Balancing

It is the process of correcting or eliminating either partially or completely the effect of dynamic force & couples on machine part.

Types of balancing —

(1) Static Balancing — The system is said to be statically balanced if centre of mass of system lies on axis of rotation. For system to be statically balanced, resultant of all dynamic forces acting on system during rotation must be zero.

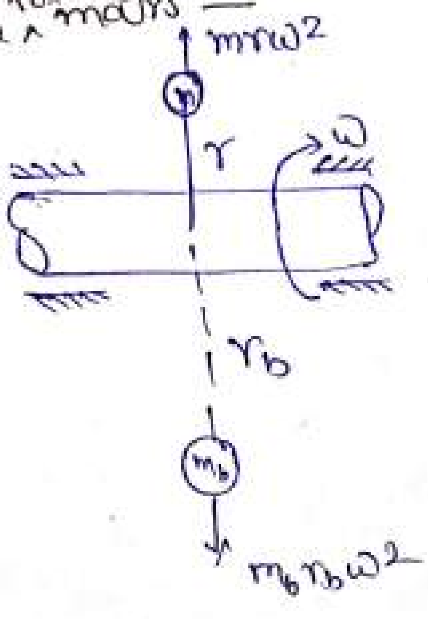
(2) Dynamic balancing — (complete balancing) —
The system is said to be dynamically balanced if it satisfies following two conditions —

(i) Resultant of all dynamic force acting on system during rotation must be zero.

(ii) Resultant couples due to all dynamic forces acting on system during rotation about any plane must be zero.

Balancing of Rotating mass —

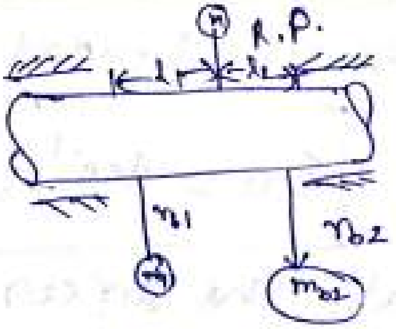
→ Balancing of single rotating mass —



$$m r \omega^2 = m_b r_b \omega^2$$

$$\Rightarrow \boxed{m r = m_b r_b}$$

Force balance —



$$m r \omega^2 = m_{b1} r_{b1} \omega^2 + m_{b2} r_{b2} \omega^2$$

$$\Rightarrow \boxed{m r = m_{b1} r_{b1} + m_{b2} r_{b2}} \quad \text{—}(1)$$

For couple balance —

$$\Sigma C_{R.P.} = 0$$

$$m_{b1} r_{b1} \omega^2 l_1 - m_{b2} r_{b2} \omega^2 l_2 = 0$$

$$\Rightarrow \boxed{m_{b1} r_{b1} l_1 = m_{b2} r_{b2} l_2} \quad \text{—}(2)$$

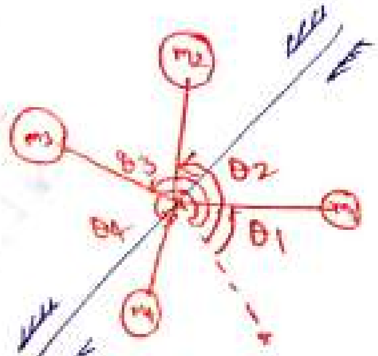
Balancing of several masses rotating in same plane —

F_N = Net unbalanced force

θ_N = dir^n of F_N from ref. axis

F_b = balancing force

θ_b = dir^n of F_b from ref. axis.

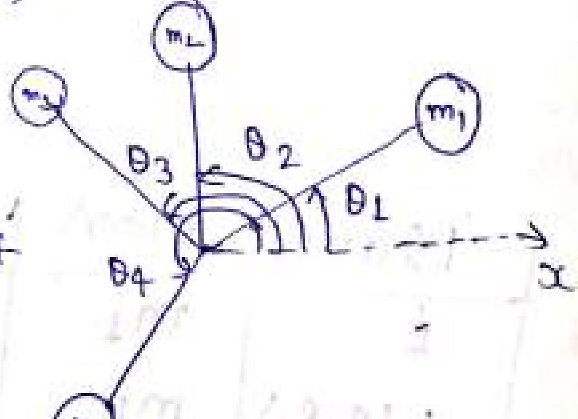


Analytical Method —

Total unbalance force in x-dir^n —

$$F_H = \Sigma F \cos\theta$$

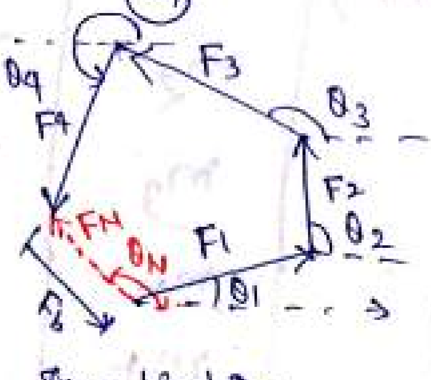
$$= F_1 \cos\theta_1 + F_2 \cos\theta_2 + F_3 \cos\theta_3 + F_4 \cos\theta_4$$



Total unbalanced ~~from~~ force in Y-dir^n —

$$F_V = \Sigma F \sin\theta$$

$$= F_1 \sin\theta_1 + F_2 \sin\theta_2 + F_3 \sin\theta_3 + F_4 \sin\theta_4$$



$\theta_b = 180 + \theta_N$

Total unbalance force

$$F_N = \sqrt{F_H^2 + F_V^2}$$

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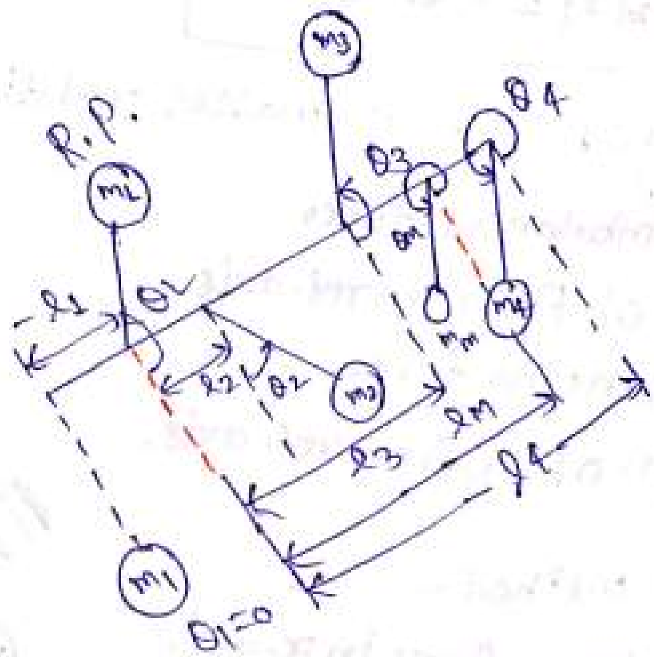
Angle of unbalanced force —

