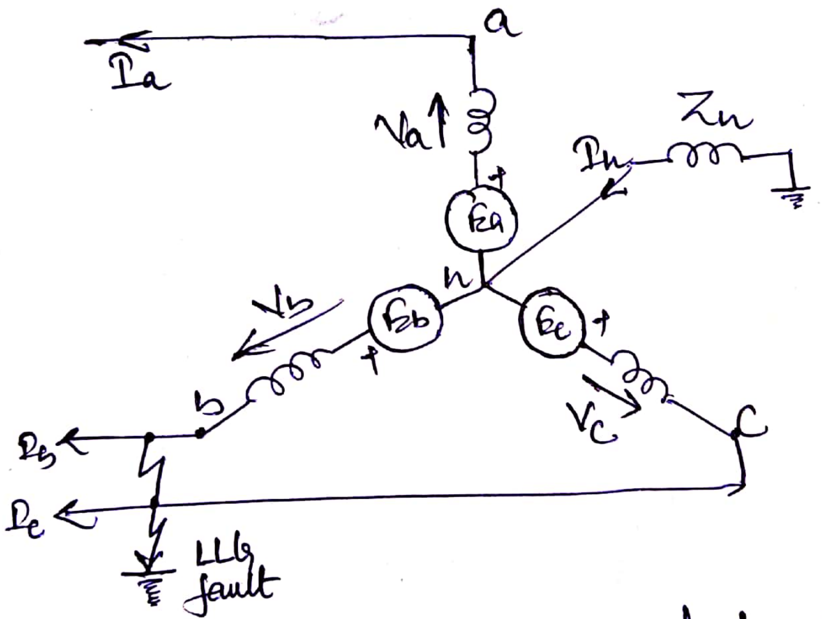
Double Line-to-Ground (LLG) fault :-



Consider a double-line to ground fault at the terminal of a unloaded generator, whose neutral is grounded through a reactance, between phase 'b' and 'c' as shown in fig.

Consider fault conditions —

$$I_a = 0 \;\&\; V_b = V_c = 0 \longrightarrow (15)$$

Symmetrical component of voltage with $V_b = V_c = 0$

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c)$$
$$= V_a/3$$

$$\cancel{V_{a1} = \frac{1}{3}(V_a + \alpha V}$$

$$V_{a1} = \frac{1}{3}(V_a + \alpha V_b + \alpha^2 V_c)$$
$$= V_a/3$$

$$V_{a2} = \frac{1}{3}(V_a + \alpha^2 V_b + \alpha V_c)$$
$$= V_a/3$$

Hence

$$V_{a0} = V_{a1} = V_{a2} = V_a/3 \longrightarrow (16)$$

Using this realation of voltage and substituting in the sequence network eqn. (2)

$$V_{a0} = V_{a1}$$

$$-I_{a0} Z_0 = E_a - V_{a1} Z_1$$

$$I_{a0} = \frac{-(E_a - I_{a1} Z_1)}{Z_0} \longrightarrow (17)$$

Similarly

$$V_{a2} = V_{a1}$$

$$-I_{a2} Z_2 = E_a - I_{a1} Z_1$$

$$I_{a2} = \frac{-(E_a - I_{a1}Z_1)}{Z_2} \longrightarrow (18)$$

Now from eqn. (15)

