AK Model

Introduction

The AK model of economic growth is an endogenous growth model used in the theory of economic growth, a subfield of modern macroeconomics. In the 1980s it became progressively clearer that the standard neoclassical exogenous growth models were theoretically unsatisfactory as tools to explore long run growth, as these models predicted economies without technological change and thus they would eventually converge to a steady state, with zero per capita growth.

The neo-classical approaches to economic growth were largely considered to be unsatisfactory due to several inherent flaws. These models view improvements in total factor productivity (technological progress) to be the ultimate source of growth in output per worker, but they do not provide an explanation as to where these improvements come from. In the language of economists, long-run growth is determined by something that is exogenous in the model. Diminishing returns to the accumulation of capital, which plays a crucial role in limiting growth in the neoclassical model, is an inevitable feature of an economy in which the other determinants of aggregate output, namely technology and the employment of labour, are both given. However, there is a class of model in which one of these other determinants is assumed to grow automatically in proportion to capital, and in which the growth of this other determinant counteracts the effects of diminishing returns, thus allowing output to grow in proportion to capital.

These models are generally referred to as AK models, because they result in a production function of the form Y = AK, with 'A' constant. The AK model is actually considered the first version of endogenous growth theory. However, the earlier version of this model go back to Harrod (1939) and Domar (1946) who assumed an aggregate production function with fixed coefficients. Frankel (1962) developed the first AK model with substitutable factors and knowledge externalities, with the purpose of reconciling the positive long-run growth result of Harrod- Domar with the factor substitutability and market clearing features of the neoclassical model. The Frankel model showed a constant savings rate, whereas Romer (1986) developed an AK model with intertemporal consumer maximization. The idea that productivity could increase as the result of learning-by-doing externalities, was put forth by Arrow (1962). Then Lucas (1988) developed an AK model where the creation and transmission of knowledge occurs through human capital accumulation. Similarly, we can cite a number of other models which have followed the AK framework. Hence, it is important here to examine this approach and its contribution to economic theory.

The Cross-Country Difference in Growth

There are large differences in per capita income across countries. Of total world income, 42 per cent goes to those who make up the richest 10 per cent of the world's population, while just 1 per cent goes to those who make up the poorest 10 per cent (World Bank, World Development Indicators, 2014). This points towards not only unequal distribution of world income across different countries but also differences in their growth rates. The key sources of these differences can be numerous depending upon the national policies and institutions. Hence, it is very important to understand how some countries can be so rich while some others are so poor as the income differences have major welfare consequences. The differences in growth rate across economies have actually widened the income inequalities. Acemoglu (2007) has indicated that even in the historically brief post- war era, the world has witnessed tremendous differences in growth rates across countries and these have ranged from negative growth rates to average rates as high as 10 per cent a year. During this period some of the countries have grown at a faster pace, some at a slower rate and some stagnated after growing for a short period. It is being believed that much of these differences in economic growth cannot be wholly attributed to the post-war era alone as during this period, the "world income distribution" has been more or less stable, with a slight tendency towards becoming more unequal.

Further, the Maddison data has suggested that much of the divergence took place during the 19th century and early 20th century. It is important to observe that the process of rapid economic growth started in the 19th, or perhaps in the late 18th century and then takes off in Western Europe, while many other parts of the world do not experience the same sustained economic growth. The high levels of income today in some parts of the world are owed to this process of sustained economic growth, and this process of differences in economic growth has also caused the divergence among nations. This divergence took place at the same time as a number of countries in the world started the process of modern and sustained economic growth. Therefore understanding modern economic growth is not only interesting and important in its own right, but it also holds the key to understanding the causes of cross-country differences in income per capita today. The endogenous growth theories largely owe these differences or divergences in economic growth to the institutions, policies, technologies along with the levels of investments and other transitional dynamics. These theories also pointed out the importance of investment in human capita along with that of physical capital to explain these divergences. These theories also point out that the technology differences across countries include both genuine differences in the techniques and in the quality of machines used in production, but also differences in productive efficiency resulting from differences in the organization of production, from differences in the way that markets are organized and from potential market failures and how the human factors handle these technologies with effects on productive efficiency.

