Phase Shift due to potential

Phase Shift.

$$\frac{d^2 u_\ell}{dr^2} + \frac{M}{\hbar^2}\left[E - V(r) - \frac{\hbar^2 \ell(\ell+1)}{M r^2}\right] u_\ell = 0$$

$$\frac{d^2 u_\ell}{dr^2} + \left\{K^2 - U(r) - \frac{\ell(\ell+1)}{r^2}\right\} u_\ell = 0$$

$u_\ell(r) \rightarrow$ radial function for the ℓ^{th} partial wave.

$$K^2 = \frac{M E}{\hbar^2}, \quad U(r) = \frac{M}{\hbar^2} V(r)$$

When scatterer is present

$$u_\ell'' + \left\{K^2 - U(r) - \frac{\ell(\ell+1)}{r^2}\right\} u_\ell = 0$$

when there is no scatterer

$$v_\ell'' + \left\{ k^2 - \frac{\ell(\ell+1)}{r^2} \right\} v_\ell = 0$$

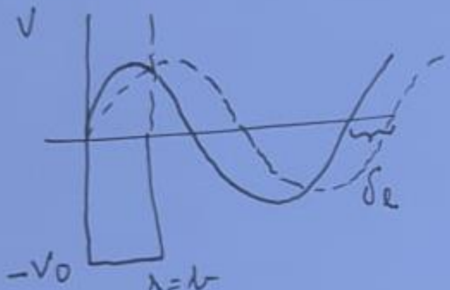
$$v_\ell \simeq \sin\left(kr - \ell\frac{\pi}{2}\right)$$

$$u_\ell \simeq \sin\left(kr - \ell\frac{\pi}{2} + \delta_\ell\right)$$

δ_ℓ determines whether the wave function $u_\ell(r)$ is ahead or lagging in phase w.r.to $v_\ell(r)$.

For attractive potential $U(r) < 0$ and hence u_ℓ'' has larger -ve value the v_ℓ''

$\Rightarrow$ wave function u_ℓ has a greater curvature in the interior region $(r < b)$ than v_ℓ (shown by dashed curve).



$\delta_\ell > 0$

$\delta_\ell < 0$

Conversely for a repulsive potential $\delta_\ell < 0$

Reference Books

- Nuclear Physics by S. N. Ghoshal
- Introductory Nuclear Physics by K. S. Krane