$$\boxed{\theta_n = \tan^{-1}\left(\frac{f_V}{f_H}\right)}$$

Since all the forces are in same plane, therefore, if force is balanced then couple will also balanced - automatically.

Balancing of several rotating mass in different plane —



Plane	mass	radius	C.F. F_C/ω^2	length from R.P. (l)	Couple C/ω^2
1	m_1	r_1	$m_1 r_1$	$-l_1$	$-m_1 r_1 l_1$
L (R.P.)	m_L	r_L	$m_L r_L$	0	0
2	m_2	r_2	$m_2 r_2$	l_2	$m_2 r_2 l_2$
3	m_3	r_3	$m_3 r_3$	l_3	$m_3 r_3 l_3$
M	m_M	r_m	$m_m r_M$	l_M	$m_M r_M l_M$
4	m_4	r_4	$m_4 r_4$	l_4	$m_4 r_4 l_4$

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Let us assume that system is balanced completely after placing two balancing mass in two different planes.

So, Couple Balance —

(ΣC) in horizontal dir^n (x-dir^n) $= 0$

$$-m_1 r_1 l_1 \cos\theta_1 + m_2 r_2 l_2 \cos\theta_2 + m_3 r_3 l_3 \cos\theta_3 + m_m r_m l_m \cos\theta_m + m_4 r_4 l_4 \cos\theta_4 = 0$$

$$\Rightarrow m_m r_m \cos\theta_m = K_1 \quad \text{—} (1)$$

(ΣC) in vertical dir^n (y-dir^n) $= 0$

$$-m_1 r_1 l_1 \sin\theta_1 + m_2 r_2 l_2 \sin\theta_3 + m_3 r_3 l_3 \sin\theta_3 + m_m r_m l_m \sin\theta_m + m_4 r_4 l_4 \sin\theta_4 = 0$$

$$\Rightarrow m_m r_m l_m \sin\theta_m = K_2 \quad \text{—} (2)$$

From eq^n (1) & (2)

$$\boxed{m_M r_M l_M = \sqrt{K_1^2 + K_2^2}}$$

Force balance —

$\Sigma F_x = 0$

$$\Rightarrow m_1 r_1 \cos\theta_1 + m_L r_L \cos\theta_L + m_2 r_2 \cos\theta_2 + m_3 r_3 \cos\theta_3 + m_m r_m \cos\theta_m + m_4 r_4 \cos\theta_4 = 0$$

$$\Rightarrow m_L r_L \cos\theta_L = K_3 \quad \text{—} (3)$$

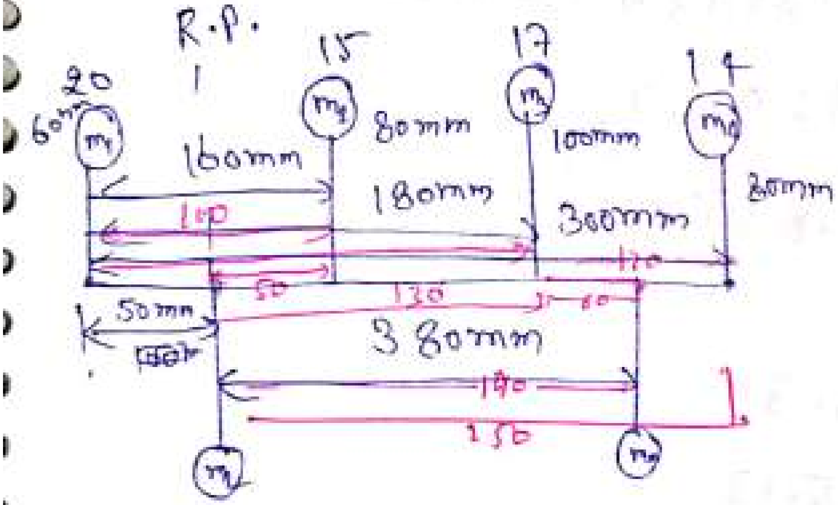
$\Sigma F_y = 0$

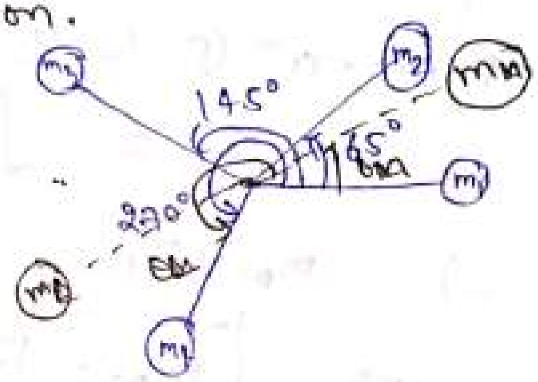
$$\Rightarrow m_1 r_1 \sin\theta_1 + m_L r_L \sin\theta_L + m_2 r_2 \sin\theta_2 + m_3 r_3 \sin\theta_3 + m_m r_m \sin\theta_m + m_4 r_4 \sin\theta_4 = 0$$

$$\Rightarrow m_L r_L \sin\theta_L = K_4 \quad \text{—} (4)$$

from (3) & (4) $\boxed{m_L r_L = \sqrt{K_3^2 + K_4^2}}$

Que) A rotating shaft carries four mass m_1, m_2, m_3 & m_4 of magnitude of 20, 15, 17 & 14 kg revolving at radius 60 mm, 80 mm, 100 mm & 80 mm respectively. The masses m_2, m_3 & m_4 revolves in planes 100 mm, 180 mm, & 300 mm respectively from plane of mass m_1 & are angularly located at 65°, 145°, & 270° respectively measured in ccw dirn from mass m_1. The shaft is to be dynamically balance by two masses, both located at 70 mm radius & revolving in planes midway b/w those of masses m_1 & m_2 & midway b/w those of masses m_3 & m_4. Determine magnitude of balancing mass & their respective angular position.





