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Sem – 4th
Sub – Hydraulics & Hydraulic Machines [CE-401]

Unit - III

Gradually Varied Flow →

→ A steady non-uniform flow in a pristmatic channel with gradual changes in its water surface elevation is termed as Gradually varied Flow (GVF).

→ In a GVF, the velocity varies along the channel and consequently the bed slope (S_o), water surface slope (S_w) and Energy slope (S_f) will all differ from each other.

Assumptions →

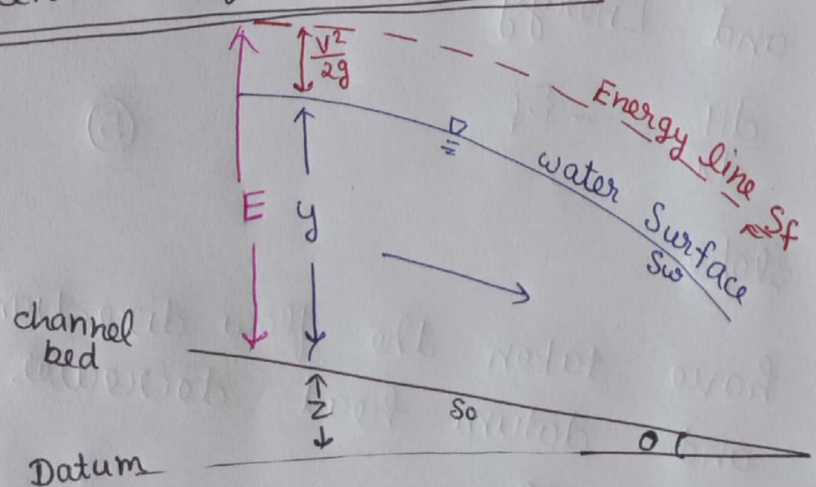
1) The pressure distribution at any section is assumed to be hydrostatic.

2) Energy slope is given by

$$V = \frac{1}{n} \cdot R^{2/3} \cdot S_f^{1/2}$$

$$\boxed{S_f = \frac{n^2 V^2}{R^{4/3}}} \quad \text{--- (1)}$$

Differential equation of GVF →



where, y = water depth
V = velocity of water
E = Specific Energy
Z = Datum head

Schematic sketch of GVF

In GVF

$V \neq$ constant $[S_f \neq S_w \neq S_o]$

$y \neq$ constant $[S_w \neq S_o]$

$Z \neq$ constant ξ

① First form of GVF equation or dynamic eq of GVF →

→ In a channel of small slope (θ can be neglected) and $\alpha = 1.0$

→ In a GVF, Total energy H is given by

$$H = Z + E = Z + y + \frac{V^2}{2g} \quad \text{---} ②$$

because the flow properties varies in horizontal direction (in x direction). So differentiating eq ② w.r.t. x

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dE}{dx} \quad \text{---} ③$$

$$\frac{dH}{dx} = \frac{dZ}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right) \quad \text{---} ④$$

→ $\frac{dH}{dx}$ = Energy slope

because we have taken the flow direction from left to right and Energy decreases in downstream direction so $\frac{dH}{dx} = -S_f$ ——— ⑤

→ $\frac{dZ}{dx}$ = Bottom slope

Because we have taken the flow direction from left to right and datum head decreases in down stream direction so

$$\frac{dZ}{dx} = -S_o \quad \text{---} ⑥$$

→ $\frac{dy}{dx}$ = water surface slope

→ $\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dx}\left(\frac{Q^2}{2gA^2}\right)$ ∵ $Q = V \cdot A$

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = \frac{d}{dA}\left(\frac{Q^2}{2gA^2}\right)\frac{dA}{dx}$$

$$= -\frac{2Q^2}{2gA^3}\left(\frac{dA}{dy}\right)\left(\frac{dy}{dx}\right)$$

$$\frac{d}{dx}\left(\frac{V^2}{2g}\right) = -\frac{Q^2 T}{\cancel{2}gA^3}\frac{dy}{dx} \quad \text{---} (7)$$

∵ $\frac{dA}{dy} = T$

where T = Top width of channel

Eq (4) would be

$$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dy}{dx} + \frac{d}{dx}\left(\frac{V^2}{2g}\right)$$

$$-S_f = -S_0 + \frac{dy}{dx} - \left(\frac{Q^2T}{gA^3}\right)\frac{dy}{dx}$$

$$\frac{dy}{dx}\left[1 - \frac{Q^2T}{gA^3}\right] = (S_0 - S_f)$$

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{Q^2T}{gA^3}}} \quad \text{---} (8)$$

if $\alpha \neq 1$

$$\boxed{\frac{dy}{dx} = \frac{S_0 - S_f}{1 - \frac{\alpha Q^2 T}{gA^3}}}$$

(2) Second form of GVF equation →

if, K = conveyance at any depth y.

