1: Introduction to Statistical Quality Control

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1.1 Introduction

In any production process, regardless of how well designed and carefully maintained it is, a certain amount of inherent or natural variability will always exist. This natural variability is the cumulative effect of many small, essentially uncontrollable causes, often called as “stable system of chance causes” or “allowable causes”. A process which is operating with only allowable causes of variation present is said to be in “Statistical control”.

Other kinds of variability may occasionally be present in the output of a process. These are large variations that are attributed to special causes like differences among machines or operators or raw material used or interaction among them. Such variability usually represents an unacceptable level of process performance. We refer to these sources of variability as “assignable causes”.

It is very typical for production process to operate in – control state, processing acceptable product for relatively long periods of time. Occasionally, however, assignable causes will occur seemingly at random, resulting in a “shift” to an out-of-control state where a larger proportion of the process output does not confirm to requirements. A major objective of statistical quality control is to quickly detect the occurrence of assignable causes or process shift so that the investigation of the process and the corrective action may be undertaken before very many non-conforming units are manufactured.

In the above type of problem, our aim is to control the production process so that the proportion of defective (or non-conforming) item is not excessively large. This is known as “process control”. In other type of problem, we like to ensure that the lots of manufactured goods do not contain an excessively large proportion of defective items. This is known as “product control or lot control”. The two are different problems, because even when the process is in control so that the proportion of defective products for the entire output over a long period will not be large, it is possible that the individual lot of items may contain excessively large number of defectives. Process control is achieved mainly through the technique of CONTROL CHARTS, whereas product control is achieved through SAMPLING INSPECTION.

1.2 Advantages of Quality Control

Every manufacturer wants the process of manufacturing to be cost effective. Also, a consistent quality of product provides a better movement in the market rather than a product with variable characteristics. So, the quality control measures re utmost requirement of any production process. Main advantages of quality control are:-

(1) With a regular use of quality control, any defect in the production process can be detected at an early stage, so that damage due to the defect is minimum.

(2) With the use of quality control techniques, we get better quality assurance at lower inspection cost.

(3) Statistical quality control provided better time management and hence waste of time and material is minimized.

(4) Wherever testing is of destructive type, by means of quality control only, one can get a sample of appropriate size so that quality can be checked with minimum waste of money.

(5) By means of quality control, any product reaches to the market with a consistent quality characteristics and so captives the market early. Also with a maintained quality, movement of the product is faster and the chances that product will come back to supplier due to poor quality are minimum.

1.3 Quality Characteristics

quality characteristic we mean any characteristic of the product which is of interest in determing its quality. Many quality characteristics are measurable qualitatively (or numerically) and may be looked upon as "variables" which may be continuous eg:- length, area, thickness, volume, density or chemical composition of a product, or discrete. e.g. the number of defects in a piece of cloth.

Often the quality characteristic cannot be measured and is expressed as an "attribute". Here each item may be classified as 'good' (or non-defective) or 'bad' (or defective). Also an item which has one or more defects is defective. Again, although a characteristic may be measurable, one may treat it as an attribute for simplicity, e.g. a manufacture producing rods may classify a rod as defective of it is too long or too short and thus avoid measuring its actual length.

1.4 Control Charts: Basic Principles

Suppose samples of a given size are taken from a process at more or less regular intervals and suppose for each sample some statistic T is computed. For example, T may be the sample mean, or sample range, or sample standard deviation or sample fraction defective. Being a sample result, T will be subject to sampling fluctuations. If the process is in control, i.e, no assignable causes of variation are present, the sampling fluctuations of T should be due to chance variation alone. Supposing in such a case

$$E(T) = \mu_T$$

and $$\text{var}(T) = \sigma_T^2$$

We may take any values of T lying outside the limits $(\mu_T - 3\sigma_T)$ and $(\mu_T + 3\sigma_T)$ as an indication of the presence of systematic variation, i.e. variation due to assignable causes the reason behind this argument is that in case T is normally distributed (and the process is stable). Then,

$$\begin{aligned} & P\left[|T-\mu_T| \le 3\sigma_T\right] \\ &= P\left[-3\sigma_T \le T-\mu_T \le 3\sigma_T\right] \\ &= P\left[-\frac{3\sigma_T}{\sigma_T} \le \frac{T-\mu_T}{\sigma_T} \le \frac{3\sigma_T}{\sigma_T}\right] \\ &= P\left[-3 \le Z \le 3\right] \end{aligned}$$

$$\text{where} - Z = \frac{T-\mu_T}{\sigma_T} \sim N(0,1)$$

$$= 2P[0 < -Z < 3]$$
$$= 2 \times 0.49865$$
$$\approx 0.9973$$

i.e. $P[|T - \sigma_T| \le 3\sigma_T] \approx 0.9973$

Even when T is non-normal, we have from the Chebychev's Inequality.

