Subject → Microwave Engg.
Branch → ECE (IIIrd Yr.)
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Scattering Matrix

Introduction

* Circuits operating at low frequencies can be treated as an interconnection of lumped passive or active components with unique voltages and currents defined at any point in the circuit.

* In this situation the circuit dimensions are small enough such that there is negligible phase delay from one point in the circuit to another.

* In addition, the fields can be considered as TEM fields supported by two or more conductors.

* This leads to a quasi static type of solution to Maxwell's eq's and to the well known Kirchoff's voltage and current laws and impedance concept of circuit theory.

* In general, the network analyzing techniques cannot be directly applied to microwave circuits, but the basic circuit and network concepts can be extended to handle many microwave analysis and design problems, of practical interest.

Microwave Network Analysis

Matrices for Network Analysis —

- Impedance Matrix (Z)
- Admittance Matrix (Y)

- Scattering Matrix (S)
- Transmission Matrix (ABCD)

Scattering Matrix

- At high frequencies, Z, Y, h & ABCD parameters are difficult to measure.

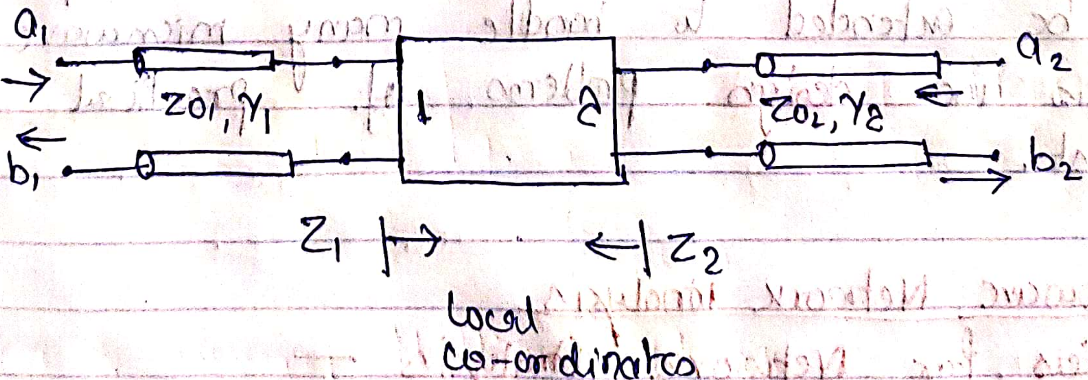
- Scattering (S) parameters are often the best representation for multi-port networks at high frequency.

- We can directly measure reflected, transmitted and incident wave with a network analyzer.

- Once the parameters are known they can be converted to any other matrix parameters.

Scattering Parameters

- S-parameters are defined assuming transmission lines are connected to each port.



On each transmission line :-

$$V_i(z_i) = V_{i0}^+ e^{-\gamma_i z_i} + V_{i0}^- e^{+\gamma_i z_i} = V_i^+(z_i) + V_i^-(z_i)$$

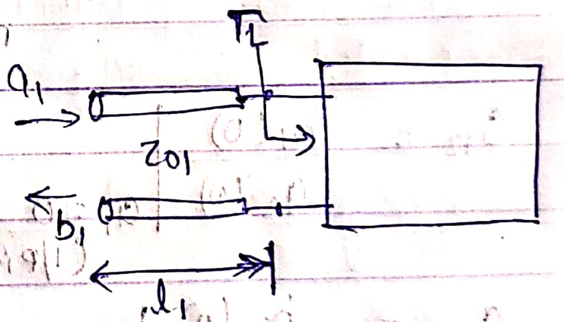
$$I_i(z_i) = \frac{V_i^+(z_i)}{Z_{0i}} - \frac{V_i^-(z_i)}{Z_{0i}} \quad ; \; i = 1,2$$

Incoming wave function $\equiv a_i(z_i) \equiv V_i^+(z_i)/\sqrt{Z_{0i}}$

Outgoing wave function $\equiv b_i(z_i) \equiv V_i^-(z_i)/\sqrt{Z_{0i}}$

One Port Network :-

* Reflection Coefficient



$$\Gamma_L = \frac{V_1^-(0)/\sqrt{Z_{01}}}{V_1^+(0)/\sqrt{Z_{01}}}$$

$$= \frac{b_1(0)}{a_1(0)} = S_{11}$$

$\Rightarrow b_1(0) = \Gamma_L \, a_1(0)$

$= S_{11} a_1(0)$

For a one-port network, S_{11} is defined to be the same as Γ_L.

* Return loss

$$RL = -20 \log |\Gamma|$$

For a two-port Network

$$b_1(0) = S_{11}\,a_1(0) + S_{12}\,a_2(0)$$
$$b_2(0) = S_{21}\,a_1(0) + S_{22}\,a_2(0)$$

$$\Rightarrow \begin{bmatrix} b_1(0) \\ b_2(0) \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1(0) \\ a_2(0) \end{bmatrix}$$

or, $[b] = [S][a]$

$\hookrightarrow$ Scattering matrix

Scattering Parameters

$$S_{11} = \left.\frac{b_1(0)}{a_1(0)}\right|_{a_2=0} \text{ (o/p is matched)}$$
$\leftarrow$ i/p reflection coefficient.

$$S_{12} = \left.\frac{b_1(0)}{a_2(0)}\right|_{a_1=0} \text{ (i/p is matched)}$$
$\leftarrow$ reverse transmission coefficient.

$$S_{21} = \left.\frac{b_2(0)}{a_1(0)}\right|_{a_2=0} \text{ (o/p is matched)}$$
$\leftarrow$ forward transmission coefficient.

$$S_{22} = \left.\frac{b_2(0)}{a_2(0)}\right|_{a_1=0} \text{ (i/p is matched)}$$
$\leftarrow$ o/p reflection coefficient.

For a General Multiport Network —

$$S_{ij} = \left.\frac{b_i(0)}{a_j(0)}\right|_{a_k=0,\ k\neq j}$$
(All ports except j are semi infinite (or matched))

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