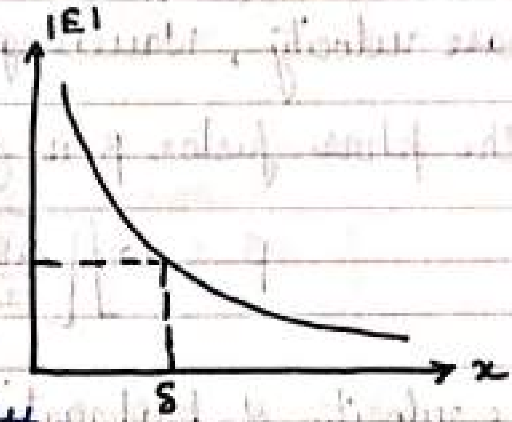
Skin depth or Depth of penetration :-

The depth of penetration is defind as the depth in which the strength of electric field associated with the electromagnetic wave reduces to $(1/e)$ times of its initial value.

The amplitude of strength of electric fields of electromagnetic wave decreases by a factor $e^{-\alpha x}$, where α is attenuation constant.



The distance for which the amplitude of electromagnetic wave decreases by a factor $(1/e)$ of its initial value is given by

$$\alpha x = 1$$

The depth of penetration or skin depth is generally denoted by δ ie

$$x = \delta$$

Therefore

$$\alpha \delta = 1$$

or

$$\delta = \frac{1}{\alpha}$$

$$\boxed{\text{Skin depth} = \frac{1}{\text{attenuation Constant}}}$$

In case of conducting media, the attenuation constant α of an electromagnetic wave is given by -

$$\alpha = \omega \left[\left(\frac{\mu \epsilon}{2}\right) \left[\left(1 + \frac{\sigma^2}{\omega^2 \epsilon^2}\right) - 1 \right] \right]^{1/2}$$

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$$\boxed{\text{Skin depth } (\delta) = \frac{1}{\omega\left[\left(\frac{\mu\varepsilon}{2}\right)\sqrt{1+\left(\frac{\sigma^2}{\omega^2\varepsilon^2}\right)}-1\right]^{1/2}}}$$

Phase velocity, Wavelength and intrinsic impedence :-

The phase factor β is given by -

$$\beta = \omega\sqrt{\left[\frac{\mu\varepsilon}{2}\right]\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]}$$

The velocity of propagation is given by -

$$V_p = \frac{\omega}{\beta} \quad \text{or} \quad V_p = \frac{\omega}{\omega\sqrt{\left(\frac{\mu\varepsilon}{2}\right)\left[\sqrt{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]}+1\right]}}$$

$$\boxed{V_p = \frac{1}{\sqrt{\left[\left(\frac{\mu\varepsilon}{2}\right)\sqrt{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]}+1\right]}}}$$

Wave length (λ)

We know that $\quad \lambda = \frac{2\pi}{\beta}$

$$\boxed{\lambda = \frac{2\pi}{\omega\sqrt{\left[\left(\frac{\mu\varepsilon}{2}\right)\sqrt{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]}+1\right]}}}$$

Intrinsic impedance -:

$$\boxed{\eta = \frac{c}{v_p} = \frac{3\times10^8}{\sqrt{\left[\left(\frac{\mu\varepsilon}{2}\right)\sqrt{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]}+1\right]}}}$$

Application of plane wave in Conducting medium -

(i) In Good dielectric -

For a good dielectric, $\frac{\sigma}{\omega\varepsilon} << 1$

for a good conductor, $\frac{\sigma}{\omega\varepsilon} >>> 1$

Attenuation constant (α)

$$\alpha = \omega\sqrt{\left(\frac{\mu\varepsilon}{2}\right)\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]}$$

using Binomial theoram as $\frac{\sigma}{\omega\varepsilon} << 1$

$$\left[1+\frac{\sigma^2}{\varepsilon^2\omega^2}\right]^{1/2} = \left[1+\frac{\sigma^2}{2\varepsilon^2\omega^2}\right]$$

So $$\alpha = \omega\sqrt{\left(\frac{\mu\varepsilon}{2}\right)\left[1+\frac{\sigma^2}{2\varepsilon^2\omega^2}-1\right]}$$

$$\alpha = \frac{\sigma}{2}\sqrt{\left(\frac{\mu}{\varepsilon}\right)}$$

$$\boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}$$

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Skin depth -

$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{\frac{\sigma}{2}\sqrt{\left(\frac{\mu}{\varepsilon}\right)}}}$$

Phase Constant (β) -:

$$\beta = \omega\sqrt{\left(\frac{\mu\varepsilon}{2}\right)\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right.}$$

$$\simeq \omega\sqrt{\left(\frac{\mu\varepsilon}{2}\right)\left[1+\frac{\sigma^2}{2\omega^2\varepsilon^2}+1\right]}$$

$$\simeq \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{4\omega^2\varepsilon^2}\right]^{1/2}$$

$$\simeq \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]$$

$$\boxed{\beta = \omega\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]}$$

Wave velocity or Velocity of propagation -:

The wave velocity is given by -

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}\left[1+\frac{\sigma^2}{8\omega^2\varepsilon^2}\right]}$$

$$v_p = \frac{1}{\sqrt{\mu\varepsilon}} \times \left[1 + \frac{\sigma^2}{8\omega^2\varepsilon^2}\right]$$

$$\boxed{v_p = \frac{1}{\sqrt{\mu\varepsilon}}\left[1 - \frac{\sigma^2}{8\omega^2\varepsilon^2}\right]}$$

In case of a perfect dielectric, the conductivity is zero. The wave velocity $v_0 = \frac{1}{\sqrt{\mu\varepsilon}}$.