Plane	mass	Radius	F_c/ω^2 (mr)	length l	C/ω^2 (mrl)
1.	20	60	1200	−50	−60,000
L(R.P.)	m_L	r_L	$m_L r_L$	0	0
2	15	80	1200	50	60,000
3	17	100	1700	130	~~221000~~ 221000
M	m_M	r_M	$m_M r_M$	~~380~~ 190	190 $m_M r_M$
4	14	80	1120	~~580~~ 250	~~453600~~ 280000

(ΣC) Horizontal –

$-60000 \cos 0 + 60000 \cos 65°$

$+ 221000 \cos 145 + 190 m_m r_m \cos\theta_m + 280000 \cos 270° = 0$

$\Rightarrow \quad m_m r_m \cos\theta_m = +1135.13$ — ①

(ΣC) vertical.

$-60000 \sin 0 + 60000 \sin 65$

$+ 221000 \sin 145 + 190 m_m r_m \sin\theta_m + 280000 \sin 270° = 0$

$\Rightarrow \quad m_m r_m \sin\theta_m = 520.32$ — ②.

From eqn ① & ② –

$m_m r_m = \sqrt{1135.13^2 + 520.32^2}$

$\Rightarrow \quad m_m \times 70 = 1248.7$

$\Rightarrow \quad \boxed{m_m = 17.83 \text{ Kg.}}$ Ans

$\tan\theta_M = \frac{520.32}{1135.13}$

$\Rightarrow \quad \boxed{\theta_m = 24.63°}$ Ans

(ΣF) Horizontal $= 0$

$\Rightarrow \quad 1200 \cos 0 + m_L r_L \cos\theta_L + 1200 \cos 65$

$+ 1200 \cos 145 + 17.83 \times 70 \cos 24.63$

$+ 1120 \cos 270° = 0$

$\Rightarrow \quad m_L r_L \cos\theta_L = -1449.13$ — ③

$\Sigma F_y = 0$

$\Rightarrow 1200 \sin 0 + m_L r_L \sin\theta_L + 1200 \sin 65^\circ + 1700 \sin 145 + 17.83 \times 70 \sin 29.63 + 11 \sin 270 = 0$

$\Rightarrow m_L r_L \sin\theta_L = -1414.46$ — (4)

From (3) & (4) -

$m_L r_L = \sqrt{(-1449.13)^2 + (1414.46)^2}$

$m_L r_L = 2025.01$

$\Rightarrow m_L = \frac{2025.01}{70}$ ($r_L = 70\,mm$)

$\Rightarrow \boxed{m_L = 28.93\,kg.}$ Ans

$\tan\theta_L = \frac{-1414.46}{-1449.13}$

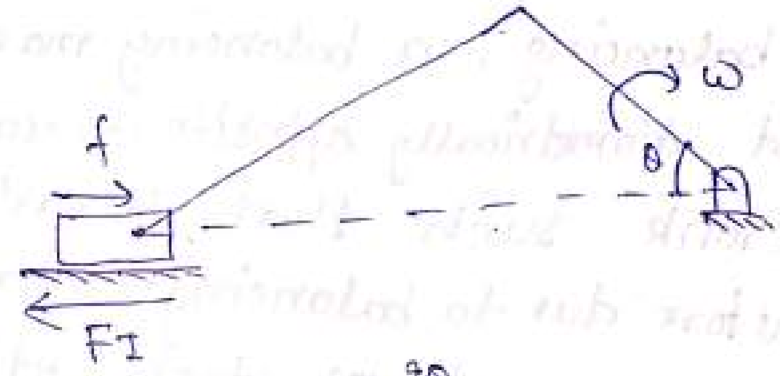
$\Rightarrow \theta_L = 44.3 + 180$

$\Rightarrow \boxed{\theta_L = 224.3^\circ}$ Ans

Balancing of Reciprocating mass

In application like I.C. engine, reciprocating compressors and reciprocating pumps, the reciprocating parts are subjected to continuous acceleration & retardation. Due to this acceleration & retardation, inertia force acts on reciprocating parts which is in a dirn opposite to acceleration.

The unbalance force due to reciprocating mass varies in magnitude & dirn whereas constant in dirn. Therefore it is not possible to balance completely the reciprocating mass.



$$F_I = mf = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$\Rightarrow F_I = mr\omega^2 \cos\theta + mr\omega^2 \frac{\cos 2\theta}{n}$$

$$\Rightarrow \boxed{F_I = F_P + F_S}$$

$F_P \rightarrow$ Primary unbalance force

$F_S \rightarrow$ Secondary unbalance force.

The primary unbalanced force is max. when $\theta = 0°$ or $180°$ i.e. twice in one rotation of crank. The secondary

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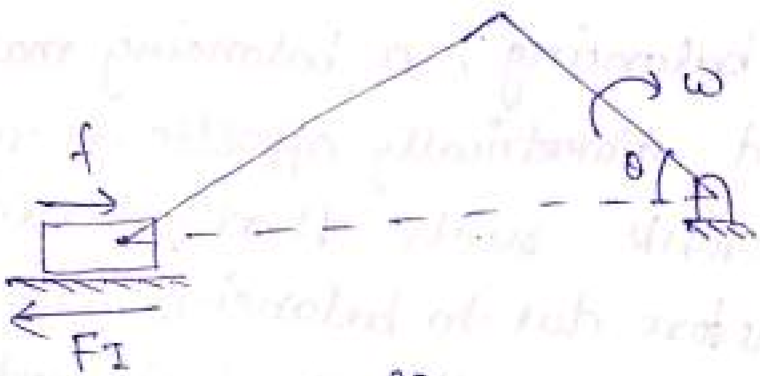


$$F_I = mf = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$\Rightarrow F_I = mr\omega^2\cos\theta + mr\omega^2\frac{\cos 2\theta}{n}$$

