Poynting Vector and poynting theoram :-

The rate of at which energy is transmitted through unit area perpendicular to the direction of propagation of electromagnetic wave is called poynting vector.

It is represented by $\bar{S}$.

The cross product of $\vec{E}$ (electric vectors) and magnetic Vector $\vec{H}$ is known as poynting vector.

$$\bar{S} = \bar{E} \times \bar{H}$$

Poynting Vector measure the electrical power per unit area and S.I unit is Weber/m^2

$$\bar{E} \times \bar{H} = \frac{\text{Volt}}{\text{length}} \times \frac{\text{Current}}{\text{length}} = \frac{\text{Power}}{\text{area}}$$

Poynting theoram :-

As the electromagnetic wave propagate from one point to other point, there is a transfer energy. To find flow of energy in the direction of propagation. We take divergence of $\bar{S}$.

$$\bar{\nabla} \cdot \bar{S} = \bar{\nabla} \cdot (\bar{E} \times \bar{H})$$

$$\bar{\nabla} \cdot \bar{S} = \frac{1}{\mu_0} \bar{\nabla} \cdot (\bar{E} \times \bar{B})$$

$$\therefore \quad \bar{\nabla} \cdot (\bar{E} \times \bar{B}) = B(\bar{\nabla} \times \bar{E}) - \bar{E}(\bar{\nabla} \times \bar{B})$$

$$\bar{\nabla}\cdot\bar{S} = \frac{1}{\mu_0}\left[B(\nabla\times E) - E(\nabla\times B)\right]$$

$$\bar{\nabla}\cdot\bar{S} = \frac{1}{\mu_0}\left[B\left(\frac{-\partial B}{\partial t}\right) - \bar{E}\left(\mu_0\varepsilon_0\frac{\partial E}{\partial t}\right)\right]$$

$\because$ $\bar{\nabla}\times\bar{E} = \frac{-\partial B}{\partial t}$ and $\bar{\nabla}\times\bar{B} = \mu_0\varepsilon_0\frac{\partial E}{\partial t}$ (Maxwell (3) & (4) equ^n in free space)

$$\bar{\nabla}\cdot\bar{S} = -\frac{1}{\mu_0}\left[\frac{B\partial B}{\partial t} + \mu_0\varepsilon_0\frac{E\partial E}{\partial t}\right]$$

But $B\frac{\partial B}{\partial t} = \frac{1}{2}\frac{\partial B^2}{\partial t}$ and $\bar{E}\cdot\frac{\partial E}{\partial t} = \frac{1}{2}\frac{\partial E^2}{\partial t}$

$$\bar{\nabla}\cdot\bar{S} = -\frac{1}{\mu_0}\left[\frac{1}{2}\frac{\partial B^2}{\partial t} + \frac{\mu_0\varepsilon_0}{2}\frac{\partial E^2}{\partial t}\right]$$

$$\bar{\nabla}\cdot\bar{S} = -\frac{1}{\mu_0}\left[\frac{1}{2}\frac{\mu_0^2\partial H^2}{\partial t} + \frac{1}{2}\mu_0\varepsilon_0\frac{\partial E^2}{\partial t}\right]$$

$$\bar{\nabla}\cdot\bar{S} = \bar{\nabla}\cdot(\bar{E}\times\bar{H}) = \frac{-\partial}{\partial t}\left[\frac{1}{2}\mu_0 H^2 + \frac{1}{2}\varepsilon_0 E^2\right]$$

$\because$ $\frac{1}{2}\mu_0 H^2$ = Magnetic energy per unit volume

$\frac{1}{2}\varepsilon_0 E^2$ = Electric energy per unit volume

Taking Volume integration both the side we get

$$\int_V \bar{\nabla}\cdot\bar{S}\,dv = \int_V \bar{\nabla}\cdot(E\times H)\,dV = \frac{-\partial}{\partial t}\int_V\left[\frac{1}{2}\mu_0 H^2 + \frac{1}{2}\varepsilon_0 E^2\right]dv$$

$$\int_V \nabla\cdot(\bar{E}\times\bar{H})\,dV = -\frac{\partial W}{\partial t}$$

W is the net energy contained in the volume of electromagnetic wave.

using gauss divergence theoram in L.H.S we get

$$\int_V \nabla\cdot(\bar{E}\times H)\,dV = \int_S (\bar{E}\times\bar{H})\,ds$$

hence

$$\boxed{\int_S (\bar{E}\times\bar{H})\,ds = -\frac{\partial W}{\partial t}}$$

Rate of transmission energy through unit area S gives the rate of flow of energy per unit volume of the surface or power flow per unit area of the surface.

Electromagnetic Wave propagation in free space:-

An electromagnetic wave is a wave of oscillation of electric and magnetic field in mutually perpendicular plane and these oscillations are perpendicular to the direction of propagation of wave.