Hence, a detailed study of "technology", physical capital and human capital as correlates of economic growth is necessary to understand both the world-wide process of economic growth and cross-country differences. It is important to examine the sources of income differences among countries that have (free) access to the same set of technologies, but do not generate sustained long-run growth. A full analysis of both cross-country income differences and the process of world economic growth requires models in which technology choices and technological progress are endogenized. Hence, we can start with simple AK model.

Simple AK Model

As we have already discussed that the first version of endogenous growth theory was AK theory, which did not make an explicit distinction between capital accumulation and technological progress. In effect it lumped together the physical and human capital whose accumulation is studied by neoclassical theory with the intellectual capital that is accumulated when innovations occur. An early version of AK theory was produced by Frankel (1962), who argued that the aggregate production function can exhibit a constant or even increasing marginal product of capital. This is because, when firms accumulate more capital, some of that increased capital will be the intellectual capital that creates technological progress, and this technological progress will offset the tendency for the marginal product of capital to diminish. In the special case where the marginal product of capital is exactly constant, aggregate output Y is proportional to the aggregate stock of capital K:

$$Y = AK$$

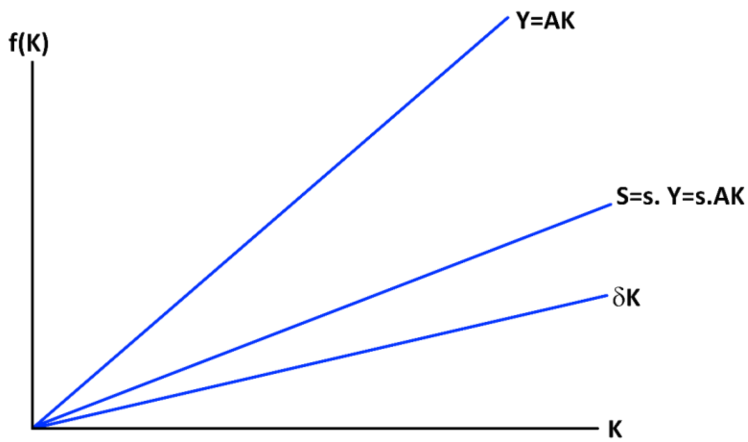
Where A is a positive constant that reflects the level of technology and 'K' here is taken in a broader sense as it includes physical as well as human capital. This model shows constant marginal product to capital (as $MPk = dY/dK = A$) indicating that long run growth is possible. Thus, AK model is a simple way of illustrating endogenous growth. Assuming a closed economy, the savings are equal to investment under conditions of full employment.

Since savings are the function f income and capital depreciates at a constant rate i.e. 'δ' the change in capital stock can be traced through following equations.

$$I = S = s.Y = s.AK$$

and, since capital depreciates at a constant rate, the change in capital stock i.e. $\dot{K}$ can be expressed as $\dot{K} = s.Y - \delta.K$. This change in capital stock can also be represented by a diagram given below.

Figure 1: The AK Model



In this figure Y-axis show output per worker while the X-axis show the capital stock. The line *Y=AK* having a constant slope shows the constant marginal productivity of capital; the line *S=s.Y* is the gross investment line while the line *δK* shows the depreciation line or the total replacement investment. The difference between the gross investment line and the replacement line i.e. area between *S=s.Y* line and *δK* line shows net investment in the economy which is positive and increasing.

