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M.Sc. (Physics) Sem - IV

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Frequency Spectrum of Continuous X-Rays:

We know that the radiated energy extended over a range of frequencies. So, this frequency spectrum can be acheived using fourier analysis of the radiation field. Moreover, the radiation take place when the entire incident electron promptly changes its direction during a collision along with one of the atoms in the metallic target. Let us consider a zero collision time and $v << c$. Then the resulting acceleration and field can be written as

$$\dot{v} = \delta(t_0 - t)\,\Delta v, \quad \int \dot{v}\, dt = \Delta v \qquad \text{---(1)}$$

Now the radiated energy,

$$E(t) = \frac{e \sin\theta}{c^2 R}\,\Delta v\,\delta(t_0 - t) \qquad \text{---(2)}$$

where t_0 is the instant at which the radiation take place. In eqn (2), the instantaneous field is reperesented in the observer's time because we have to consider a frequency spectrum in terms of observer frequency ω.

The fourier transform

$$E(t) = \int_0^{+\infty} E(\omega)\, e^{-i\omega t}\, d\omega. \qquad \text{---(3)}$$

where, $\omega \rightarrow$ is the angular frequency, $\omega = 2\pi\nu$.

The inverse is,

$$E(\omega) = \frac{1}{\pi}\int_{-\infty}^{+\infty} E(t)\, e^{i\omega t}\, dt \qquad \text{---(4)}$$

From eqn (2) and (4) we have,

$$E(\omega) = \frac{e \sin\theta}{\pi c^2 R}\cdot \Delta v\, e^{i\omega t_0} \qquad \text{---(5)}$$

(P-1)

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So, the total energy radiated is given by the integral of the Poynting vector over the surface of a sphere as well as over the time during which the change of velocity take place.

Thus, for $v << c$,

$$-U = \int \left(-\frac{du}{dT}\right) \cancel{dt}\, dT = \frac{c}{4\pi} \iint E^2\, dt\, d\sigma \quad \text{——— (6)}$$

According to the Parseval's formula:

$$\int_{-\infty}^{+\infty} |E|^2\, dt = \pi \int_0^{\infty} |E(\omega)|^2\, d\sigma\, d\omega \quad \text{——— (7)}$$

Hence, for the energy radiated in a frequency interval $d\omega$,

$$-U(\omega)\, d\omega = \pi \frac{c}{4\pi} \int_{\sigma} |E(\omega)|^2\, d\omega \quad \text{——— (8)}$$

$$= \frac{e^2 (\Delta v)^2}{4\pi^2 c^3 R^2}\, d\omega \int_0^{2\pi} d\phi \int_0^{\pi} \sin^2\theta\, R^2 \sin\theta\, d\theta$$

$$= \frac{4e^2}{3c} \left(\frac{\Delta v}{c}\right)^2 \frac{d\omega}{2\pi} \quad \text{——— (9)}$$

Therefore, eqn (9) reveals the spectrum of radiation is independent of frequency, ω as shown in fig 1.

This spectrum extends to high values of ω, due to the assumption of zero collision. Thus, a fourier analysis of a finite collision-time process will damp the very high frequency components but not abruptly.

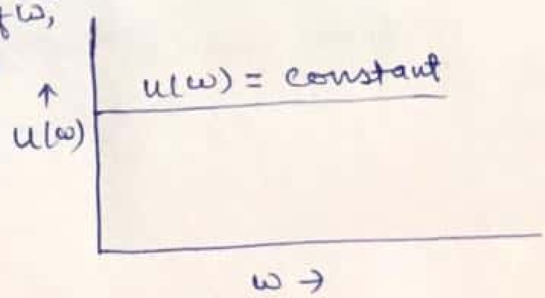

Fig 1: Frequency spectrum of continuous X-rays (according to the classical theory)

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