In region of free space where there is no charge or current, Maxwell's equations read as-

(i) $\nabla\cdot E = 0$ (ii) $\nabla\cdot B = 0$ (iii) $\nabla\times E = -\frac{\partial B}{\partial t}$ (iv) $\nabla\times B = \mu_0\varepsilon_0\frac{\partial E}{\partial t}$

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Apply curl in equ^n (iii) we get -

$$\nabla \times (\nabla \times E) = -\nabla \times \left(\frac{\partial B}{\partial t}\right)$$

using vector product rule we get -

$$\nabla(\nabla \cdot E) - \nabla^2 E = -\frac{\partial}{\partial t}(\bar{\nabla} \times \bar{B})$$

using equ^n (i) and (iv) we get

$$\cancel{-}\nabla^2 E = \cancel{-}\mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} \quad \text{——— (V)}$$

Now apply curl to equ^n (iv)

$$\nabla \times (\nabla \times B) = \nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial E}{\partial t}\right)$$

$$\nabla(\nabla \cdot B) - \nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times E)$$

using equ^n (ii) and (iii) we get

$$-\nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right)$$

$$\nabla^2 B = \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} \quad \text{——— (vi)}$$

Equ^n 5 and 6 are the plane electromagnetic wave equ^n governing electromagnetic field $\bar{E}$ and $\bar{B}$ in free space.

The equ^n 5 and 6 are identical in form. The general form of these equ^n may be written as

$$\nabla^2 u = \mu_0 \varepsilon_0 \frac{\partial^2 v}{\partial t^2} \quad \text{——— (vii)}$$

No.

Date

where u be the scaler and vector product quantity.

The well known equ^n of a wave propagating with a velocity v is given by

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} = 0 \quad \text{——— (8)}$$

Comparing equ^n (7) and (8) shows that the fields vector $\bar{E}$ and $\bar{B}$ are propagated in free space as wave speed equal to

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{(4\pi \times 10^{-7}\, Wb/Am)(8.85 \times 10^{-12} C^2 N.m^2)}}$$

$$v = 2.99 \times 10^{8}\, m/sec. = c = \text{velocity of light}$$

hence electromagnetic wave propagate with the speed of light or the velocity of the electromagnetic wave in vaccum is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

From this revolutionizing result, we conclude that light is an electromagnetic wave.

Substituting $\frac{1}{\sqrt{\mu_0\varepsilon_0}} = c$ in equ^n (5) and (6) we get-

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \quad \text{——— (9)}$$

$$\nabla^2 B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = 0 \quad \text{——— (10)}$$

and general solution of equ^n:-

$$\boxed{\nabla^2 u - \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} = 0}$$

Transverse nature of light or solution of electromagnetic wave :-

We know that plane electromagnetic wave indicates electromagnetic field $\bar{E}$ and $\bar{B}$ in free space -

$$\nabla^2 \bar{E} - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0 \quad \text{——— (1)}$$

and

$$\nabla^2 B - \mu_0 \varepsilon_0 \frac{\partial^2 B}{\partial t^2} = 0 \quad \text{——— (2)}$$

Solution of above equ^n -

$$\bar{E}(r,t) = E_0 e^{i\bar{k}\bar{r} - i\omega t} \quad \text{——— (3)}$$

$$\bar{B}(r,t) = B_0 e^{i\bar{k}\bar{r} - i\omega t} \quad \text{——— (4)}$$

where $\omega = 2\pi n$ and $k = 2\pi / \lambda$.

In free space

$$\bar{\nabla}\cdot\bar{E} = 0 \quad (5) \quad \& \quad \bar{\nabla}\cdot\bar{B} = 0 \quad \text{—— (6)}$$

From equ^n (3) taking divergence both the side -

$$\bar{\nabla}\cdot\bar{E} = \nabla\cdot\left(E_0 e^{ik\cdot r - i\omega t}\right)$$

$$\vec{\nabla}\cdot\vec{E} = \left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\left(E_0 e^{i\vec{k}\cdot\vec{r}-i\omega t}\right) \quad \text{——} (9)$$

