FRAMES

1. Let $X \rightarrow$ complex Hilbert space with $\dim X = n < \infty$ (no. of elements in The basis be n).
2. Let $a_1, a_2, \dots, a_r \in X$, $r > n$.
3. Let $T : X \longrightarrow \mathbb{C}^r$ (r dim complex plane), such that $Tx \in \mathbb{C}^r := Y$. & $(Tx)_j \rightarrow j^{th}$ element of Tx i.e,
$$(Tx)_j = \langle Tx, e_j \rangle$$
where $\{e_1, e_2, \dots, e_r\}$ is the basis of Y.
4. We define,
$$\langle Tx, e_j \rangle := \langle x, a_j \rangle$$
i.e, the j^{th} element of Tx be $\langle x, a_j \rangle$. Then
$$Tx = \sum_{j=1}^{n} (Tx)_j \, e_j = \sum_{j=1}^{n} \langle x, a_j \rangle e_j$$

Since, ~~Let~~ X be n dim. so, $U = \text{Image}(T) = \{Tx \mid x \in X\}$ is almost n dimensional i.e, U is a subspace of $\mathbb{C}^r = Y$.

"Def: If T is injective then the given collection $a_{\mathcal{J}} = (a_1, a_2, \dots, a_r)$ of vectors $a_j \in X$ is called a FRAME for the finite dim. HS X and the mapping T is called The FRAME operator".

§ Let $\langle x, y \rangle = \sum_{i=1}^{r} x_i \bar{y}_i$ be the inner product on Y, then Y is also a Hilbert space (?). Thus the mapping T is a mapping from the HS X to a HS Y so $\exists$/s a mapping T^*, the adjoint of T, from Y to X i.e, $T^* : Y \rightarrow X$ such that
$$\langle x, T^* y \rangle_X = \langle Tx, y \rangle_Y \quad \forall\, x \in X, y \in Y.$$
So in particular, $\langle x, T^* e_j \rangle = \langle Tx, e_j \rangle = \langle x, a_j \rangle$
$$\Longrightarrow T^* e_j = a_j, \quad j = 1, 2, \dots r.$$