**For the students of**

**M. Com. (Applied Economics) Sem. IV**

**Paper: Research Methodology (Unit III)**

Note: Study material may be useful for the courses wherever Research Methodology paper is being taught.

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# Topic: Tests of Significance for Large and Small Samples

## I. Test of Significance of difference for Large Samples (Z test)

### (I) Hypothesis Testing for Single Population Mean

**Two-Tailed Test:** Let $\mu$ be the hypothesized value of the population mean to be tested. For this the null and alternative hypotheses for two tailed-tests are defined as:

$H_0$: $\mu = \bar{X}$ or $\mu - \bar{X} = 0$

$H_1$: $\mu \neq \bar{X}$

And;

If standard deviation $\sigma$ of the population is known, then based on the central limit theorem, the sampling distribution of mean $\bar{x}$ would follow the standard normal distribution for a large sample size. Then z-test statistic is given by;

Test-statistic: z = $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

Or z = $\frac{\bar{x}-\mu}{S.E.}$

Here S.E. stands for Standard Error of Mean = $\frac{\sigma}{\sqrt{n}}$

If the population standard deviation **σ** is not known , then a sample standard deviation 's' is used to estimate **σ**. The value of the z-test statistic is given by;

$$z = \frac{\bar{x}-\mu}{s/\sqrt{-n}}$$

The decision on the basis of critical value and calculated value for the two-tailed test can be presented in the following form:

- Reject $H_0$ if $z_{calculated} \leq -z_{\alpha/2}$ or $z_{calculated} \geq z_{\alpha/2}$
  i.e. Null Hypothesis is rejected if calculated value is less than (-) critical value or greater than (+) critical value in case of two tailed test.
- Accept $H_0$ if $-z_{\alpha/2} <$ z Calculated $< z_{\alpha/2}$
  Where $z_{\alpha/2}$ is the table value (also called Critical Value) of z at a chosen level of significance i.e.α.

**Left-Tailed Test:** Large sample (n>30) hypothesis testing about a population mean for a left-tailed test is of the form;

- Null Hypothesis ($H_0$) : $\mu \geq \bar{x}$ and $H_a$: $\mu < \bar{x}$

Test-statistic: z = $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

Decision rule:

- Reject $H_0$ if $z_{calculated} \leq -z_{\alpha}$ (Table value of z at α)

**Right-Tailed Test:** Large sample (n>30) hypothesis testing about a population mean for a left-tailed test is of the form;

- $H_0$ : μ ≤ $\bar{x}$ and $H_1$: μ > $\bar{x}$ ( Right-tailed test)

Test-statistic: z = $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

Decision rule:

Reject $H_0$ if $z_{calculated} \geq z_{\alpha}$ (Table value of z at α)

Accept $H_0$ if $z_{calculated} < z_{\alpha}$

**Illustration 1**: A company producing fluorescent light bulbs claims that the average life of the bulbs is 1570 hours. The mean life time of sample of 200 fluorescent light bulbs was found to be 1600 hours with a standard deviation of 150 hours. Test for the company at 1% level of significance, whether the claim that the average life of the bulbs is 1570 hours is acceptable.

**Solution:** Let us take the null hypothesis that mean life time of bulbs is 1570 hours,

i.e. $H_0$ : $\bar{X}$ = μ and $H_1$: $\bar{X}$ ≠μ (Two- tailed test)

Given: n= 200, $\bar{x}$ = 1600 hours, s = 150 hours, and α = 0.01. Thus using the z-test statistic;

z = $\frac{\bar{x}-\mu}{s/\sqrt{n}}$ = $\frac{1600-1570}{150/\sqrt{200}}$ = $\frac{30}{150/14.14}$ = 10.60

Since the calculated value $z_{calculated}$ = 10.60 which is more than its critical value $z_{\alpha}$ = 2.58, the $H_0$ is rejected. Hence, we conclude that the mean life time of bulbs produced by the company is not 1570 hours. In other words the difference between sample mean and universe Mean is significant, i.e. : $\bar{X}$ ≠μ

**Illustration 2:** A packaging device is set to fill detergent powder packets with a mean weight of 5 kg, with a standard deviation of 0.31 kg. The weight of packets can be assumed to be normally distributed. The weight of packets is known to drift upwards over a period of time due to machine fault, which is not tolerable. A random sample of 100 packets is taken and weighed. The sample has mean weight of 5.03 kg. Can we conclude that the mean weight produced by the machine has increased? Use a 5 per cent level of significance.