$$P[|x - E(x)| \le \varepsilon] \ge 1 - \frac{v(x)}{\varepsilon^2}$$

Let $\varepsilon = 3\sigma_T$

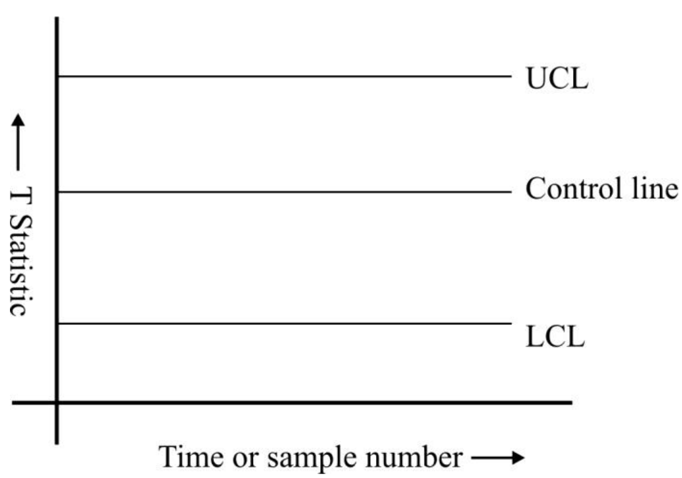
$$\therefore P[|T - \mu_T| \le 3\sigma_T] \ge 1 - \frac{\sigma_T^2}{9\sigma_T}$$

$$P[|T - \mu_T| \le 3\sigma_T] \ge 1 - \frac{1}{9}$$

$$P[|T - \mu_T| \le 3\sigma_T] \ge \frac{8}{9}$$

Thus if the observed T_i (where i stands for the i^{th} sample) lies between the limits $(\mu_T - 3\sigma_T)$ and $(\mu_T + 3\sigma_T)$ it is taken to be a fairly good indication of non existence of assignable causes of variation at the time i^{th} sample was taken. If the observed T_i wanders or false outside the limits, one suspects the existence of assignable cause of variation and the process is supposed to be out of control. The obvious action is then to stop the process and to hunt for and remove the assignable causes.

The theory of control charts was developed by Dr. Walter Shewhart. It is a horizontal chart where time or sample number is plotted on the abscissa and the values of statistic T are plotted as ordinates. There is a 'central line' corresponding to the mean value μ_T, a 'lower control limit' (LCL) corresponding to $(\mu_T - 3\sigma_T)$ and an upper control limit (UCL) corresponding to $(\mu_T + 3\sigma_T)$ as shown in the following diagram.



If μ_T and σ_T are not known, it is possible to estimate them. If several samples are taken, the mean of T is estimated from the mean of sample and standard deviation of T is estimated from the within sample variation of the samples.

According to Dr. Shewhart a control chart may serve, first, to define the goal or standard for a process that the management might strive to attain, second it may be used as an instrument for attaining that goal; and, third, it may serve as a means of judging whether the goal has been attained. It is thus an instrument to be used in specification, production and inspection and when so used, bring these three phases of industry into an independent whole. Let us elaborate this.

If the sample values of T are plotted for a significant range of output and time and if these values all fall within the control limits and show no systematic pattern, we say that the process is in control at the level indicated by the chartered thus, a control chart may be used to specify the goal of management.

The control chart may also be used to attain certain goals with respect to process quality. The central line and control limits may be standard values chosen by the management such that they want the process to be in control at that level of quality. Sample data are plotted on the chart and if departures from the in control state are investigated and corrected, then eventually the process may be brought into control at the target or standard values.

Lastly, a control chart may be used for judging whether the state of control has been attained. If the sample values of T all fall within the control limits without varying in a non-random manner within the limits, then the process may be judged to be within control at the level indicated by the chart. Likewise, if a process has been judged to be in control and new sample results continue to fall within the limits on the chart (without being in any non random pattern) the process may be judged to be continuing in a state of statistical control at the given level.