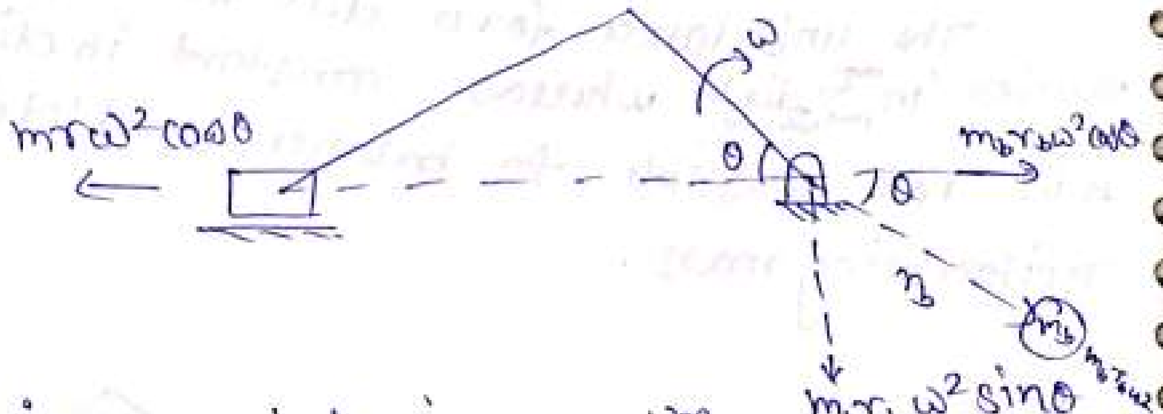
$$\Rightarrow \boxed{F_I = F_P + F_S}$$

$F_P \rightarrow$ Primary unbalance force

$F_S \rightarrow$ Secondry unbalance force.

The primary unbalanced force is max. when $\theta = 0°$ or $180°$ i.e. twice in one rotation of crank. The secondry unbalance force will be max. When

$\theta = 0°, 90°, 180° \; \& \; 270°$ i.e four times in one rotation of crank. However, magnitude of secondary unbalance force is $\frac{1}{n}$ times that of primary unbalance force. Therefore, in case of low or moderate speed engines, secondary force is neglected.



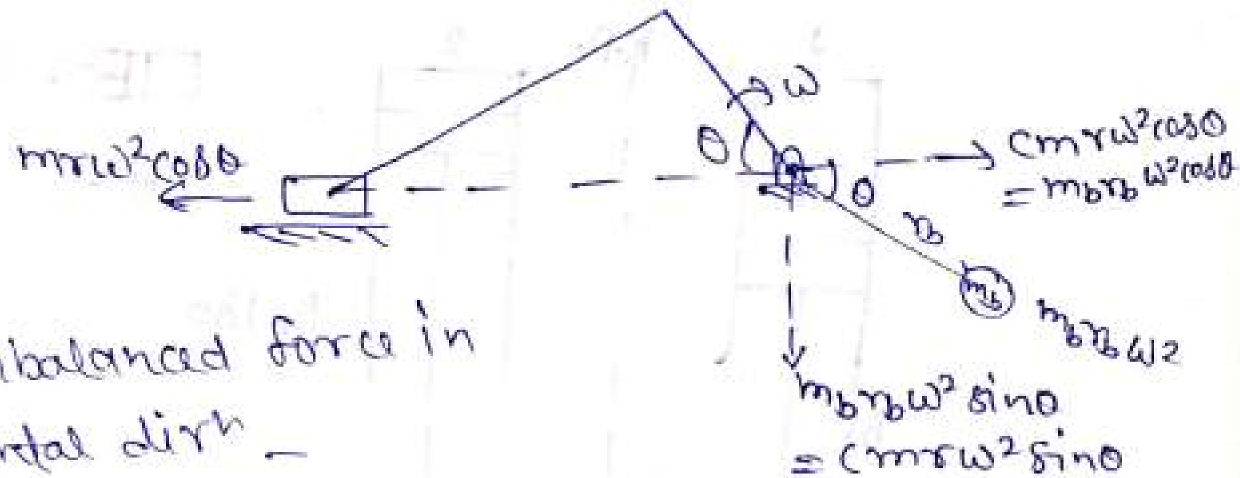
For balancing, a balancing mass m_b placed diametrically opposite at radius r_b to crank such that $mr\omega^2 = m_b r_b \omega^2$.

Therefore due to balancing mass m_b, primary unbalanced force along line of stroke gets completely balance but there is an another vertical component of mass m_b resulted which have same max. magnitude as that of primary unbalance force. Therefore effect of balancing mass is that earlier primary unbalance force along line of stroke gets changed into vertical unbalanced force of same magnitude. Hence this method of balancing is not prefered.

For single cylinder reciprocating engine, only partial balancing is done such that -

$$c m r \omega^2 = m_b r_b \omega^2.$$

where c is a fraction ($c < 1$)