- Solution: Let us take the null hypothesis that mean weight produced by the machine has not increased $H_0$ : $\bar{X} \leq \mu$ and $H_a$: $\bar{X} > \mu$ (Right-tailed test)

  Given n= 100, $\bar{x}$ = 5.03 kg, **σ**= 0.31 kg, and α = 5 per cent. Thus using the z-test statistic;

$$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \frac{5.03-5}{0.31/\sqrt{100}} = \frac{0.03}{0.031} = 0.96$$

  Since calculated value $z_{calculated}$ =0.96 which is less than its critical value $z_{\alpha}$ = 1.645 at α = 0.05, the null hypothesis, $H_0$ is accepted. Hence we conclude that mean weight produced by the machine has not increased.

## Test of Hypothesis for the Difference between Two Population Means

If two random samples with $\bar{X}_1, \sigma_1, n_1,$ and $\bar{X}_2,\ \sigma_2,\ n_2$ are drawn from different populations, then the Z statistic takes the following form:

$$Z= \frac{\bar{X}_1-\bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$$

**Illustration 3**: A potential buyer wants to decide which of the two brands of electric bulbs he should buy as he has to buy them in bulk. As a specimen, he buys 200 bulbs of each of the two brands –A and B. On using these bulbs, he finds that brand A has a mean life of 1,400 hours with a standard deviation of 60 hours and brand B has mean life of 1,250 hours with a standard deviation of 50 hours. Do the two brands differ significantly in quality? Use α=0.05.

**Solution:** Let us take the null hypothesis that the two brands do not differ significantly in quality:

$H_0$ : $\mu_1 = \mu_2$

$H_a$ : $\mu_1 \neq \mu_2$ (Two-tailed test)

Where $\mu_1$ = Mean life of Brand A bulbs, and $\mu_2$ = mean life of brand B bulbs.

We now construct the Z statistic.

$$Z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{1400 - 1250}{\sqrt{\frac{(60)^2}{200} + \frac{(50)^2}{200}}} = \frac{150}{\sqrt{\frac{3600}{200} + \frac{2500}{200}}} = \frac{150}{\sqrt{18 + 12.5}}$$

$$= \frac{150}{\sqrt{30.5}} = \frac{150}{5.52} = 27.17$$

As we are given at $\alpha$=0.05, the value of Z for a two- tailed test is 1.96. Since the calculated value of Z (27.17) falls in the rejection region, we reject the null hypothesis and, therefore, conclude that the bulbs of two brands differ significantly in quality.

## Student's t-Distribution (Test of Significance for Small Samples)

If the sample size is less than 30 i.e., $n < 30$, the sample may be regarded as small sample.

The greatest contribution to the theory of small samples was made by Sir William Gossett and R.A. Fisher. Gosset published his discovery in 1905 under the pen name 'students' and it is popularly known as t-test or students' t-distribution or students' distribution.

The following are some important applications of the t-distribution:

**I) Test of Hypothesis about the population mean;**

The formula is $t = \frac{\bar{x} - \mu}{S} \sqrt{n}$

Where $S = \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}}$

**Illustration 1:** For a random sample of size 20 from a normal population, the mean is 12.1 and the standard deviation is 3.2. Is it reasonable to suppose that the population mean is 14.5? Test at 5% significance level.

(Given $t_{0.05}$ at 19 d.f= 1.729)

Solution:

Here in this problem Degree of Freedom (v) is required to compute:

v = n-1,     20-1=19

Let us take the null hypothesis that there is no significant difference between the sample mean and population mean.