Intrinsic Impedance :-

The intrinsic impedance η is given by.

$$\eta = \sqrt{\left[\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}\right]}$$

$$= \sqrt{\frac{(j\omega\mu)}{j\omega\varepsilon\left(1 + \frac{\sigma}{j\omega\varepsilon}\right)}} = \sqrt{\frac{\mu}{\varepsilon}} \times \frac{1}{\left[1 + \frac{\sigma}{j\omega\varepsilon}\right]^{1/2}} = \sqrt{\left(\frac{\mu}{\varepsilon}\right)}\left[1 + \frac{\sigma}{j\omega\varepsilon}\right]^{-1/2}$$

$$\eta = \sqrt{\left(\frac{\mu}{\varepsilon}\right)}\left[1 - \frac{\sigma}{2j\omega\varepsilon}\right] = \sqrt{\left(\frac{\mu}{\varepsilon}\right)}\left[1 + \frac{j\sigma}{2\omega\varepsilon}\right]$$

$$\eta = \sqrt{\left(\frac{\mu}{\varepsilon}\right)}\left[1 + \frac{j\sigma}{2\omega\varepsilon}\right].$$

for a dielectric with $\sigma = 0$ the intrinsic impedance

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0$$

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$$\boxed{\eta = \eta_0 \left[1 + \frac{\sigma j}{2\omega\varepsilon}\right] = \left(\eta_0 + \frac{j\sigma\eta_0}{2\omega\varepsilon}\right)}$$

So for a dielectric for $\sigma \neq 0$, a reactive component $\left(\frac{\sigma\eta_0}{2\omega\varepsilon}\right)$ is added to intrinsic impedence of medium (η_0) This is the main effect of loss.

Application to good Conductor :-

The wave equ^n for a conducting medium is given by -

$$\frac{\partial^2 E_x}{\partial z^2} - \gamma^2 E_x = 0$$

where γ is propagation constant $\gamma = \alpha + j\beta$ where α is attenuation constant and β is phase constant.

$$\alpha = \omega \sqrt{\left(\frac{\mu\varepsilon}{2}\right)\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]}$$

$$\beta = \omega \sqrt{\left(\frac{\mu\varepsilon}{2}\right)\left[\sqrt{1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]}$$

For a good conductor σ and ε are nearly independent of frequency and $\frac{\sigma}{\omega\varepsilon} >> 1$, $\frac{\sigma}{\omega\varepsilon}$ is known as loss tangent.

We know that $\gamma^2 = (j\omega\mu\sigma - \omega^2\mu\varepsilon)$

$$\gamma = \left[j\omega\mu\sigma - \omega^2\mu\varepsilon\right]^{1/2}$$

$$\gamma = \sqrt{(j\omega\mu\sigma)} \qquad \text{for good conductor } \frac{\sigma}{\omega\varepsilon} >> 1$$

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(iii) Phase Constant (β) :-

Comparing the imaginary part of equ^n ① and ② we get.

$$\boxed{\beta = \sqrt{\frac{\omega \mu \sigma}{2}}}$$

Thus α and β are equal. β is also large, i.e. phase shift is very large.

(iv) Wave Velocity :-

$$V_p = \frac{\omega}{\beta} = \frac{\omega}{\sqrt{\left(\frac{\omega \mu \sigma}{2}\right)}} = \sqrt{\left(\frac{2\omega}{\mu\sigma}\right)}$$

$$= \sqrt{\left(\frac{4\pi f}{\mu\sigma}\right)} \qquad \because \omega = 2\pi f$$

$$\boxed{V_p = \sqrt{\frac{2\omega}{\mu\sigma}}}$$

as $V_p \propto \frac{1}{\beta}$, the wave velocity in good conductors is very small.

(v) Intrinsic impedance :-

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$\eta \simeq \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\boxed{\eta = \sqrt{\left(\frac{\omega\mu}{\sigma}\right)} \angle 45^\circ}$$ where $\sqrt{j} = 1\angle 45^\circ$

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$$\gamma = \sqrt{\omega\mu\sigma}\sqrt{j} = \sqrt{\omega\mu\sigma}\angle 45^\circ$$

$$= \sqrt{(\omega\mu\sigma)}\,(\cos 45^\circ + \sin 45^\circ)$$

$$\boxed{\gamma = \sqrt{\left(\frac{\omega\mu\sigma}{2}\right)} + j\sqrt{\left(\frac{\omega\mu\sigma}{2}\right)}} \quad \text{——— (1)}$$

(i) Attenuation Constant (α)

We know that

$$\gamma = \alpha + j\beta \quad \text{——— (2)}$$

Comparing equ^n (1) and (2) we get

$$\boxed{\alpha = \sqrt{\left(\frac{\omega\mu\sigma}{2}\right)}} \quad \text{——— (3)}$$

In case of good conductor σ is very large, Thus α is very large. This means that the wave is attenuated greatly as it progresses through good conductor.

(ii) Skin depth :-

$$\boxed{\delta = \frac{1}{\alpha} = \frac{\sqrt{2}}{\sqrt{\omega\mu\alpha}}} \quad \text{——— (4)}$$

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The refractive index of conducting medium.

$$n = \frac{c}{v} = \frac{c}{\sqrt{\left(\frac{2\omega}{\mu\sigma}\right)}} = c\sqrt{\left(\frac{\mu\sigma}{2\omega}\right)}$$

$$\boxed{n = c\sqrt{\left(\frac{\mu\sigma}{2\omega}\right)}}$$