

Critical damping coefficient (C_c)

$$2\zeta \cdot \omega_n = \frac{C}{m}$$

$$2 \times 1 \times \omega_n = \frac{C_c}{m}$$

$$\Rightarrow \boxed{\zeta = \frac{C}{C_c}}$$

$$\boxed{\zeta = \frac{\text{Actual damping coefficient}}{\text{Critical damping coefficient}}}$$

* The displacement ratio of end to 3rd cycle to start of 8th cycle is 2.5

$$\frac{x_3}{x_7} = 2.5$$

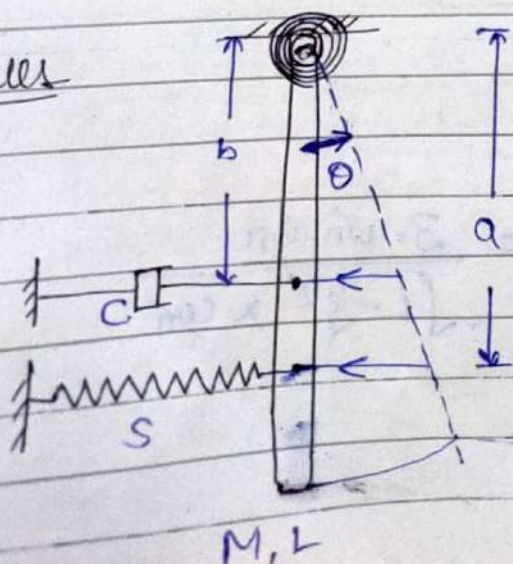
* The displacement ratio of 3rd cycle to 8th cycle.

Either $\frac{x_2}{x_7} = 2.5$ or $\frac{x_3}{x_8} = 2.5$

(both start of the cycle)

(both end of the cycle)

Ques



$$\begin{aligned} & Mg \cdot \frac{L}{2} \cdot \theta \\ & S \cdot a^2 \\ & S \theta \cdot \theta \\ & C b^2 \cdot \theta \\ & I \cdot \ddot{\theta} \end{aligned}$$

$$I \cdot \ddot{\theta} + (cb^2) \cdot \dot{\theta} + (sa^2 + s_0 + \frac{Mgl}{2}) \cdot \theta = 0$$

$$m \cdot \ddot{x} + c \cdot \dot{x} + sx = 0$$

Ques. 1 The damping coefficient in vib. equa. will be

cb^2 Ans

Q.2 $\omega_n = ?$

$$\omega_n = \sqrt{\frac{sa^2 + s_0 + \frac{Mgl}{2}}{I}}$$

Q.3 $\xi = ?$

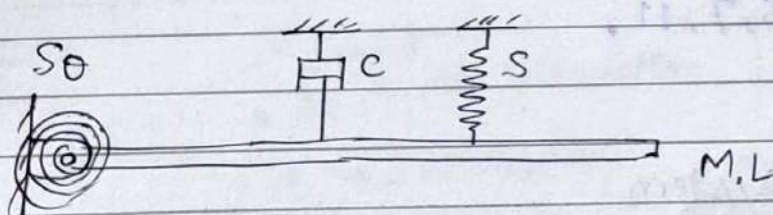
$$2\xi\omega_n = \frac{cb^2}{I} \Rightarrow \xi = ?$$

Q.4 $C_c = ?$

$$2\omega_n = \frac{C_c \cdot b^2}{I} \Rightarrow C_c = ?$$

Q.5 $\omega_d = ?$

$$\omega_d = \sqrt{1 - \xi^2} \cdot \omega_n = \text{---}$$



Mg torque is not to be considered

Ques. 5 (W.B.)