$H_0 = \bar{x} - \mu = 0$

$t = \frac{\bar{x}-\mu}{S}\sqrt{n}$

$\bar{X}$= 12.1; μ= 14.5; S=3.2; n= 20

$t = \frac{12.1-14.5}{3.2} \times \sqrt{20}$

$= -0.75 \text{ X } 4.47 = 3.35$

For 19 degree of freedom $t_{0.05}$ =1.729

The calculated value of t is greater than its corresponding table value, hence the null hypothesis is rejected. It can be concluded that there is significant difference between the sample mean and population mean and it is not due to sampling fluctuation.

**ii) Test of significance of the difference between two means (independent samples)**

The formula is:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S} \times \sqrt{\frac{n_{1.}n_2}{n_{1+}n_2}}$$

$\bar{X}_1$= Mean of the first sample

$\bar{X}_2$= Mean of the second sample

$n_1$= Number of observations in the first sample

$n_2$= Number of observations in the second sample

S= Combined Standard deviation

The value of S is calculated by the following formula:

$$S = \sqrt{\frac{(X_1 - \bar{X}_1)^2 + (X_2 - \bar{X}_2)^2}{n_{1} + n_{2-2}}}$$

**Illustration 2**: The heights of six randomly chosen soldiers are in inches: 76, 70, 68, 69, 69 and 69. Those of 6 randomly chosen sailors are 68, 64, 65, 69, 72, 70. Discuss in the light of these data that whether soldiers are, on the average, taller than sailors. Use t-test.

Solution:

- Let us take the null hypothesis that there is no difference in height of soldiers and sailors. Ho: $\bar{X}_{1=\bar{X}_2}$
- Level of Significance: it can be taken 5% i.e. α=0.05

**Applying t-test**;

| Height $X_1$ | $X_1 - \bar{X}_1$ | $(X_1 - \bar{X}_1)^2$ | Height $X_2$ | $X_2 - \bar{X}_2$ | $(X_2 - \bar{X}_2)^2$ |
|---|---|---|---|---|---|
| 76 | +6 | 36 | 68 | 0 | 1 |
| 70 | 0 | 0 | 64 | -4 | 16 |
| 68 | -2 | 4 | 65 | -3 | 9 |
| 69 | -1 | 1 | 69 | +1 | 1 |
| 68 | -2 | 4 | 72 | +4 | 16 |
| 69 | -1 | 1 | 70 | +2 | 4 |
| $\sum X_1$ = 421 | | $\sum(X_1 - \bar{X}_1)^2$ =10 | $\sum X_2$ = 408 | | $\sum(X_2 - \bar{X}_2)^2$=47 |

$\bar{X}_1$= 421/6= 70

$\bar{X}_2$= 408/6= 68

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S} \text{ x } \sqrt{\frac{n_{1.} n_2}{n_{1+} n_2}}$$

$$S = \sqrt{\frac{\sum(X_1 - \bar{X}_1)^2 + \sum(X_2 - \bar{X}_2)^2}{n_1 + n_{2-2}}}$$

$$= \sqrt{\frac{10 + 47}{6+6-2}} = \sqrt{\frac{57}{10}} = 2.38$$

$$= \frac{70-68}{2.38} \text{ x } \sqrt{\frac{6x6}{6+6-2}} = \frac{2}{2.38} \text{ x } \sqrt{\frac{36}{10}} = 0.84 \text{ x } \sqrt{3.6} = 0.84 \text{ x } 1.897 = 1.59$$

Here degree of freedom d.f. = $n_1 + n_{2-2}$

= 6+6-2 = 10

For 10 degree of freedom, $t_{0.05}$= 2.23

Decision: Since the calculated value of t is less than corresponding table value, hence there is no reason to doubt the null hypothesis. Thus null hypothesis is accepted and it is concluded that the soldiers are not, on an average, taller than sailors.