The growth of capital stock can be found by dividing both sides of the equation showing change in capital stock with 'K', we get;

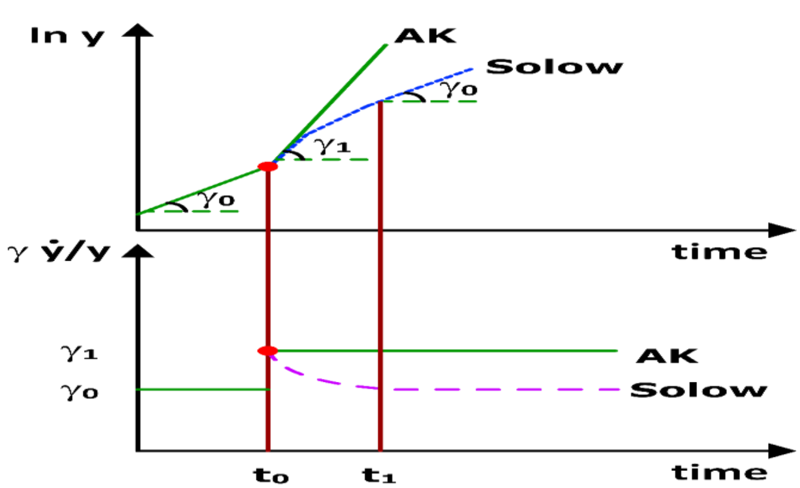
$$\frac{\dot{K}}{K} = s.\frac{Y}{K} - \delta$$

Since, Y=AK, i.e. Y/K =A, therefore, above equation can be rewritten as

$$\frac{\dot{K}}{K} = s.A - \delta$$

As, growth of output is equal to the growth of capital stock, Further, assuming that s.A > δ, the growth of capital stock as well as growth of output i.e , showing that the economy will be ever increasing as compared to the Solow model.

The AK Model and the Solow Model Compared for Rising Saving Rate



(**Miguel Lebre de Freitas, Introduction to Economic Growth)**

Figure 2 compares the impact of rate of change in savings upon the growth of income. The top part of the diagram shows the levels of income and the bottom part shows the growth rate of the same. In the upper part, we can see that a once for all increase in saving rate in t0 time period leads to an ever growing income curve (shown as ln y) in case of AK model while in case of Solow model, the income increases initially but ultimately reaches at the same level after t1. This can be observed through the angle 'γ'. In case of Solow type growth path, as savings increase or say, due to exogenous change in technology in t0 time period, the income curve immediately and its slope rises as we can see that the size of angle 'γ' increases from γ0 to γ1 but after t1 time period, it again comes back to the previous level i.e. γ0. However, in case of AK type growth the increase in income is forever, shown by an ever increasing curve and once for all increase in the size of angle γ0 to γ1. The growth path can better be elaborated through growth rate of income in the lower segment of the diagram.

AK Model with Human Factor

In its more realistic form, we can also add labour as an input along with capital. In this context, first of all, we can discuss Arrow's model with knowledge spillovers. In this model, the production function for final output can be written as

$$Y = B.K^{\alpha}L^{1-\alpha} \qquad (1)$$

which is a Cobb-Douglas type production function showing constant returns to scale with inputs K and L. In a model with technology and population growth as exogenous factors, the population, equal to labour input L, can be normalized to one and the individual firm takes total factor productivity B as given. However, we suppose that B is in fact endogenously determined. Specifically, the accumulation of capital generates new knowledge about production in the economy as a whole. In particular, we assume that

$$B = AK^{1-\alpha} \qquad (2)$$

where, A is constant and is greater than zero i.e. A > 0

That is, an incidental by-product of capital accumulation by firms in the economy is the improvement of the technology that firms use to produce. Technological progress, modelled as a by-product of capital accumulation, is external to the firm. Combining the two preceding equations gives

$$Y = A.K.L^{1-\alpha} \qquad (3)$$

This is exactly the AK model above, noting that L = 1. However, in further formulation of the AK model, we can include human capital as a separate variable having a positive effect upon the level of output. Thus, more skilled labour force will be assumed to produce more output than an unskilled individual, and the total stock of such "skills" is called human capital. Crucially, human capital can be accumulated through education. Thus, both types of capital can be accumulated—this turns out to imply that the model has similar properties to the AK model. In this perspective, we can have a production function of the following type: $Y_t = A_t.K_t{}^{\alpha}H_t{}^{1-\alpha}$ (4)