Total unbalanced force in horizontal dirn -

$$F_H = mr\omega^2\cos\theta - cmr\omega^2\cos\theta$$

$$\boxed{F_H = (1-c)\, mr\omega^2\cos\theta}$$

Unbalance vertical force -

$$\boxed{F_V = cmr\omega^2\sin\theta}$$

Net unbalance force =

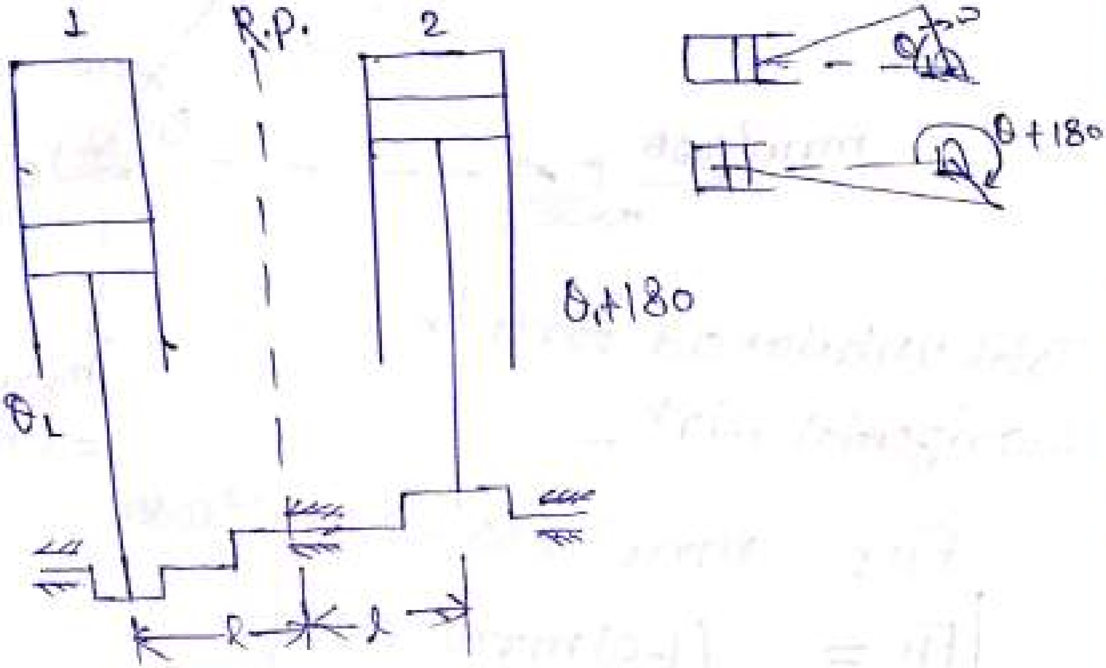
$$\boxed{F = \sqrt{F_H^2 + F_V^2}}$$

Magnitude of total unbalance force will be min. at $\boxed{c = \frac{1}{2}}$

Balancing of Reciprocating mass in multi cylinder engine —

(1) Two - cylinder In-line engine —

If all the cylinders are on same side of crank then it is said to be in-line engine.



Total Primary unbalanced

$$\text{force} = mr\omega^2 \cos\theta_1 + mr\omega^2 \cos(\theta_1+180)$$
$$= 0$$

$$\text{Total Primary couple} = mr\omega^2 \cos\theta_1 \times l - mr\omega^2 \, l \cos(180+\theta_1)$$
$$= 2mr\omega^2 l \cos\theta_1$$

$$\text{Total Secondry force} = \frac{mr\omega^2}{n}\cos 2\theta_1 + \frac{mr\omega^2}{n}\cos(360+2\theta_1)$$
$$= \frac{2mr\omega^2}{n}\cos 2\theta_1$$

$$\text{Total secondry couple} = \frac{mr\omega^2}{n}\cos 2\theta_1 \times l - \frac{mr\omega^2}{n}\cos(360+2\theta_1) \times l$$

$$= 0$$

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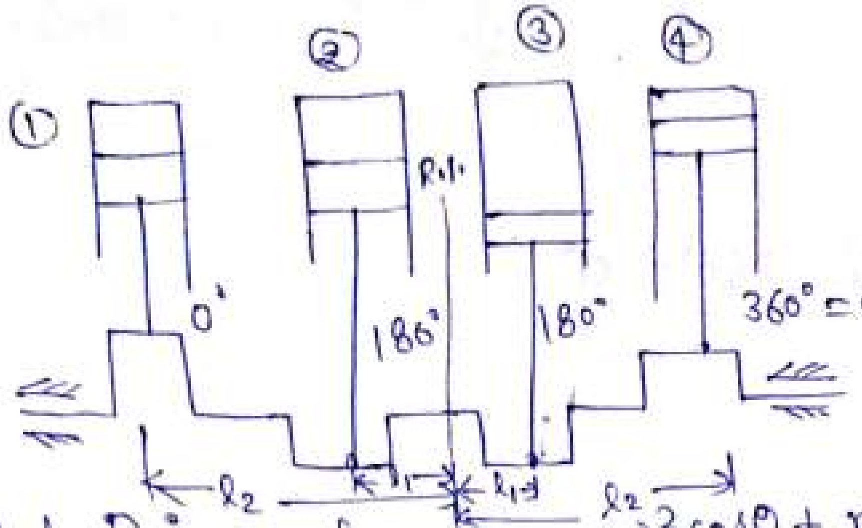
(2) Four cylinder engine — [Firing order = 1-2-4-3 or 1-3-4-2 with crank angle 180°]



Total Primary force $= mr\omega^2\cos 0 + mr\omega^2\cos 180 + mr\omega^2\cos 180 + mr\omega^2\cos 360$ (or $\cos 0$)

$= 0$

Total Primary Couple

$= mr\omega^2\cos 0° \times l_2 + mr\omega^2\cos 180 \times l_1 - mr\omega^2\cos 180 \times l_1 - mr\omega^2\cos 360°$ (or $\cos 0$)

$= 0$

Total Secondry Force $= \frac{mr\omega^2}{n}\cos 0 + \frac{mr\omega^2}{n}\cos 360 + \frac{mr\omega^2}{n}\cos 360 + \frac{mr\omega^2}{n}\cos 0°$

$= 4\frac{mr\omega^2}{n}$

Total Secondry Couple

$= \frac{mr\omega^2}{n}\cos 0 \times l_2 + \frac{mr\omega^2}{n}\cos 360 \times l_1 - \frac{mr\omega^2}{n}\cos 360 \times l_1 - \frac{mr\omega^2}{n}\cos 0 \times l_2$

$= 0$

(3) Six- Cylinder Engine — (1-4-2-6-3-5 with crank angle diff. at 120°)

