⇒ Optimulty Test for Transportation Problem :-

There are basically two methods
1) Modified distribution method (MODI)
2) Stepping stone method

⇒ **Modified distribution method** :- The modified distribution method is also known as MODI method or u-v method, which provides a minimum ~~basic~~ cost solution (optimum solution) to the transportation problem. Following are the steps invalved in this method.

step1: Find out the basic feasible solution of the transportation problem using any of the following method.
i) North-east corner method.
ii) Least cost method.
iii) Vogel's approximation method.

step2:- Introduce dual variables corresponding to the row constraints & the column constraints. If there are m supply points and n destination points then there will be m+n dual variables. Dual variable corresponding to row ~~variable~~ are represented by u_i, $i=1,2 \ldots m$, whereas the dual variable corresponding to column are represented by v_j, $j=1,2,\ldots n$. The value of dual variable is calculated from the equation

$$u_i + v_j = c_{ij} \quad \text{if } x_{ij} > 0$$

Step 3: Any basic feasible solution has $m+n-1$ occupied cell $x_{ij}>0$. Thus, there will be $m+n-1$ no. of equations to determine $m+n$ dual variables. One of the dual variables can be chosen arbitrarily. It is also to be noted that as the primal constraints are equations, the dual variables are unrestricted in sign.

Step 4: Calculate $c_{ij} - u_i - v_j$ for all unoccupied cells (i.e. for $x_{ij}=0$). If all $c_{ij} - u_i - v_j \geq 0$, then by the theorem of complementary slackness it can be shown that the corresponding solution of the transportation problem is optimum. If one or more $c_{ij}-u_i-v_j$ is less than zero ($c_{ij} - u_i - v_j < 0$) we select the cell with the least value of $c_{ij}-u_i-v_j$ and allocate as much as posible subject to the row and column constraints. The allocations of the number of adjacent cell are adjusted so that a basic variable becomes nonbasic.

Step 5: A fresh set of dual variables are calculated and repeat the entire procedure.

Let us consider a problem and discuss the above method in detail.

Problem :- The transportion cost per unit of a product is given beloco. Find out the optimal transportation cost.

				Supply
	8	9	7	40
	4	3	5	25
	8	5	6	35
demand	30	30	40	

Solution:

8	9	7	40
4	3	5	25
8	5	6	35
30	30	40	100 / 100

100 → Hence balenced problem

First determine basic feasible solution as discussed in the previous section.

1) North- east corner method

8 (30)	9 (10)	7	40
4	3 (20)	5 (5)	25
8	5	6 (35)	35
30	30	40	

Cost $= 8\times30 + 9\times10 + 3\times20 + 5\times5 + 6\times35$
$=$ Rs 625/-

2) Solution by Least cost method

8 (30)	9	7 (10)	40
4	3 (25)	5	25
8	5 (5)	6 (30)	35
30	30	40	

Cost $= 8\times30 + 7\times10 + 3\times25 + 5\times5 + 6\times30$
$=$ Rs 590/-

3) Solution by Vogel's Approximation method :- (Penalty method)

8 (5)	9	7 (35)	40
4 (25)	3	5	25
8	5 (30)	6 (5)	35
30	30	40	

Cost $= 8\times5 + 7\times35 + 4\times25 + 5\times30 + 6\times5$
$=$ Rs 565/-

<u>Note</u>: There is no need to obtain basic feasible solution by all methods. We have to use just one method to get basic feasible solution and then proceed to get optimal solution by the given method.

Now let us consider the basic feasible solution by by least cost method.

	v_1	v_2	v_3	supply
u_1	8 (30)	9	7 (10)	40
u_2	4	3 (25)	5	25
u_3	8	5 (5)	6 (30)	35
	30	30	40	

Now calculate the dual variables u_i & v_j using

$u_i + v_j = C_{ij}$

$u_1 + v_1 = 8$ $\quad$ $u_3 + v_2 = 5$

$u_1 + v_3 = 7$ $\quad$ $u_3 + v_3 = 6$

$u_2 + v_2 = 3$

For occupied cells only

Choose any one of dual variables arbitrarily is zero, let $u_1 = 0 \Rightarrow u_1 = 0, \; u_2 = -3, \; u_3 = -1$

$v_1 = 8, \; v_2 = 6, \; v_3 = 7$

Now ~~for unallocated positions calculate $c_{ij} - u_i - v_j$~~

	$v_1 = 8$	$v_2 = 6$	$v_3 = 7$	
$u_1 = 0$	8 (30)	9	7 (10)	40
$u_2 = -3$	4	3 (25)	5	25
$u_3 = -1$	8	5 (5)	6 (30)	35
	30	30	40	

~~$8 - 0 - 8 = 0$~~

~~$7 - 0 - 7 = 0$~~

~~$3 - (-3) - 6 = 0$~~

~~$5 - (-1) - 6 = 0$~~

~~$6 - (-1) - 7 = 0$~~

Now compute $C_{ij} - u_i - v_j$ for unallocated positions

$A_{12} \rightarrow \quad 9 - 0 - 6 = 3$

$A_{21} \rightarrow \quad 4 - (-3) - 8 = -1$

$A_{23} \rightarrow \quad 5 - (-3) - 7 = 1$

$A_{31} \rightarrow \quad 8 - (-1) - 6 = 3$

	(3)	
(-1)		(1)
(3)		

We observe that there is a -ve value $(C_{ij} - u_i - v_j)$ here and hence there is a gain and so we can put something in that cell. So put maximum amount (as much as posible) in that cell.

8 (30) - (25)	9	7 (10) + (25)	40
4 (25)	3 (25) - (25)	5	25
8	5 (5) + (25)	6 (30) - (25)	35
30	30	40	

Now balencing the supply and demand we have put as much as posible in the -ve cell. We can put there the amount of 25. Hence put there 25 and adjust other cell accordingly.

Hence New table obtained is!

8 (5)	9	7 (35)	40
4 (25)	3	5	25
8	5 (30)	6 (5)	35
30	30	40	

this is a new solution we obtained. Now we ~~have~~ need to check whether this solution is the best solution or whether some more gain is posible.

* We have to repeat this complete procedure untill all $C_{ij} - u_i - v_j$ in unoccupied cell are greater than or equal to zero.

So check again

	v_1	v_2	v_3	
u_1	8 (5)	9	7 (35)	40
u_2	4 (25)	3	5	25
u_3	8	5 (30)	6 (5)	35
	30	30	40	

$u_1 + v_1 = 8$
$u_1 + v_3 = 7$
$u_2 + v_1 = 4$
$u_3 + v_2 = 5$
$u_3 + v_3 = 6$

Let $u_1 = 0 \Rightarrow u_1 = 0, \; u_2 = -4, \; u_3 = -1$
$v_1 = 8, \; v_2 = 6, \; v_3 = 7$

Now evaluate $c_{ij} - u_i - v_j$ for unallocated positions.

$A_{12} \rightarrow 9 - 0 - 6 = 3$

$A_{22} \rightarrow 3 - (-4) - 6 = 1$

$A_{23} \rightarrow 5 - (-4) - 7 = 2$

$A_{31} \rightarrow 8 - (-1) - 8 = 1$

All these values of $c_{ij} - u_i - v_j > 0$ and hence solution is optimum.

Cost $= 8 \times 5 + 7 \times 35 + 4 \times 25 + 5 \times 30 + 6 \times 5$
$=$ Rs 565/- (optimal cost)