$C_4 \cdot C_4 = C_4^2$

$m_x C_4 = \sigma_v$

$m_y C_4 = \sigma_u$

$\sigma_u C_4 = m_x$

$\sigma_v C_4 = m_y$

$m_x m_x = E$

$m_x m_y = C_4^2$

$m_y C_4^2 = m_x$ etc.

The all above operations shows that the all elements (symmetry transformations) obey the properties of Group. The above operation can be understood as:

$$\sigma_u C_4 \begin{vmatrix} a & b \\ d & c \end{vmatrix} = \sigma_u \begin{vmatrix} d & a \\ c & b \end{vmatrix} = \begin{vmatrix} d & c \\ a & b \end{vmatrix} \quad -(i)$$

$$m_x \begin{vmatrix} a & b \\ d & c \end{vmatrix} = \begin{vmatrix} d & c \\ a & b \end{vmatrix} \quad (ii)$$

From eq^n (i) and eq (ii) it is clear that

$\sigma_u C_4 = m_x$

Inverse Property:-

$$(C_4)^{-1} \begin{vmatrix} a & b \\ d & c \end{vmatrix} = \begin{vmatrix} b & c \\ a & d \end{vmatrix} \text{ and } C_4^3 \begin{vmatrix} a & b \\ d & c \end{vmatrix} = \begin{vmatrix} b & c \\ a & d \end{vmatrix}$$

Hence $(C_4)^{-1} = C_4^3$

It shows that inverse property is also hold.

Associative Property:

$$\sigma_{u} C_{4} \neq C_{4} \sigma_{u} \neq m_x$$

Proof:

$$\sigma_{u} C_{4} \begin{vmatrix} a & b \\ d & c \end{vmatrix} = \sigma_{u} \begin{vmatrix} d & a \\ c & b \end{vmatrix} = \begin{vmatrix} d & c \\ a & b \end{vmatrix} = m_y \begin{vmatrix} a & b \\ d & c \end{vmatrix} \quad (i)$$

$$C_{4} \sigma_{u} \begin{vmatrix} a & b \\ d & c \end{vmatrix} = C_{4} \begin{vmatrix} a & d \\ b & c \end{vmatrix} = \begin{vmatrix} b & a \\ c & d \end{vmatrix} = m_y \begin{vmatrix} a & b \\ d & c \end{vmatrix}$$

Hence $\sigma_{u} C_{4} \neq C_{4} \sigma_{u}$

From above discussion it is clear that associative Property is obeyed but commutative law does not hold. Thus it is a group but not abelian group.

The multiplication Table:-

All possible multiplications of group elements of symmetric operations of square can be represented by a table. This type of table is known as group multiplication table. The order of operation is from right to left in successive operation such as ABC.... Thus, in the product $C_4 m_x$, m_x is the first operation and C_4 is second operation. The entry for $C_4 m_x$ would be found in the table in the column corresponding to m_x and the row corresponding to C_4.

The multiplication Table for the Group C_{4v}.

Second operation \ First operation →	E	C_4	C_4^2	C_4^3	m_x	m_y	σ_u	σ_v
E	E	C_4	C_4^2	C_4^3	m_x	m_y	σ_u	σ_v
C_4^3	C_4^3	E	C_4	C_4^2	σ_v	σ_u	m_x	m_y
C_4^2	C_4^2	C_4^3	E	C_4	m_y	m_x	σ_v	σ_u
C_4	C_4	C_4^2	C_4^3	E	σ_u	σ_v	m_y	m_x
m_x	m_x	σ_v	m_y	σ_u	E	C_4^2	C_4^3	C_4
m_y	m_y	σ_u	m_x	σ_v	C_4^2	E	C_4	C_4^3
σ_u	σ_u	m_x	σ_v	m_y	C_4	C_4^3	E	C_4^2
σ_v	σ_v	m_y	σ_u	m_x	C_4^3	C_4	C_4^2	E

The rearrangement theorem:-

Each elements of the group occurs once and only once in each column. This is known as the rearrangement theorem. The arrangement of elements in a row (column) is different from that in every other row (column).

Proof:- Suppose an element D occurs twice in a column corresponding to the element A. This means that there exist two elements say B and C, such that

$$BA = D \quad \text{and} \quad CA = D.$$

Multiplying from the right by A^{-1}, we get

$$B = DA^{-1}, \quad C = DA^{-1}$$

This shows that $B = C$, which is contrary to the hypothesis