~~ R, L or C)

Ans – $f = 50\,Hz$, $L = 0.2758\,H$, $R = 50\,\Omega$

Q.1 A 120V, 100 Watt lamp is to be connected to a 220V, 50Hz AC supply. What value of pure inductance should be connected in series in order to run the lamp on rated voltage.

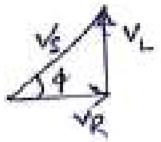
Solution :-

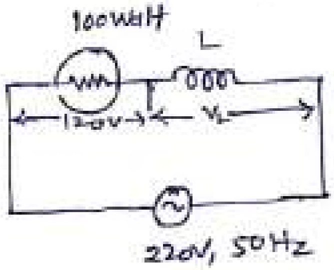
$V_R = 120V$

$V_L = ?$

$V_S = V_R + jV_L$

$V_S = \sqrt{V_R^2 + V_L^2}$





$220V = \sqrt{120^2 + V_L^2}$

$V_L = 184.39$ volt

⇒ Current through the lamp and inductance is same.

Current Through lamp $I = \frac{P}{V} = \frac{100}{120}$ A

$V_L = IX_L = I \times 2\pi f L$

$L = \frac{V_L}{I \times 2\pi f}$

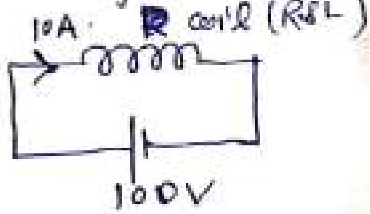
$L = \frac{184.39}{\frac{100}{120} \times 2 \times \pi \times 50}$

$\boxed{L = 0.7046 H}$

Q.2 A coil connected to 100V DC supply draws 10 A and the same coil connected 100V, AC voltage of frequency 50 Hz draws 5A. Calculate the parameters of the coil and power factor.

Sol. coil means a resistance and inductance both.

let impedance of coil $Z = R + jX_L$

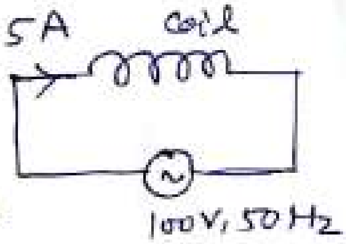


$X_L = 2\pi f L$

① in case of dc $f = 0$, Hence $X_L = 0$

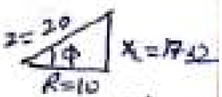
Hence resistance of coil $R = \frac{V}{I} = \frac{100}{10}$

$R = 10\Omega$



② in case of AC supply impedance of coil

$|Z| = \frac{V}{I} = \frac{100}{5} = 20\Omega$



$\sqrt{R^2 + X_L^2} = 20 \Rightarrow X_L = 17.32\Omega$

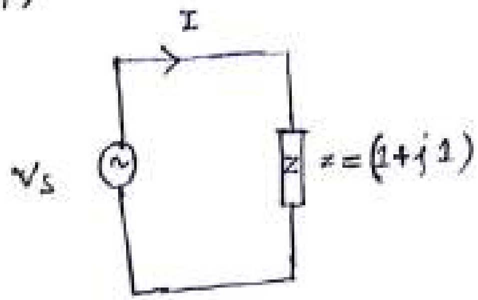
$\Rightarrow L = X_L/2\pi f = 0.05H$

$\cos\phi = R/Z$

$\cos\phi = 0.5$

Q.3 A Load having impedance of $(1+j1)\ \Omega$ is connected to an ac voltage represented as $V = 20\sqrt{2}\cos(\omega t + 10^\circ)$ volt. Find the current in load expressed in the form of

$$i = I_m \sin(\omega t + \phi)$$



$Z = 1 + j1 = |Z| \angle \phi$

$|Z| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\phi = \tan^{-1}\left(\frac{1}{1}\right) = 45^\circ$

$Z = \sqrt{2} \angle 45^\circ$

voltage across the load $V = 20\sqrt{2}\cos(\omega t + 10^\circ)$

$= 20\sqrt{2}\sin(\omega t + 100^\circ)$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \angle 100^\circ$$

$$V_{rms} = \frac{20\sqrt{2}}{\sqrt{2}} \angle 100^\circ = 20 \angle 100^\circ$$

Current through load $I = \frac{V}{Z} = \frac{20 \angle 100^\circ}{\sqrt{2} \angle 45^\circ}$

$I = 14.144 \angle 55^\circ$ (rms

$I_{rms} = 14.144$

$I_m = 14.144 \times \sqrt{2} = 20$ and $\phi = 55^\circ$

$$\boxed{i = 20\sin(\omega t + 55^\circ)}$$

Q4 A emf is given by $100\sin(314t - \frac{\pi}{4})$ volt is applied to a circuit and the current is $20\sin(314t - \frac{\pi}{2})$ A. Find (i) frequency (ii) circuit elements.

45°

Sol. $\omega t = 314t$

$2\pi f = 314$

$\boxed{f = 50\text{Hz}}$

$E = \frac{100}{\sqrt{2}} \angle -\frac{\pi}{4}$ $\quad I = \frac{20}{\sqrt{2}} \angle -\frac{\pi}{2}$

$Z = \frac{E}{I} = \dfrac{\frac{100}{\sqrt{2}} \angle -\frac{\pi}{4}}{\frac{20}{\sqrt{2}} \angle -\frac{\pi}{2}} = \cancel{100 \angle 60} = 5 \angle (90^\circ - 45^\circ)$

$= 5 \angle 45^\circ)$

$Z = R + jX_L = 5\cos 45^\circ + j5\sin 45^\circ$

$Z = \frac{5}{\sqrt{2}}(1+j)$

$R = 5/\sqrt{2}\,\Omega$, $X_L = 5/\sqrt{2}\,\Omega \Rightarrow L = \frac{X_L}{2\pi}$

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