$$m = 7.5 \text{ kg}$$

$$T_d = \frac{35}{60} \text{ sec}$$

$$\omega_d = \frac{2\pi}{T_d} = 10.7711 \text{ rad/sec}$$

$$\frac{x_1}{x_7} = 2.5$$

$$\frac{x_1}{x_2} \cdot \frac{x_2}{x_3} \cdot \frac{x_3}{x_4} \cdot \frac{x_4}{x_5} \cdot \frac{x_5}{x_6} \cdot \frac{x_6}{x_7} = 2.5$$

$$\Rightarrow e^{6\delta} = 2.5 \Rightarrow 6\delta = \ln 2.5$$

$$\Rightarrow 6 \times 2\pi \zeta = \ln 2.5 \Rightarrow \zeta = 0.0242$$

$$\omega_d = \frac{\sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}} \cdot \omega_n$$

$$\Rightarrow \omega_n = 10.7743 \text{ rad/sec}$$

$$(i) \omega_n = \sqrt{s/m} \rightarrow ?$$

$$s = 870.64 \text{ N/m}$$

$$(ii) 2\zeta\omega_n = c/m$$

$$c = 3.911 \text{ N/m/sec}$$

$$(iii) \zeta = \frac{c}{c_c}$$

$$c_c = 161.6141 \text{ N/m/sec}$$

*Objective - 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 16, 19, 24, 25
26, 28, 30, 34, 37, 38, 39, 40, 41

Conventional - 1, 2, 4, 5, 7, 11.

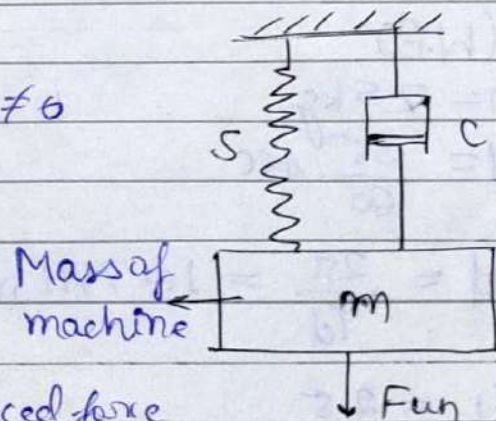
Forced damped system

In the running system $F_{un} \neq 0$

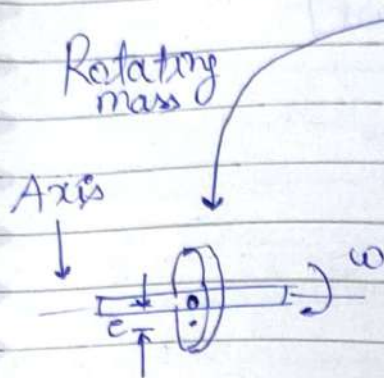
$$F_{un} = F_0 \sin \omega t \text{ (Harmonic force)}$$

$F_0 \rightarrow$ Max. value

$\omega \rightarrow$ Frequency of unbalanced force



$$F_{un} = F_0 \sin \omega t$$



$$m_{rotor} \cdot e \cdot \omega^2$$

$$= m_{rotor} \cdot e \omega^2 \sin \omega t$$

$$= \underbrace{(m_{rotor} \cdot e \cdot \omega^2)}_{F_0} \underbrace{\sin \omega t}_{\text{frequency}}$$

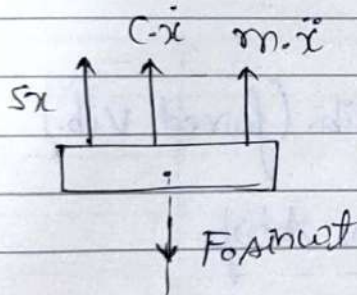
Reciprocating mass

$$F_{un} = m_{reci} \cdot \omega^2 \cdot \sin \omega t$$

$$= \underbrace{m_{reci} \cdot \omega^2}_{F_0} \underbrace{\sin \omega t}_{\text{frequency}}$$

FBD

mass $m \rightarrow$ displaced



$$m \ddot{x} + c \dot{x} + s x = F_0 \sin \omega t$$

$$\Rightarrow \ddot{x} + (2\zeta \omega_n) \dot{x} + \omega_n^2 x = \frac{F_0}{m} \sin \omega t$$

Solⁿ of this equaⁿ will be
 $x = C.F. + P.I.$

C.F.
 $C.F. \begin{cases} \zeta > 1 \\ \zeta = 1 \\ \zeta < 1 \end{cases} \begin{matrix} \text{no vibration} \\ \text{no vibration} \\ \text{vibration} \end{matrix} \rightarrow 0 \text{ After some time}$