K_o = conveyance corresponding to normal depth y_o

$$Q = K\sqrt{S_f}$$

$$K = \frac{Q}{\sqrt{S_f}} \quad \text{------} \quad (9)$$

$$Q = K_o\sqrt{S_o}$$

$$K_o = \frac{Q}{\sqrt{S_o}} \quad \text{------} \quad (9a)$$

$$\frac{eq\,(9)}{eq\,(9a)} \Rightarrow \frac{K}{K_o} = \frac{Q}{\sqrt{S_f}} \div \frac{Q}{\sqrt{S_o}} = \frac{Q \times \sqrt{S_o}}{\sqrt{S_f} \times Q} = \sqrt{\frac{S_o}{S_f}}$$

$$\left(\frac{K_o}{K}\right)^2 = \frac{S_f}{S_o} \quad \text{------} \quad (10)$$

if, Z = Section factor at depth y

$$Z^2 = \frac{A^3}{T} \quad \text{------} \quad (10a)$$

Z_c = Section factor at the critical depth y_c

$$Z_c^2 = \frac{A_c^3}{T_c} = \frac{Q^2}{g} \quad \text{------} \quad (10b) \qquad \because \frac{Q^2}{g} = \frac{A_c^3}{T_c}$$

$$\frac{eq\,(10b)}{eq\,(10a)} \Rightarrow \frac{Z_c^2}{Z^2} = \frac{Q^2}{g} \times \frac{T}{A^3}$$

$$\frac{Q^2 T}{g A^3} = \frac{Z_c^2}{Z^2} \quad \text{------} \quad (11)$$

eq 8 can be written as →

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2 T}{g A^3}}$$

$$= \frac{S_o\left[1 - \frac{S_f}{S_o}\right]}{1 - \frac{Q^2 T}{g A^3}}$$

$$\boxed{\frac{dy}{dx} = \frac{S_o\left[1 - \left(\frac{K_o}{K}\right)^2\right]}{\left[1 - \left(\frac{Z_c}{Z}\right)^2\right]}} \quad \text{--- (12)}$$

from eq (10) & (11)

(3) Third form of GVF equation →

if, Q_n = Normal discharge at a depth y
Q_c = Critical discharge at the same depth y

$$Q_n = K\sqrt{S_o} \quad \text{--- (13)}$$

$$Q_c = Z\sqrt{g} \quad \text{--- (14)}$$

$$Q = Z_c\sqrt{g} \quad \text{--- (14a)}$$

$$Q = K\sqrt{S_f} \quad \text{--- (14b)}$$

$$\frac{\text{eq (14b)}}{\text{eq (13)}} \Rightarrow \frac{Q}{Q_n} = \frac{K\sqrt{S_f}}{K\sqrt{S_o}}$$

$$\frac{S_f}{S_o} = \left(\frac{Q}{Q_n}\right)^2 \quad \text{--- (14c)}$$

$$\frac{\text{eq (14a)}}{\text{eq (14)}} \quad \frac{Q}{Q_c} = \frac{Z_c\sqrt{g}}{Z\sqrt{g}} = \frac{Z_c}{Z}$$

$$\frac{Z_c}{Z} = \frac{Q}{Q_c} \quad \text{--- (14d)}$$

eq (8) can be written as

$$\frac{dy}{dx} = \frac{S_o - S_f}{1 - \frac{Q^2 T}{g A^3}} = \frac{S_o\left[1 - \frac{S_f}{S_o}\right]}{1 - \frac{Q^2 T}{g A^3}}$$ from eq (14C)

$$\frac{dy}{dx} = S_o \frac{\left[1 - \left(\frac{Q}{Q_n}\right)^2\right]}{\left[1 - \left(\frac{Z_c}{Z}\right)^2\right]}$$ from eq (11) from eq (14C)

$$\boxed{\frac{dy}{dx} = S_o \frac{\left[1 - \left(\frac{Q}{Q_n}\right)^2\right]}{\left[1 - \left(\frac{Q}{Q_c}\right)^2\right]}} \quad \text{—— (15)}$$

(4) Fourth form of GVF equation →

eq (3) ⇒ $$\frac{dH}{dx} = \frac{dz}{dx} + \frac{dE}{dx}$$ ∵ eq (5), (6)

$$-S_f = -S_o + \frac{dE}{dx}$$

$$\boxed{\frac{dE}{dx} = S_o - S_f} \quad \text{—— (16)}$$

Limitation of GVF equation →

(1) Valid only for steady state flow.
(2) Based on one dimensional flow consideration [can only calculate average cross sectional water velocity].