Where Kt is physical capital and Ht is human capital. So, growth is determined by;

$$\frac{Y'_t}{Y_t} = \frac{A'_t}{A_t} + \alpha\frac{K'_t}{K_t} + (1-\alpha)\frac{H'_t}{H_t}$$

Assuming that like physical capital, human capital also depreciates for given attainments. This can be understood in this way that if a person does not updates its knowledge, the knowledge accumulated so far depreciates in a dynamic economy. For simplifying the analysis, let us assume that both the physical as well as human capital depreciates at the same rate, then we can easily derive the equations for accumulation of each type of capital stock.

In sum, AK model gives a new framework for the long run growth of the economies. However, there are still some reasons to doubt the predictions about long-run growth generated by this class of models. The first line of criticism is related with the non-accumulable factors. In the real world, there are factors of production that are in fixed supply, such as land, or that cannot simply be accumulated indefinitely such as energy. Remember that the AK model results are of a knife-edge variety: Any move away from all factors being accumulable, and we move back to the Solow model results. Moreover, similar treatment to all type of human capital is also criticised by many as they say that the strict parallel between human capital and physical capital in the model just described is probably not completely accurate. For instance, not all expenditures on education will produce the same effect on output. The marginal boost to aggregate output of primary teaching is altogether different to that of higher education; training the non- skilled informally and the formal training of the professional also differ in their marginal returns. By clubbing all these different types of human capital together hardly proposes an effective policy suggestion for countries with a varied structure of human capital. Another source of the difficulties faced by the AK model is that it does not make an explicit distinction between capital accumulation and technological progress. In effect it just lumps together the physical and human capital.

Discussion of AK Model

Non-Accumulable Factors: In the real world, there are factors of production that are in fixed supply, such as land, or that cannot simply be accumulated indefinitely such as energy. Remember that the AK model results are of a knife-edge variety: Any move away from all factors being accumulable.

Treatment of Human Capital: The strict parallel between human capital and physical capital in the model just described is probably not completely accurate. For instance, not all expenditures on education will produce the same effect on output. The marginal boost to aggregate output of teaching someone how to read and write is presumably greater than that of masters in economics! Thus, there may be limits to which one can increase growth just by boosting educational enrollment.

The AK model and Policy Debates

- The fact that savings rate can affect the growth rate (and in a big way) made the AK model very popular in policy discussions.
- It makes government policy potentially very important for growth.
- In a famous paper, Lucas (1990) called tax cuts on savings as the "largest genuinely free lunch I have seen in 25 years in this business."
- Even today when candidates fiercely debate tax policy, an important part of discussion revolves around growth

- King and Rebelo (1990, JPE): The “welfare effect ”of a 10 percent increase in income tax is 40 times larger in an (AK) endogenous growth model (65% of consumption) than it is in a neoclassical growth model (1.6% of consumption)
- Stokey and Rebelo (1995) and Lucas (1990) argue that if endogenous growth models are calibrated to plausible values the effect on welfare is not likely to be large
- Because if tax differences are so important for growth, how come countries like Sweden with extremely high tax rates grow as fast as the US?

Shortcomings of the AK model

- Growth is the outcome of accidents---actions that are completely unintentional.
- Externalities must be substantial: For example, the capital bought by an investor contributes twice as much to others’ production than to his/her own. Same for human capital: Your education benefits others more than it benefits you.
- Alternatively stated, the Social return on many types of investments far exceed their private return. If externalities are really that big, individuals will typically find a way to capitalize on them (A doctor will not distribute advise on the street, etc.)