After some time value of $C.F. \rightarrow 0$
 then $x = P.I.$

$$\text{P.I.} \quad P.I. = \frac{F_0 \sin \omega t}{D^2 + (2\zeta \omega_n) D + \omega_n^2} \quad \left[\begin{array}{l} D \rightarrow \text{operator} \\ D^2 = -\omega^2 \end{array} \right]$$

$$P.I. = \frac{F_0/m \sin \omega t}{[(\omega_n^2 - \omega^2) + 2\zeta \omega_n D]} \times \frac{[(\omega_n^2 - \omega^2) - 2\zeta \omega_n D]}{[(\omega_n^2 - \omega^2) - 2\zeta \omega_n D]}$$

$$\begin{aligned}
 P.I. &= \frac{F_0/m \left[\overbrace{(\omega_n^2 - \omega^2)}^{R \cdot \cos \phi} \sin \omega t - \overbrace{(2\zeta \omega \omega_n)}^{R \sin \phi} \cos \omega t \right]}{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega \omega_n)^2} \\
 &= \frac{F_0}{m} \frac{\sin(\omega t - \phi)}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta \omega \omega_n)^2}}
 \end{aligned}$$

$$P.I. = \frac{F_0/s}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta \omega}{\omega_n}\right\}^2}} \cdot \sin(\omega t - \phi)$$

$\rightarrow A$
 Amplitude

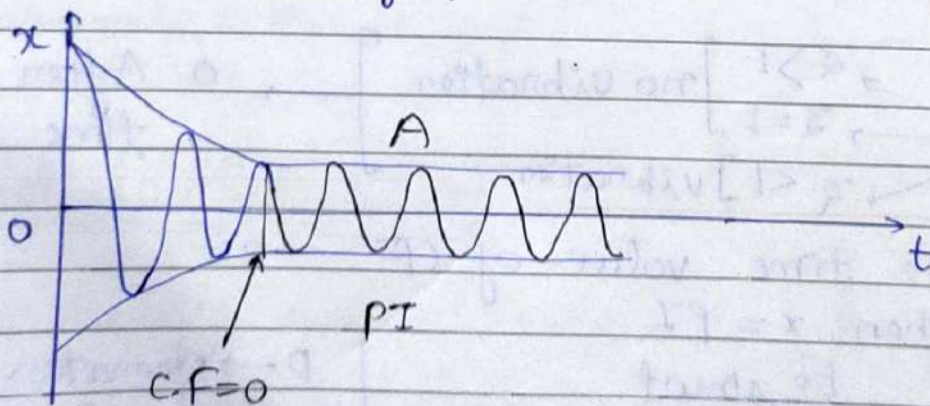
Independent of time

After some time

$$x = P.I. = A \cdot \sin(\omega t + \phi)$$

$A \rightarrow$ amplitude of Steady State Vib. (forced vib.)

Vibration in running system will never stop



After some time: $x = A \cdot \sin(\omega t - \phi)$
 where,

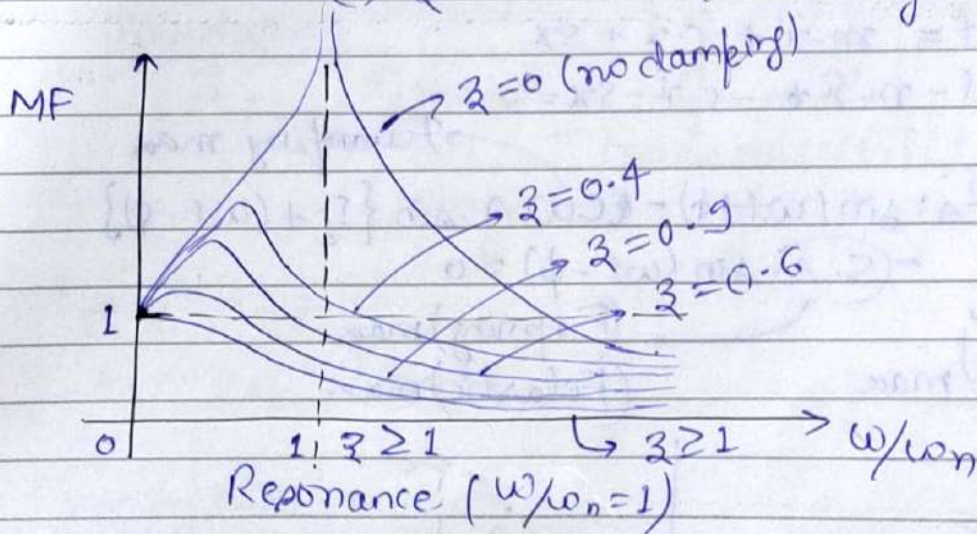
$$A = \frac{F_0/s}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}}$$

