Dynamical Similarity, Reynold's number:

In practical cases, ~~such~~ it is usually necessary to carry out experiments on models and to relate their behaviour to that of the full-scale body. ~~Generally speaking~~, This requires not only that the model and the full-scale body should be geometrically similar, but also, as far as possible ~~that~~ one should ensure that the two possess dynamical similarity in the sense that the ratios of corresponding dynamical quantities at the corresponding stations should be same. Two fluid motions are said to be dynamically similar if, with similar geometrical boundaries, the velocity field are geometrically similar, i.e, if they have geometrically similar stream-lines. For two flows about similar geometric boundaries with different fluids, different velocities and different linear dimensions to be dynamically similar, the following condition must be fulfilled: at all geometrically similar point the forces acting on a fluid particle must bear a fixed ratio at every instant of time. Reynold was the first to consider the laws of similar flows. The method used involves constructing dimensionless equations and forms part of the science of dimensional analysis.

Let us consider the Navier-stoke's equation of motion of a viscous incompressible fluid in the x-direction

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \quad (1)$$

Suppose L, V, P denote a characteristic length, velocity and pressure respectively. Then we can write

$x = Lx'$, $y = Ly'$, $z = Lz'$, $u = V.u'$, $v = V.v'$, $w = Vw'$, $p = Pp'$

where primed quantities are pure numbers having no dimensions.

equation (1) Can be written as

$$\frac{V}{LV^{-1}}\frac{\partial u'}{\partial t'} + \frac{V^2}{L}u'\frac{\partial u'}{\partial x'} + \frac{V^2}{L}v'\frac{\partial u'}{\partial y'} + \frac{V^2}{L}w'\frac{\partial u'}{\partial z'} = X - \frac{P}{\rho L}\frac{\partial p'}{\partial x'}$$

$$+ \frac{\nu V}{L^2}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2}\right)$$

$$\frac{\partial u'}{\partial t'} + u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} + w'\frac{\partial u'}{\partial z'} = \frac{LX}{V^2} - \frac{P}{\rho V^2}\frac{\partial p'}{\partial x'} + \frac{\nu}{LV}\left(\frac{\partial^2 u'}{\partial x'^2} + \frac{\partial^2 u'}{\partial y'^2} + \frac{\partial^2 u'}{\partial z'^2}\right)$$

Since L.H.S. is entirely dimensionless, Hence the R.H.S must likewise be so. It follows that the three quantities

$$\frac{LX}{V^2}, \quad \frac{P}{\rho V^2}, \quad \frac{\nu}{VL}$$

must be dimensionless quantities. In order to produce a faithful model of a given incompressible viscous flow it is essential to keep these three numbers Constant.

The first no. $\frac{LX}{V^2}$ tells us how to scale the body forces.

The second no. $\frac{P}{\rho V^2}$ ensures dynamical similarity in the two flows at paints where viscosity is unimpartant, such paint would occur at stations remote from the boundaries. Reciprocal of this no. is called euler's no. (Eu)

The third no. ensures dynamical similarity at corresponding points near the boundaries where viscous effects are important. Its reciprocal is called the Reynalds number and is denote by R so that

$$R = \frac{VL}{\nu} = \frac{\text{Inertia force}}{\text{viscous force}} = \frac{\text{Mass . Acc.}}{\text{Stress . Area}} = \frac{\text{Volume . density . Acc}}{\text{Stress . Area}} = \frac{\text{Area} \times \text{Linear dim . dens . Acc.}}{\text{Stress . Area}} = \frac{L . \rho}{}$$

In actual practice if Reynalds no. is Constant and other two no. are not, it means our model is faithful only near the boundary where viscous forces are effective. If it is possible to make $\frac{P}{\rho V^2}$ no. also constant then model is also faithful at stations remote from the boundary.

~~the Reynold no. is useful to~~

Significance of Reynolds number:

1) Reynolds no. is useful for dynamical similarity of two ~~flows~~ viscous incompressible flow. In order to make two flows dynamically similar near the boundary, Reynolds number must be same for the two flows.

2) Since Reynolds number is inversely proportional to the viscosity of ~~the~~ fluid. Thus large viscosity corresponds to low Reynolds number and small viscosity to high Reynolds number.

3) If the value of R. ~~exceed~~ exceeds a certain critical value, the flow becomes turbulent near the boundaries.

Example: A one-sixth scale model of an automobile is to be tested in a wind tunnel at a speed corresponding to the ~~the~~ speed of 60 Km/h of the prototype (full-~~scale~~ scale ~~new~~ body) what will be the speed of wind If only inertial and viscous forces are to be modelled.

The modelling rule is

$$\left(\frac{\rho VL}{\mu}\right)_m = \left(\frac{\rho VL}{\mu}\right)_p$$

Since, model and prototype both are tested in same fluid (air) so $\rho_m = \rho_p$ & $\mu_m = \mu_p$

so, $V_m \cdot L_m = V_p \cdot L_p$

$$V_m = \frac{L_p}{L_m} \cdot V_p = 6 \times V_p = 6 \times 60 = 360 \text{ km/h}$$

So model should be tested at a wind speed of 360 Km/h

Example: ④

To make some estimations about an aircraft traveling at 10 m/sec it is proposed to test a one-twentieth scale model of it in water. what should its velocity be in water? only inertial and viscous forces are modelled.

For dynamical similarity we need

$$\left(\frac{\rho V L}{\mu}\right)_m = \left(\frac{\rho V L}{\mu}\right)_p$$

$$V_m = V_p \frac{L_p}{L_m} \cdot \frac{\rho_p}{\rho_m} \cdot \frac{\mu_m}{\mu_p}$$

$$= 10 \times 20 \times \left(\frac{1.226}{10^3}\right) \cdot \left(\frac{1.14 \times 10^{-3}}{1.78 \times 10^{-3}}\right)$$

$$V_m = 15.7 \text{ m/s}$$