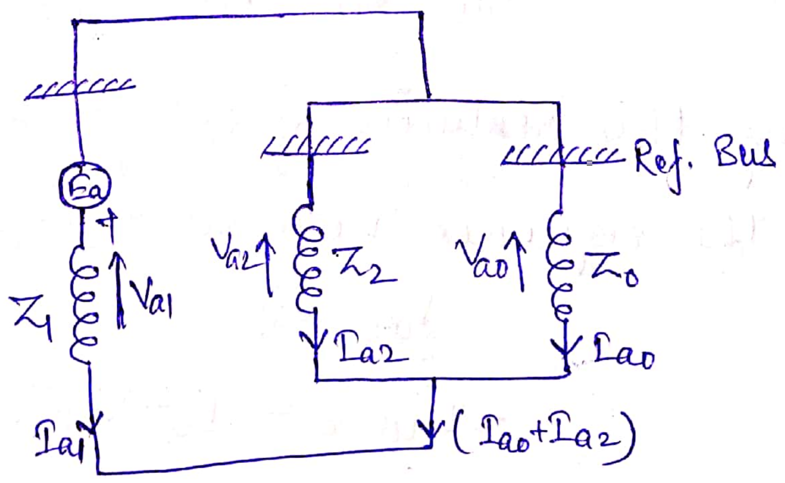
$$I_a = I_{a1} + I_{a2} + I_{a0} = 0$$

Now substitute the value of I_{a0} & I_{a2} from eqn. (17) & (18)

$$I_{a1} - \frac{(E_a - I_{a1}Z_1)}{Z_2} - \frac{(E_a - I_{a1}Z_2)}{Z_0} = 0$$

After rearranging the terms ——.

$$\boxed{I_{a1} = \frac{E_a}{Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}}} \longrightarrow (19)$$



Eqn. (19) implies that, to simulate the LLG fault, the three sequence networks are connected such that the positive N/W is connected in series with the parallel combination of the negative and zero sequence networks as shown in above figure.

LL fault with impedance Z_n —

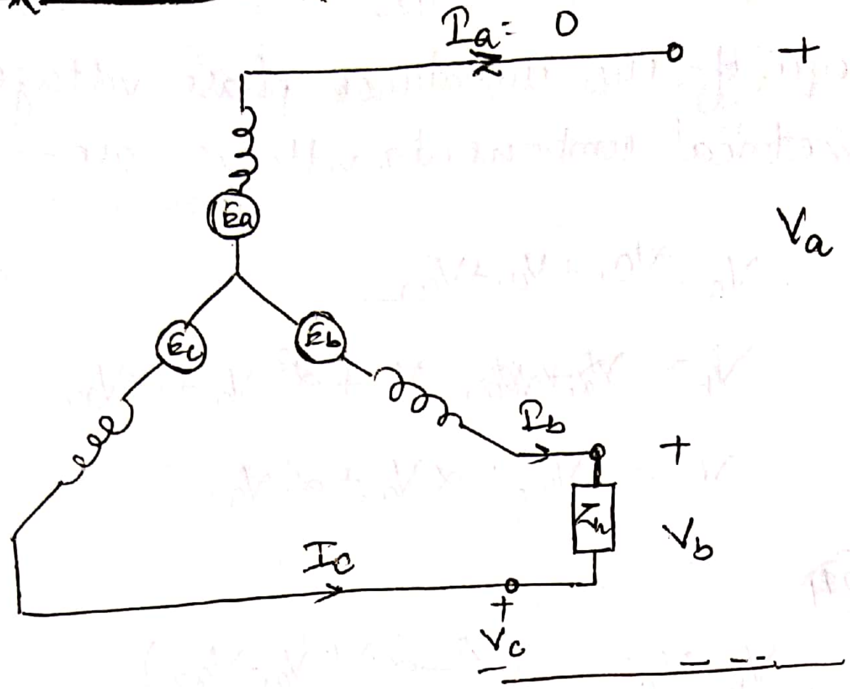


Figure shows a 3-ϕ generator with fault through an impedance Z_f between phases b and c. Assuming the generator ~~is~~ is initially on noload, the boundary conditions at the fault point are —

$$\left.\begin{aligned} V_b - V_c &= Z_f I_b \\ I_b + I_c &= 0 \text{ or } I_b = -I_c \\ I_a &= 0 \end{aligned}\right\} \longrightarrow (20)$$

Substituting for $I_a = 0$, and $I_c = -I_b$, the symmetrical components of currents ~~from~~ —

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix} \longrightarrow (21)$$

After solving the above eqn.