1.5 Operating Characteristics of a Control Chart

There is a close connection between control chart and hypothesis testing. Essentially the control chart is a test of the hypothesis that the process is in a state of statistical control. A sample point falling within the control limits is equivalent to accepting the hypotheses of statistical control and a point falling outside the limits is equivalent to rejecting the hypothesis of statistical control. Just as in hypothesis testing, we may think of the probability of a type I error of the control chart (concluding that the process is out of control when it is really in control) and the probability of type II error of the control chart (concluding that the process is in control when it is not).

It is occasionally useful to use the operating characteristic (OC) curve of control chart to display the probability of type II error. An OC curve shows the probability (or risk) of inferring that process is in control at the designated level, when the process is actually open at a different level. This would be an indication of the ability of a control chart to detect process shifts of different magnitudes.

1.6 Choice of Control Limits

Specifying the control limits is one of the critical decisions that must be made in designing control chart. By moving the control limits further from the central line we decrease the risk of type I error but, simultaneously, increase the risk of type II error. If we move the control line closes to the central line, the risk of type I error increases while the risk of type II error decreases.

In the above discussion, we have considered 3σ control limits. We could as well use 0.001 probability limits, k*, such that

$$P\left[\left|T-\mu_T\right| > k^*\sigma_T\right] = 0.002$$

If T is normally distributed, k* = 3.09 some analysts suggest using two sets of limits on control chart. The outer limits, at 3σ, all the usual action limits that is when a point falls outside these limits, a search for an assignable cause is made and a corrective action is taken if necessary. The inner limits, usually at 2σ, are called ‘warning limits’. When probability limits are used the action limits are 0.001 limits and the warning limits are 0.025 limits. If one or more points fall between the warning limits and action limits or very close to the warning limits, then we should be suspicious that the process may not be operating properly. One possible action then is to increase the sampling frequency and use the additional data in conjunction with the suspicious points to investigate the state of control of the process.

1.7 Sample Size and Sample Frequency

In designing a control chart, one must specify both the sample size and sampling frequency. In general, larger samples will make it easier to detect small shifts in the process. This can be seen from the OC curves of a control chart for different sample sizes. If the process shift is relatively large, than those that would be employed if the shift of interest is small.

The most desirable situation from the point of view of detecting shift would be to take large sample very frequently. However, this is not economically feasible, i.e. either we take small samples at short intervals or large samples at larger intervals. Current industry practices to favour the former among the two.

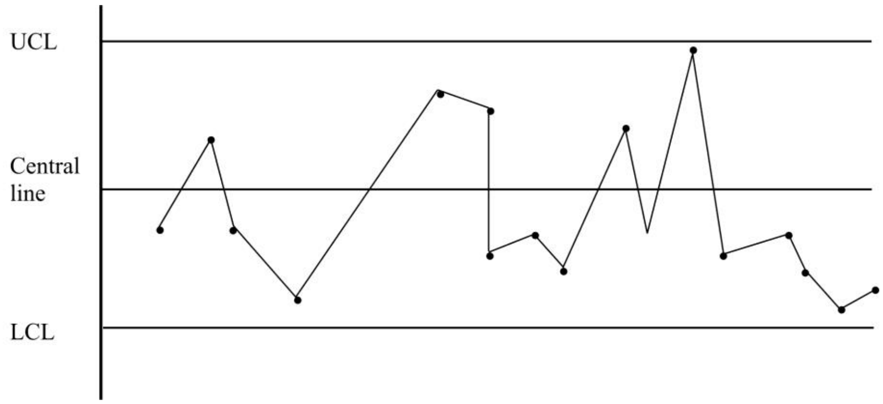
1.8 Rational Subgroups

A fundamental idea in the use of control charts is the collection of sample data according to what Shewart called ‘rational subgroup’ concept generally, this means that the subgroups or sample should be so selected that if assignable causes are present, the choice for difference between the subgroup will be maximized, while the chance for difference with in a subgroup will be minimized. In other words, the products within a subgroup should be very homogeneous, while the differences between subgroups will indicate the presence of systematic variations.

The most obvious bases for selection of subgroups is the order of production. Each subgroup will then consist of products of a machine or a homogeneous group of machines for a short period of time, so that there cannot be any unmarkable change in the cause system within the period. When there are two or more machines having different pattern of variation, it may be necessary to have different subgroups for different machines. It may, therefore be necessary to have different subgroups for different machines or different operators or different shifts etc.