Magnification factor (MF) = $\frac{A}{(F_0/s)}$
 ↳ Strength of A

$$MF = \frac{1}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left[\frac{2\zeta\omega}{\omega_n}\right]^2}}$$

depends upon (i) ω/ω_n
 (ii) ζ

Nature of A curve → Same
 Nature of MF curve → Same



Q.1 If underdamping ($\uparrow$) $\Rightarrow \zeta (\downarrow)$

$\hookrightarrow MF \uparrow \Rightarrow A \uparrow$

Running life $\downarrow \downarrow$

② $(A)_{\max}$ at $(MF)_{\max}$

at $\omega/\omega_n < 1$ (viscous damping)

(under damped)

Performance Parameter of V. I. System

Transmissibility

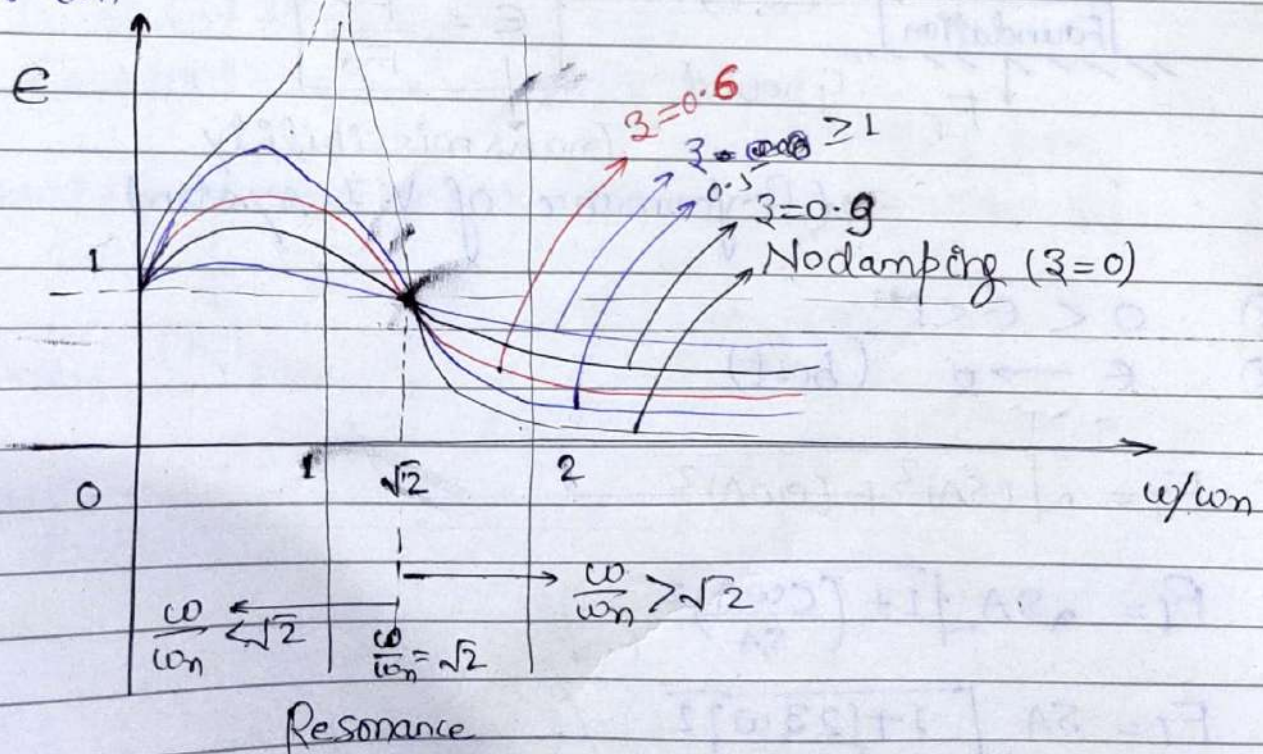
$$E = \frac{\sqrt{1 + \left\{ \frac{2\zeta\omega}{\omega_n} \right\}^2}}{\sqrt{\left\{ 1 - \left(\frac{\omega}{\omega_n} \right)^2 \right\}^2 + \left\{ \frac{2\zeta\omega}{\omega_n} \right\}^2}}$$