$\theta_1 = 0°$

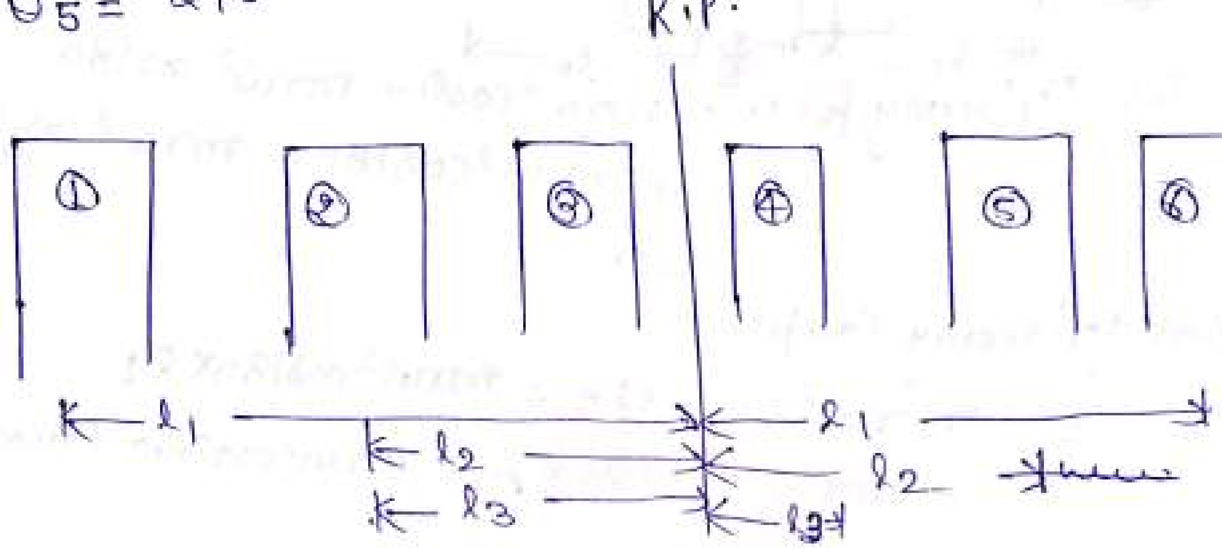
$\theta_4 = 120°$

$\theta_2 = 240°$

$\theta_6 = 240° + 120° = 360° = 0°$

$\theta_3 = 120°$

$\theta_5 = 240°$



Total Primary force $= mr\omega^2 (\cos 0° + \cos 240° + \cos 120° + \cos 120° + \cos 240° + \cos 0)$

$= 0$

Total secondary force $= \frac{mr\omega^2}{n} [\cos 0 + \cos 480 + \cos 240 + \cos 240 + \cos 480 + \cos 0]$

$= 0$

Total Primary couple $= mr\omega^2 [\cos 0° \times l_1 + \cos 240 \times l_2 + l_3 \cos 120° - (l_3 \cos 120° + l_2 \cos 240° + l_1 \cos 0°)]$

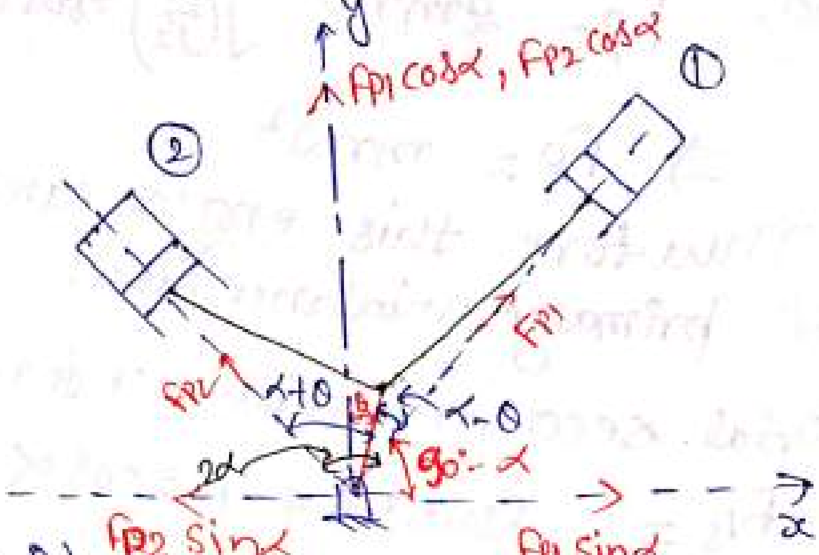
$= 0$

Total Secondary couple $= \frac{mr\omega^2}{n} [l_1 \cos 0° + l_2 \cos 480 + l_3 \cos 240° - (l_3 \cos 240 + l_2 \cos 480 + l_1 \cos 0°)]$

$= 0$

Therefore, a six-cylinder engine can be completely balanced.

Balancing of V- Engine —



$$F_{P1} = mr\omega^2 \cos(\alpha-\theta)$$

$$F_{P2} = mr\omega^2 \cos(\alpha+\theta)$$

$$F_{S1} = \frac{mr\omega^2}{n} \cos 2(\alpha-\theta)$$

$$F_{S2} = \frac{mr\omega^2}{n} \cos 2(\alpha+\theta)$$

Total Primary unbalance force along Y-axis

$$F_{VP} = F_{P1}\cos\alpha + F_{P2}\cos\alpha$$

$$\boxed{F_{VP} = 2mr\omega^2 \cos^2\alpha \cos\theta}$$

Total Primary unbalance force along X-axis

$$F_{HP} = F_{P1}\sin\alpha - F_{P2}\sin\alpha$$

$$F_{HP} = \cancel{2mr\omega^2 \cos^2\alpha \cos\theta}\ 2mr\omega^2 \sin^2\alpha \sin\theta$$

Total Primary unbalance force

$$F_P = \sqrt{F_{VP}^2 + F_{HP}^2} = 2mr\omega^2 \sqrt{\left[\sin^4\alpha \sin^2\theta + \cos^4\alpha \cos^2\theta\right]}$$

$$\Rightarrow \cancel{F_P = 2\sqrt{2}\, mr\omega^2}$$

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If V-angle $2\alpha = 90^\circ$ is taken then

$$\boxed{\alpha = 45^\circ}$$

So, $F_P = 2mr\omega^2 \sqrt{\left(\frac{1}{\sqrt{2}}\right)^4 \cos^2\theta + \left(\frac{1}{\sqrt{2}}\right)^4 \sin^2\theta}$

$\Rightarrow F_P = mr\omega^2$

Therefore this engine can be balanced completely for primary unbalance force.