1.9 Analysis of Pattern on Control Charts

A control chart may indicate an out of control condition either when one or more points fall beyond the control limits, or when the points falling within the control limits exhibit some non-random or systematic pattern of behaviour for example, see the following control charts;

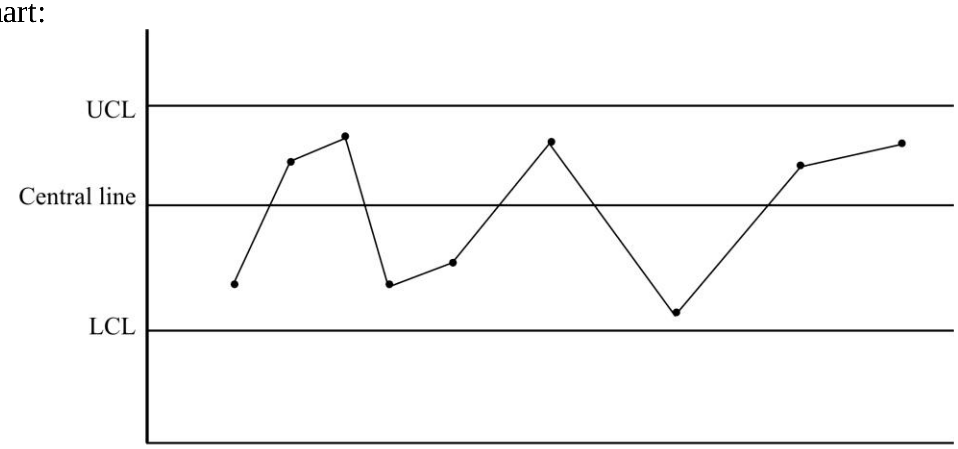


Although all the points fall within the control limits, the points do not indicate statistical control because their pattern is very non-random in appearance. Most of the points fall because the central line. We observe that there are cases of long run up and long run down.

We define run as a sequence of observation of the same type (increasing or descending). In addition to runs up and down, we could define the types of observations as those above and below the central line, respectively, so that two in a row above the central line would a run of length two.

A run of length 8 or more points has a very low probability of occurrence in a random sample of points. Consequently, any type of run of length 7 or more should be taken as a signal of an out of control condition. For example 8 consecutive points on one side of the central line would indicate that the process is out of control.

Other types of patterns may also indicate an out of control situation. So the following chart:



Note that the points exhibit a cyclic behavior, although they fall within the control limits. Such a pattern would indicate a problem with the process, such as operator fatigue, raw material deliveries, etc. While the process is not out of control, the yield may be improved by elimination or reduction of sources of variability causing cyclic behavior.

The problem is one of pattern recognition that is recognizing systematic or non-random patterns on the control chart and identifying the reason for this behavior. The ability to interpret a particular pattern in tern of assignable causes requires experience and knowledge of the process.

Several different criteria may be applied simultaneously to the control chart to determine whether the process is out of control. Some such criteria, used in practice are as follows.

One or more points outside the control limits.

A run of at least seven or eight points, where the run may be of any kind.
Two of three consecutive points outside the 2σ warning limits, but still inside the control limits.
Four of five consecutive points beyond the 1σ limits.
An unusual or non-random pattern on the data
One or more points near a warning or control limits.
Suppose an analyst uses k-test criteria, criteria i having type I error or false alarm probability for the decision based on all k tests is

$$\alpha = 1 - \prod_{i=1}^{k}(1-\alpha_i)$$

assuming independence of tests.

1.10 Rate of Detection of change in average level (Average Run Length or A.R.L. Function)

The A.R.L is the average number of sample points, which must be plotted on a chart before a point indicates an out of control situation. This indicates how quickly the chart will detect any shift in the process average.

The OC function of the chart gives the probability of deciding that the process is in control, Pa as a function of process overage μ Let.

$$p = 1 - P_a$$

be the probability of deciding that the process is out of control.
If N is the number of points necessary before taking a decision is out of control,

$$\begin{aligned} \text{A.R.L.} = E(N) &= \sum_{n=1}^{\infty} n(1-p)^{n-1} \\ &= p\left[1-(1-p)^{-2}\right] \\ &= p[1-1+p]^2 = \frac{p}{p^2} \end{aligned}$$

This, as a function of process average is, gives the A.R.L curve of the chart for true average μ but should be small for shift in μ.