But $\vec{k}\cdot\vec{r} = (\hat{i}k_x+\hat{j}k_y+\hat{k}k_z)\cdot(\hat{i}x+\hat{j}y+\hat{k}z)$

$$\vec{k}\cdot\vec{r} = (xk_x+yk_y+zk_z)$$

equ^n (9) becomes -

$$\vec{\nabla}\cdot\vec{E} = \left(\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}E_{0x}+\hat{j}E_{0y}+\hat{k}E_{0z}\right)e^{i(xk_x+yk_y-zk_z-\omega t)}$$

$$\vec{\nabla}\cdot\vec{E} = \frac{\partial}{\partial x}\left[E_{0x}e^{i(xk_x+yk_y+zk_z)-\omega t)}\right. + \frac{\partial}{\partial y}\left[E_{0y}e^{i(xk_x+yk_y+zk_z)-\omega t)}\right.$$
$$+\frac{\partial}{\partial z}\left(E_{0z}e^{i(xk_x+yk_y+zk_z-\omega t)}\right.$$

$$\cancel{\vec{\nabla}\cdot\vec{E} = E_{0x}e^{i(k_xE_{0x}+k_yE_{0y}+k_zE_{0z}}}$$

$$\cancel{\vec{\nabla}\cdot\vec{E} = E_{0x}e^{i(xk_x+yk_y+zk_z-\omega t)}(ik_x)} + \cancel{\frac{\partial}{\partial y}}$$

$$\vec{\nabla}\cdot\vec{E} = E_{0x}e^{i(xk_x+yk_y+zk_z-\omega t)}(ik_x) + E_{0y}e^{i(xk_x+yk_y+zk_z-\omega t)}\cdot(ik_y)$$
$$+E_{0z}e^{i(xk_x+yk_y+zk_z-\omega t)}(ik_z)$$

$$\vec{\nabla}\cdot\vec{E} = i(k_xE_{0x}+k_yE_{0y}+k_zE_{0z})e^{i(xk_x+yk_y+zk_z-\omega t)}$$

$$\vec{\nabla}\cdot\vec{E} = i(k_xE_{0x}+k_yE_{0y}+k_zE_{0z})e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\vec{\nabla}\cdot\vec{E} = i(\hat{i}k_x+\hat{j}k_y+\hat{k}k_z)\cdot(\hat{i}E_{0x}+\hat{j}E_{0y}+\hat{k}E_{0z})e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$\nabla\cdot E = i\vec{k}\cdot E_0 e^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

No.

Date

$$\boxed{\bar{\nabla}\cdot\bar{E} = i\,\bar{k}\cdot\bar{E}} \quad \text{——— (8)}$$

for free space, Maxwell's first equⁿ-

$$\bar{\nabla}\cdot\bar{E} = 0 \quad \text{——— (9)}$$

Comparing equⁿ (8) and (9) we get -

$$i(\bar{k}\cdot\bar{E}) = 0$$

$$(\bar{k}\cdot\bar{E}) = 0$$

Proceeding in the same manner, we shall get -

$$\bar{k}\cdot E = 0 \quad \& \quad \bar{k}\cdot\bar{H} = 0$$

So $\bar{E}$ and $\bar{H}$ are perpendicular to the direction of propagation of electromagnetic wave.

Velocity of propagation of electromagnetic wave -

We know that the well known equⁿ of a wave propagating with a velocity v is given by -

$$\nabla^2 u - \frac{1}{v^2}\frac{\partial^2 v}{\partial t^2} = 0$$

and the general equⁿ of wave propagation is

$$\nabla^2 v - \mu_0\varepsilon_0\frac{\partial^2 v}{\partial t^2} = 0$$

Comparing both the equⁿ

No.

Date

$$\mu_0 \varepsilon_0 = \frac{1}{v^2}$$

$$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$$

$$v = 2.99 \times 10^{8} \, m/sec = c$$

hence electromagnetic wave propagate with the speed of light in vaccum.

So $\boxed{c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}}$

⇒ Show that at every instant, the ratio of electric field and magnetic field of e.m wave is equal to speed of light c

The simoultaneously sinusoidally varying electric and magnetic field of electromagnetic wave are given as-

$$E = E_0 \cos(kx - \omega t) \quad \text{——} (1)$$
$$B = B_0 \cos(kx - \omega t) \quad \text{——} (2)$$

where E_0 and B_0 are the maximum value of electric and magnetic fields associated with em wave

$$\frac{\omega}{K} = \frac{2\pi f}{2\pi/\lambda} = f\lambda = c \quad \text{——} (3)$$

where c is the velocity of light

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Date

Taking partial differentiation of equ^n 1 and 2 we get -

$$\frac{\partial E}{\partial x} = -kE_0 \sin(kx - \omega t) \quad \text{—— (4)}$$

$$\frac{\partial B}{\partial t} = -\omega B_0 \sin(kx - \omega t) \quad \text{—— (5)}$$

According to Faraday's law

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$-kE_0 \sin(kx - \omega t) = -\omega B_0 \sin(kx - \omega t)$$

$$kE_0 = \omega B_0$$

$$\frac{E_0}{B_0} = \frac{\omega}{k} \quad \text{—— (6)}$$

from equ^n 3 and 6 we get -

$$\frac{E_0}{B_0} = c \quad \text{—— (7)}$$

Considering equ^n 1 2 and 7 we get -

$$\boxed{\frac{E}{B} = \frac{E_0}{B_0} = c}$$