$$\left.\begin{aligned} I_{a0} &= 0 \\ I_{a1} &= \frac{1}{3}(\alpha - \alpha^2) I_b \\ I_{a2} &= \frac{1}{3}(\alpha^2 - \alpha) I_b \end{aligned}\right\} \longrightarrow (22)$$

from equ. no. 22

$$I_{a1} = -I_{a2} \longrightarrow (23)$$

From the equ. of the unbalance phase voltages in terms of the symmetrical components voltages are —

$$\left.\begin{aligned} V_a &= V_{a0} + V_{a1} + V_{a2} \\ V_b &= V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2} \\ V_c &= V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2} \end{aligned}\right\} \longrightarrow (24)$$

from equ. (24)

$$V_b - V_c = (\alpha^2 - \alpha)(V_{a1} - V_{a2})$$
$$= Z_n I_b$$

$$(\alpha^2 - \alpha)(V_{a1} - V_{a2}) = Z_n I_b \longrightarrow (25)$$

from equ. (2)

$$(\alpha^2 - \alpha)\left[(E_a - Z_1 I_{a1}) - (-Z_2 I_{a2})\right] = Z_n I_b$$

$$(\alpha^2 - \alpha)\left[E_a - Z_1 I_{a1} + Z_2 I_{a2}\right] = Z_n I_b$$

from equ. (23)

$$(\alpha^2 - \alpha)\left[E_a - (Z_1 + Z_2) I_{a1}\right] = Z_n I_b$$

from equ. (22)

$$E_a - (Z_1 + Z_2) I_{a1} = \frac{3 Z_n I_{a1}}{(\alpha - \alpha^2)(\alpha^2 - \alpha)} \longrightarrow (26)$$

After solution the value of $(\alpha - \alpha^2)(\alpha^2 - \alpha) = 3$

from equ. no. 26

$$\boxed{I_{a1} = \frac{E_a}{Z_1+Z_2+Z_n}} \longrightarrow (27)$$

with fault impedance Z_n

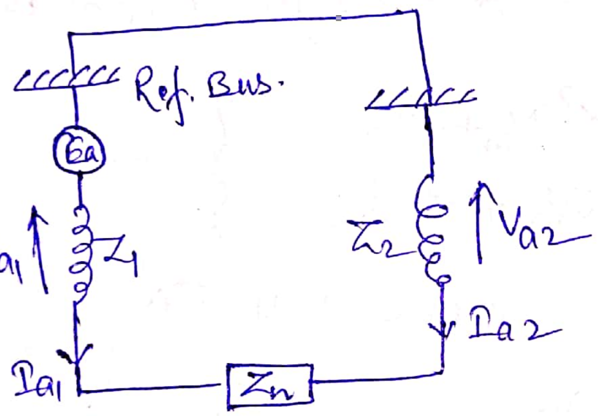
The phase currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{bmatrix}$$

The fault current is

$$I_b = -I_c = (\alpha^2 - \alpha) I_{a1}$$

$$\boxed{I_b = -j\sqrt{3}\, I_{a1}} \longrightarrow (28)$$



LLG Fault with fault impedance Z_n :-

Figure shows a 3-ϕ generator with a fault on phases 'b' and 'c' through an impedance Z_n to ground. Assuming the generator is initially on no-load, the boundary conditions at the fault point are —

$$V_b = V_c = Z_f(I_b + I_c) \longrightarrow (29)$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0 \longrightarrow (30)$$