Depends upon

- (i) ω/ω_n
- (ii) ζ

* If $\frac{\omega}{\omega_n} = 0$
 * If $\frac{\omega}{\omega_n} = \sqrt{2}$

} $E = 1$ (for any value of ζ)



1) If $\uparrow$ underdamping $\Rightarrow \zeta \downarrow$

ζ will $\uparrow$ if $\frac{\omega}{\omega_n} < \sqrt{2}$

ζ will $\downarrow$ if $\frac{\omega}{\omega_n} > \sqrt{2}$

ζ will remain const. if $\frac{\omega}{\omega_n} = \sqrt{2}$

2) Vibration isolation is effective
When $\frac{\omega}{\omega_n} > \sqrt{2} \Rightarrow \zeta < 1$

Ques. For effective vib. iso. ω_n must be

(i) ω (ii) 2ω (iii) $\omega/4$ (iv) 10ω

$$\rightarrow \frac{\omega}{\omega_n} > \sqrt{2} \Rightarrow \omega_n < \omega/\sqrt{2}$$

3) The zone in which isolation is effective

$$\text{If } \frac{\omega}{\omega_n} > \sqrt{2} \Rightarrow \boxed{\zeta \propto \zeta}$$

$\rightarrow$ Damping becomes detrimental
(Harmful)

(W.B $\rightarrow 10$)

Ques. $m = 17 \text{ kg}$
 $S = 1000 \text{ N/m}$

$m_{\text{steel}} = 2 \text{ kg}$
 $\eta = 75/2000$
 $N = 500 \text{ rpm}$

$$F_0 = 205.6167 \text{ N}$$

$$\omega = 52.3598 \text{ rad/sec} \quad F_0 = 2 \times \frac{75}{2000} \times \left(\frac{2\pi \times 500}{60} \right)^2 \text{ N}$$

$$\omega_n = \sqrt{\frac{S}{m}} = \sqrt{\frac{1000}{17}}$$

$$\omega = \frac{2\pi \times 500}{60} \text{ rad/sec}$$

$$\omega_n = 7.6696 \text{ m/sec}$$

measure
natural
vibration
frequency

← Frahm-Reed tachometer
→ Based on transverse vibration

$$\frac{\omega}{\omega_n} = 6.8268, \quad \zeta = 0.20$$

$$(i) A = \frac{F_0/s}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left(\frac{2\zeta\omega}{\omega_n}\right)^2}} = 4.5 \times 10^{-3} \text{ m} = 4.5 \text{ mm}$$

