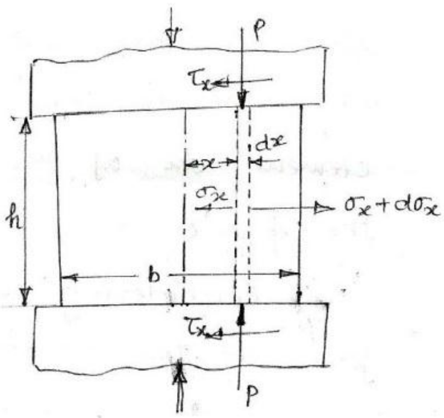
FORGING OF A SLAB

Assumptions-



1. The material being torged is plastically rigid.
2. The coefficient of friction between the w/p and the dies is same throughout.
3. The thickness of the work piece is small in comparison to other dimensions.
4. Condition of plane strain exists during torging i.e. the width of block remains constant.

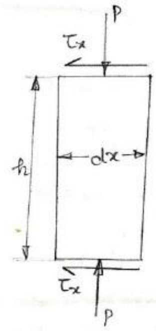
figure shows a typical open die plane strain forging of that strip consider the size of strip undergoing torging operation as (b x h x w). The stress acting on an element dx of the strip are shown in the figure

Considering the equilibrium of the forces in the $x - direction$, we get

$$(\sigma_x + d\sigma_x)hw - \sigma_x.hw - 2\tau_x dx.w = 0$$

$$\cancel{\sigma_x} hw + hwd\sigma_x - \cancel{\sigma_x}.hw - 2\tau_x dx.w = 0$$

$$hd\sigma_x - 2\tau_x dx = 0 - - - ❶$$



Assuming σ_x and P as principle stresses and taking. Tresca's yield theory we have for plane strain

$$\sigma_1 - \sigma_3 = 2K = \sigma_y \ (yield\ stess)$$

$$\sigma_1 = \sigma_x \quad \& \quad \sigma_3 = -P$$

$$\sigma_x - (-P) = 2K$$

$$\sigma_x + P = 2K$$

$$d\sigma_x = -dP$$

Now substitute the value of $d\sigma_x$ in eq^{-n} ❶

$$h(-dP) - 2\tau_x dx = 0$$

$$dP = -\frac{2\tau_x}{h} dx - - - - ❷$$

1) SLIDING FRICTION

with constant coefficient of friction μ,

$$\tau_x = \mu_p$$

Now substituting the value of τ_x in eq^h ❷

$$dP = -\frac{2\mu P}{h} dx$$

$$\frac{dP}{P} = -\frac{2\mu}{h} dx$$

After Integrating, We get

$$lnP = \frac{-2\mu}{h}x + c - - - - ❸$$

Now at $x = \frac{b}{2}, \sigma_x = 0\ (stress\ free\ surface)$

$p = 2K\ (from\ yield\ criteria)$

$$\ln 2K = \frac{-2\mu}{h}.\frac{b}{2} + C$$

$$C = ln\ 2K + \frac{2\mu}{h}.\frac{b}{2}$$

Now put the value of c in eq^h ❸

$$ln\ P = \frac{-2\mu}{h}x + ln2K + \frac{2\mu}{h}.\frac{b}{2}$$

$$ln\frac{P}{2K} = \frac{-2\mu}{h}\left(x - \frac{b}{2}\right)$$

$$\boxed{\frac{P}{2k} = e^{\frac{-2\mu}{h}\left(x-\frac{b}{2}\right)}}$$

$$P = 2K.e^{\frac{-2\mu}{h}\left(x-\frac{b}{2}\right)} \text{ --------------}❹$$

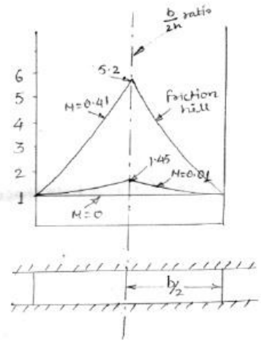
$$\frac{P}{2k} = \frac{P}{\sigma_y} = e^{\frac{-2\mu}{h}\left(x-\frac{b}{2}\right)} \text{ -------------}❺$$

Also $\sigma_x = 2k - P$

$$\sigma_x = 2k - 2ke^{\frac{-2\mu}{h}\left(x-\frac{b}{2}\right)} \qquad \text{from ❹}$$

$$\sigma_x = 2k\left[1 - e^{\frac{-2\mu}{h}\left(x-\frac{b}{2}\right)}\right]$$

equation ❺ is plotted in dimensionless from the pressure increases exponentially towards the centre of the part and also that it increases with the $\frac{b}{2h}$ ratio and increasing friction because of its shape the pressure distribution curve is referred to as friction hill



Hence Putting $x = 0$

$$\left(\frac{P}{2k}\right)_{max} = e^{\left(\frac{2\mu}{h}\frac{b}{2}\right)}$$

And $$(\sigma_x)_{max} = 2k\left[1 - 2k^{\frac{2\mu}{h}\frac{b}{2}}\right]$$

Not total forging load

$$f = 2\omega \int_0^{b/2} P dx$$

and Avg pressure

$$P_a = \frac{f}{2\omega\left(\frac{b}{2}\right)} = \int_0^{b/2} \frac{Pdx}{b/2}$$

$$= \frac{\sigma_y}{b/2} \int_0^{b/2} e^{\frac{-2\mu}{h}\left(x-\frac{b}{2}\right)} dx$$

$$= \frac{\sigma_y}{b/2} \cdot \frac{h}{2\mu} \left[-e^{\frac{-2\mu}{h}\left(x-\frac{b}{2}\right)}\right]_0^{b/2}$$

$$\frac{\sigma_y h}{\mu b}\left[-1 + e^{\frac{\mu b}{h}}\right]$$

Assuming $\frac{\mu b}{h}$ to be small and expanding as a series

$$P_a = \frac{\sigma_y h}{\mu b}\left[-1 + 1 + \frac{\mu b}{h} + \frac{1}{2}\left(\frac{\mu b}{h}\right)^2 - \cdots\right]$$

$$= \sigma_y \left[1 + \frac{\mu b}{2h} + \cdots\right]$$

$$\boxed{P_a = \sigma_y \left[1 + \frac{\mu b}{2h}\right]}$$

2) STIKING FRICTION:

If the friction is high it may reach the sticking friction then the sticking friction extents over the whole work piece/die interface; for the condition

$$\tau_x = k$$

Now from eq^n ❶

$$h d\sigma_x - 2K dx = 0$$

$$-h dP - 2k dx = 0$$

$$\frac{dP}{2k} = -\frac{dx}{h}$$

integrating

$$\frac{P}{2k} = -\frac{x}{h} + c$$

Now at $x = b/2$; $P = 2k$ $(Since\ \sigma_x = 0)$

$$C = 1 + \frac{b}{2h}$$

hence

$$\boxed{\frac{P}{2k} = -\frac{x}{h} + \left(1 + \frac{b}{2h}\right)}$$

The above equation predicts a linear variations of P from the outer edge to the centre line or max value will be

$$\left(\frac{P}{2k}\right)_{max} = 1 + \frac{b}{2h} \quad (at\ centre\ x = 0)$$

3) MIXED FRICTION CONDITION :

$$\frac{P}{2k} = \frac{\left(\frac{b}{2} - x\right)}{h} + \frac{1}{2\mu}\left[1 - ln\frac{1}{2\mu}\right]$$