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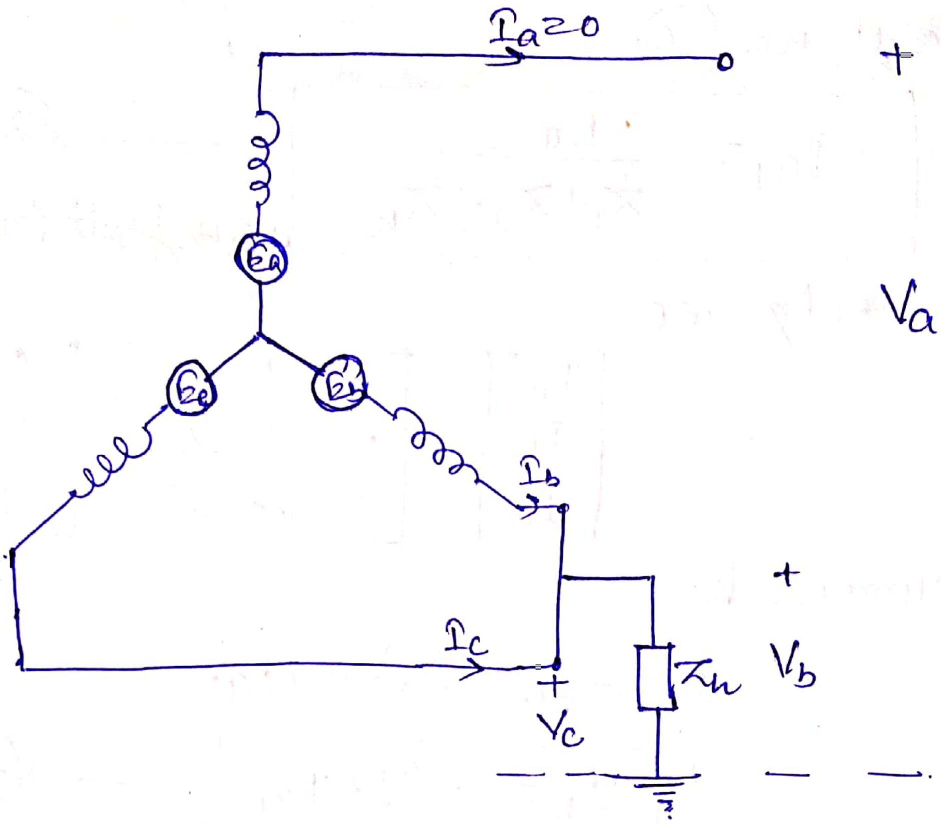

Relation b/w phase voltages & its symmetrical components

$$\left.\begin{aligned} V_b &= V_{a0} + \alpha^2 V_{a1} + \alpha V_{a2} \\ V_c &= V_{a0} + \alpha V_{a1} + \alpha^2 V_{a2} \end{aligned}\right\} \longrightarrow (31)$$

since $V_b = V_c$, from above eqn. (31)

$$V_{a1} = V_{a2} \longrightarrow (32)$$

Put the symmetrical components of currents I_b & I_c in eqn. (29)

$$V_b = Z_n(I_a^0 + \alpha^2 I_{a1} + \alpha I_{a2} + I_{a0} + \alpha I_{a1} + \alpha^2 I_{a2})$$

$$= Z_n(2I_{a0} - I_{a1} - I_{a2}) \quad \left\{3I_{a0} - \underbrace{I_{a0} - I_{a1} - I_{a2}}_{0}\right\}$$

$$V_b = 3Z_n I_{a0} \longrightarrow (33)$$

Put the value of V_b & V_{a2} from eqn. (33) & (32) in eqn. (31)

$$3Z_n I_{a0} = V_{a0} + (\alpha^2 + \alpha) V_{a1}$$

$$3Z_n I_{a0} = V_{a0} - V_{a1} \longrightarrow (32)$$

from equ. (2)

$$3Z_n I_{a0} = -Z_0 I_{a0} - [E_a - Z_1 I_{a1}]$$

$$(3Z_n + Z_0) I_{a0} = -(E_a - Z_1 I_{a1})$$

$$\boxed{I_{a0} = \frac{-(E_a - Z_1 I_{a1})}{Z_0 + 3Z_n}} \longrightarrow (33)$$

Put the value of V_{a1} & V_{a2} in equ. (32) from equ. (2)

$$E_a - Z_1 I_{a1} = -Z_2 I_{a2}$$

$$\boxed{I_{a2} = \frac{-(E_a - Z_1 I_{a1})}{Z_2}} \longrightarrow (34)$$

Put the value of I_{a0} & I_{a2} from equ. (33) & (34) in equ. (30) & solve for I_{a1} —

$$\boxed{I_{a1} = \frac{E_a}{Z_1 + \frac{Z_2(Z_0 + 3Z_n)}{Z_2 + Z_0 + 3Z_n}}}$$

With fault impedance Z_n.

Finally the fault current $I_f = I_b + I_c = 3I_{a0}$