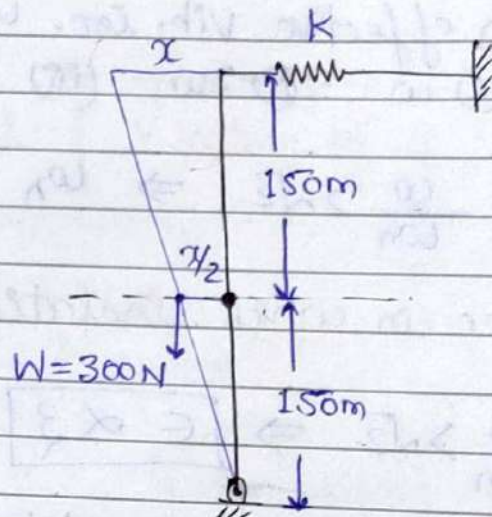
$$(ii) C = \frac{\sqrt{1 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}}{\sqrt{\left\{1 - \left(\frac{\omega}{\omega_n}\right)^2\right\}^2 + \left\{\frac{2\zeta\omega}{\omega_n}\right\}^2}} = 1.2 = \frac{F_T}{205.6167}$$

$$\Rightarrow \boxed{F_T = 13 \text{ N}}$$

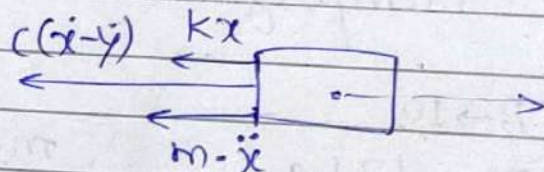
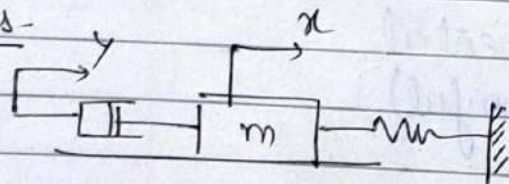
W.B. Ques.

$$K \times 0.3 = W \times \frac{x}{2}$$

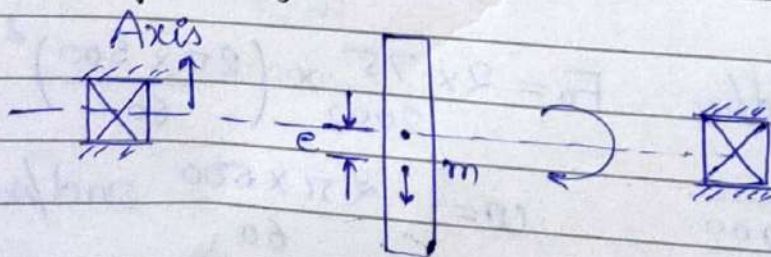
$$\Rightarrow K = 500 \text{ N/m}$$



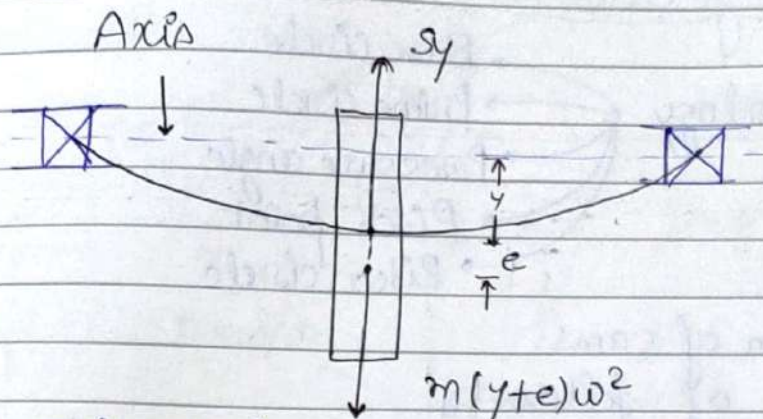
Ques.



Whirling of shafts



After some time



$$S y = m(y+e)\omega^2$$

$$\Rightarrow S y = m y \omega^2 + m e \omega^2$$

$$\Rightarrow \boxed{y = \frac{m e \omega^2}{S - m \omega^2}}$$

$$y = \frac{m \omega^2 \cdot e}{m \omega^2 \left[\left(\frac{\omega n}{\omega} \right)^2 - 1 \right]} = \frac{e}{\left[\left(\frac{\omega n}{\omega} \right)^2 - 1 \right]}$$

$$\boxed{y = \frac{e}{\left(\frac{\omega n}{\omega} \right)^2 - 1}}$$

How to ↑ the running life? ↑↑

↳ If vibration ↓↓