Total secondry unbalance force along Y-axis –

$F_{VS} = F_{S1}\cos\alpha + F_{S2}\cos\alpha$

$= 2\frac{mr\omega^2}{n}\cos\alpha \cdot \cos 2\alpha \cdot \cos 2\theta$

Total secondry unbalance force along X-axis –

$F_{HS} = F_{S1}\sin\alpha - F_{S2}\sin\alpha$

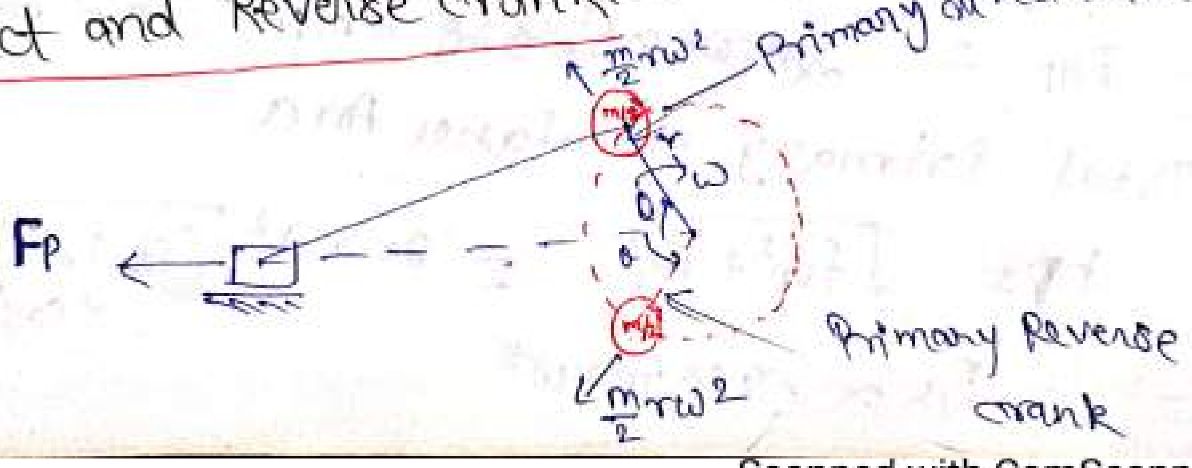
$F_{HS} = 2\frac{mr\omega^2}{n}\sin\alpha \sin 2\alpha \sin 2\theta$

Total secondry unbalance force –

$$\boxed{F_S = \sqrt{F_{VS}^2 + F_{HS}^2}}$$

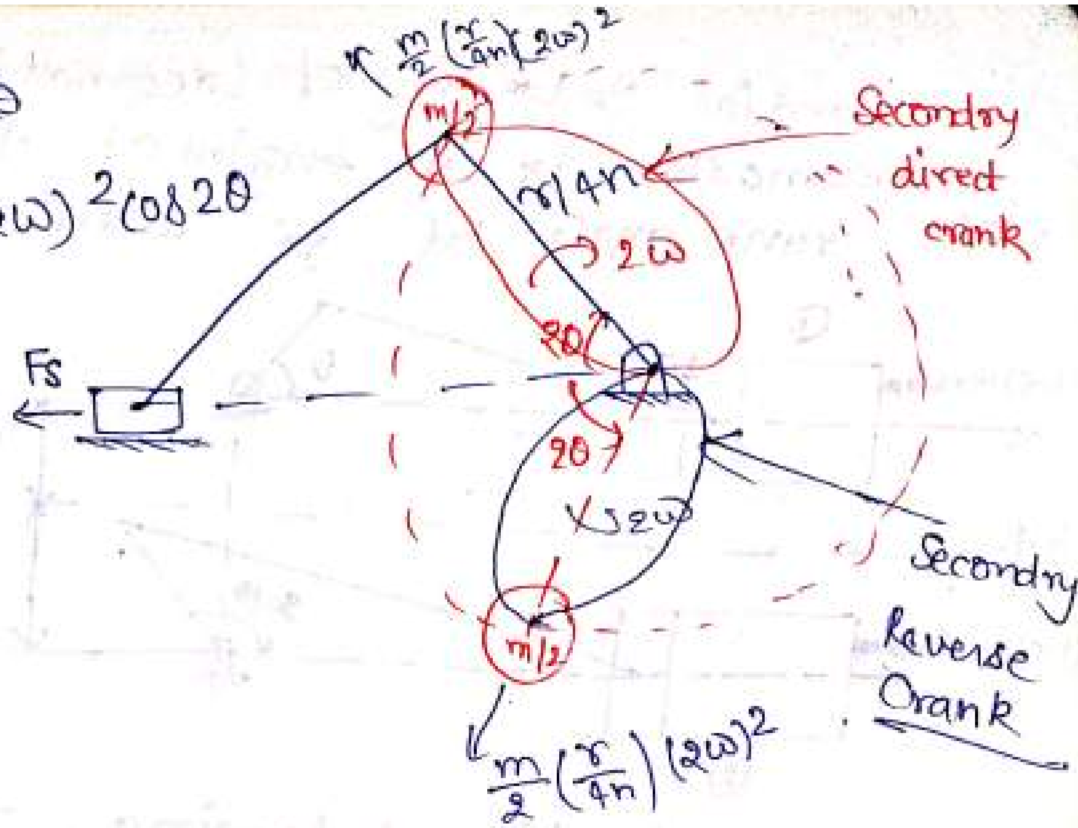
~~For~~ ~~Vero~~

Direct and Reverse Crank –



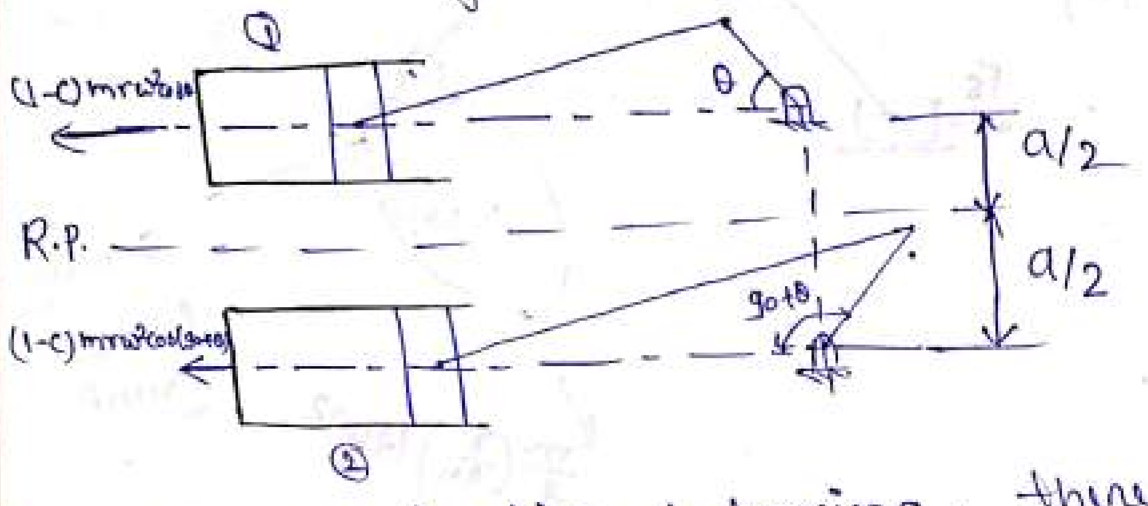
$$F_S = \frac{mr\omega^2}{n}\cos 2\theta$$

$$\Rightarrow F_S = m\left(\frac{r}{4n}\right)(2\omega)^2\cos 2\theta$$



Partial Balancing of Locomotive —

A locomotive is a engine of two-cylinder with crank angle at 90° with each other.



Due to partial balancing, there is an unbalanced primary force along line of stroke & also an unbalanced primary force ⊥r to line of stroke.

The effect of an unbalanced primary force along line of stroke is to produce —

(i) variation in tractive force

(ii) Swaying couple

The max. magnitude of unbalanced force along ⊥r to line of stroke is k/a hammer blow.

(i) variation in Tractive force —

It is the summation of total unbalanced force along line of stroke.

$$F_T = (1-c)\,mr\omega^2\left[\cos\theta + \cos(90+\theta)\right]$$

$$\Rightarrow \boxed{F_T = (1-c)\,mr\omega^2(\cos\theta - \sin\theta)}$$

For. max. F_T —

$\frac{d}{d\theta}(F_T) = 0$ gives $\boxed{\theta = 135°, 315°}$

So, $\boxed{F_{Tmax} = \pm\sqrt{2}(1-c)\, mr\omega^2}$

(ii) Swaying couple —

It is the couple about reference plane due to unbalanced force along line of stroke.

$$\text{Swaying couple} = (1-c)\, mr\omega^2 \cos\theta \times \frac{a}{2} - (1-c)\, mr\omega^2 \cos(90+\theta) \times \frac{a}{2}$$

$$\boxed{\text{Swaying couple} = (1-c)\, mr\omega^2 \times \frac{a}{2}\left[\cos\theta + \sin\theta\right]}$$

For max. S.C.

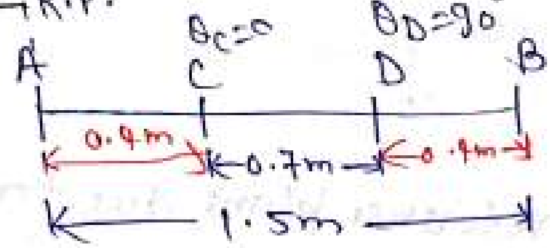
$$\frac{d(S.C)}{d\theta} = 0 \Rightarrow \boxed{\theta = 45^\circ \;\&\; 225^\circ}$$

$$\boxed{(\text{Swaying couple})_{max} = \pm\frac{a}{\sqrt{2}}(1-c)\, mr\omega^2}$$

... locomotive has inside

Que A two cylinder uncoupled locomotive has inside cylinders 70cm apart; the cranks are at 90° & are 0.3m long. mass of revolving part per cylinder is 160kg & mass of reciprocating part per cylinder is 180kg. The whole of revolving & $\frac{2}{3}$ of reciprocating parts are to be balanced & balance mass are to placed in plane of rotation of driving wheel at a radius of 80cm. Driving wheels are of 2m dia. & 1.5 m appart. Find balancing mass & its angular position.

Soln:- R.P.



$r = 0.3m$

$m_{rev.} = 160 kg.$

$m_{rec.} = 180 kg$

$r_b = 80cm.$

Total mass to be balanced

$= 160 + \frac{2}{3} 180 = 280 kg.$

Let m_A & m_B are the balancing mass, placed at θ_A & θ_B from cylinder C.

Couple balancing about plane A —

$\Sigma C_H = 0$

$280 r \cos\theta_C \times 0.4 + 280 r \cos\theta_D \times 1.1 + m_B \times r_b \times \cos\theta_B \times 1.5 = 0$

$\Rightarrow 280 \times 0.3 \cos 0^\circ + 0.4 + 28 \times 0.3 \cos 90^\circ \times 1.1 + m_B \times 0.8 \cos\theta_B \times 1.5 = 0$

$\Rightarrow m_B \cos\theta_B = -28$ — ①

$\Sigma C_V = 0$

$\Rightarrow 280 \times 0.3 \sin 0 \times 0.4 + 280 \times 0.3 \sin 90 \times 1.1 + m_B \times 0.8 \times \sin\theta_B \times 1.5 = 0$

$\Rightarrow m_B \sin\theta_B = -77$ — ②

From eqn ① & ②

$\boxed{m_B = 81.93 kg. \quad \theta_B = 250^\circ}$

Ans

Force balancing —

$\Sigma F_H = 0$

$m_A \times 0.8 \times \cos\theta_A + 280 \times 0.3 \cos 0^\circ + 280 \times 0.3 \cos 90^\circ$

$+ 81.9 \times 0.8 \cos 250 = 0$

$\Rightarrow \quad m_A \cos\theta_A = -76.98 \quad — (3)$

$\Sigma F_V = 0$

$\Rightarrow \quad m_A \times 0.8 \sin\theta_A + 280 \times 0.3 \sin 0 + 280 \times 0.3 \sin 90^\circ$

$+ 81.9 \times 0.8 \sin 250 = 0$

$\Rightarrow \quad m_A \sin\theta_A = -28.04 \quad — (4)$

From (3) + (4) —

$\boxed{m_A = 81.93 \text{ kg.}}$

$\boxed{\theta_A = 200^\circ}$ Ans