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Lectures 1, 2, 3 for
MSc (Physics) - Optoelectronics students
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'Interference Experiments'
'Optical data Processing'

Photodetectors are sensitive only to $\vec{E}$ (and not $\vec{B}$).

$$\vec{E}(\vec{r},t) = A' \cos(\bar{k}\cdot\bar{r} - \omega t) = \frac{A'}{2}\left[e^{i(\bar{k}\cdot\bar{r}-\omega t)} + e^{-i(\quad)}\right]$$

let $\frac{A'}{2} = A$

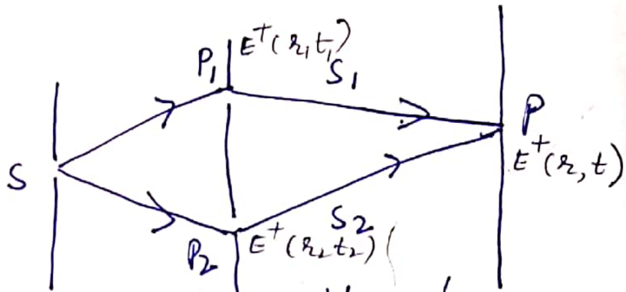
$$= A\left[\;'' \quad + \quad ''\;\right]$$

$$= \vec{E}^{+}(\bar{r},t) + E^{-}(\;)$$

$$E^{-}(\bar{r},t) = E^{+*}(\bar{r},t) \quad (\text{c.c.})$$

Since classical measuring devices respond to Real $\vec{E}$ only
$\therefore$ In classical theory either $E^{+}(\bar{r},t)$ or $E^{-}(\bar{r},t)$ is taken to represent the field.

## Interference Experiments




Young's

Field at P at time $t = \overset{+}{E}(r,t)$
= certain linear superposⁿ of the fields present at the two pinholes at earlier times

$P_1P = S_1$
$P_2P = S_2$

At $P_1$, $E^{+}(r_1,t_1)$ at instant $t_1$
$P_2$ $\quad E^{+}(r_2,t_2)$ " " $t_2$ ] earlier than t

$$t = t_1 + \frac{S_1}{c}, \qquad t_1 = t - \frac{S_1}{c}$$

$$t = t_2 + \frac{S_2}{c} \qquad t_2 = t - \frac{S_2}{c}$$

Field at pt. P at time t is

$$E^{+}(r,t) = \lambda_1 E^{+}(r_1,t_1) + \lambda_2 E^{+}(r_2,t_2)$$

$\lambda_1, \lambda_2 \rightarrow$ depend upon the geometry of arrangement. & are complex quantities

The photodetector responds to modulus of square of some complex field amplitude. So

$$|E^{+}(r,t)|^2 = E^{+}(r,t)\, E^{-}(r,t)$$

$$= [\lambda_1 E^{+}(r_1,t_1) + \lambda_2 E^{+}(r_2 t_2)] \cdot [\lambda_1^{*} E^{-}(r_1 t_1) + \lambda_2^{*} E^{-}(r_2 t_2)]$$

$\because a + a^{*} = 2\,\text{Re}\,a$
$(a+i\omega)+(a-i\omega) = 2a = 2\,\text{Re}\,a$

$$= \lambda_1\lambda_1^{*} E^{+}(r_1 t_1) E^{-}(r_1 t_1) + \lambda_1\lambda_2^{*} E^{+}(r_1 t_1) E^{-}(r_2 t_2)$$
$$+ \lambda_2\lambda_1^{*} E^{+}(r_2 t_2) E^{-}(r_1 t_1) + \lambda_2\lambda_2^{*} E^{+}(r_2 t_2) E^{-}(r_2 t$$

$$= |\lambda_1|^2 E^{+}(r_1 t_1)\bar{E}(r_1 t_1) + |\lambda_2|^2 E^{+}(r_2 t_2)\bar{E}(r_2 t_2) + 2\,\text{Re}\{\lambda_1^{*}\lambda_2 E^{-}(r_1 t \cdot E^{+}(r_2 t_2)$$

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Op. light detector have long response-times and are capable of measuring avg. intensity.

Take ensemble average 

$$I = \langle |E^+(r,t)|^2 \rangle = |\lambda_1^2| \langle |E^+(r_1,t_1)|^2 \rangle + |\lambda_2^2| \langle |E^+(r_2 t_2)|^2 \rangle + 2\,\mathrm{Re}\,\lambda_1^* \lambda_2 \langle E^-(r_1,t_1)\, E^+(r_2 t_2) \rangle$$

$$= |\lambda_1|^2 G^{(1)}(r_1 t_1, r_1 t_1) + |\lambda_2|^2 G^{(1)}(r_2 t_2, r_2 t_2) + 2\,\mathrm{Re}\,\lambda_1^* \lambda_2 G^{(1)}(r_1 t_1, r_2 t_2)$$

1st order correlation fn:

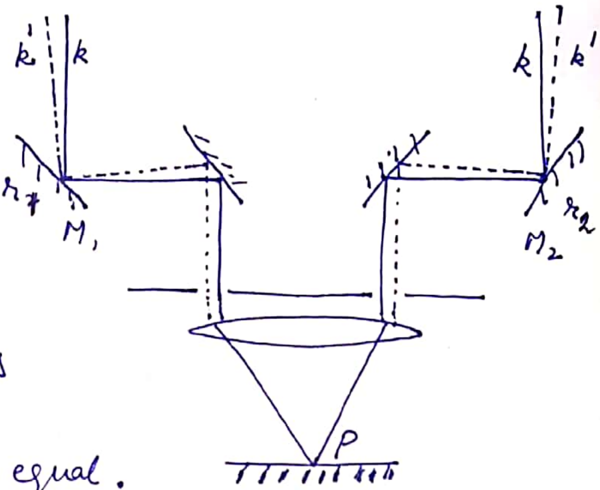
$$G^{(1)}(rt, r't') = \langle E^-(rt)\, E^+(r't') \rangle$$

∵ vector prop. of the field are not imp. in this expt. ∴ vector or tensor indices of the fields and G are omitted.

If plane of poln plays any role then we can't omit.

Diffn effects of slits ignored.

## Michelson Stellar Interferometer




Field incident upon a detector is a superposition of 2 waves (plane).

Propn vector is only slightly different.

(e.g. monochromatically filtered light from the two members of a double star)

Assume freq. of both waves are equal. Then field can be expressed as

$$E^+(\bar{r}, t) = A e^{i(\bar{k}\cdot\bar{r} - \omega t)} + B e^{i(\bar{k}'\cdot \bar{r} - \omega t)}$$

Field at P at time t is, in effect, = sum of the two fields falling on $M_1$ + $M_2$ at $t'$ (if op. paths $M_1P = M_2P$)

Field at $M_1$ and $M_2$ can be written as

$$E^+(r_1 t') = A e^{i(k\cdot r_1 - \omega t')} + B e^{i(k'\cdot r_1 - \omega t')}$$

and $$E^+(r_2 t') = A e^{i(k\cdot r_2 - \omega t')} + B e^{i(k'\cdot r_2 - \omega t')}$$

Now, Resultant intensity at point P is

$$E^+(r,t) = E^+(r_1 t') + E^+(r_2 t')$$

Therefore $E^-(rt) =$

Since detector responds to modulus of square of field amplitude (i.e. intensity), so

$$|E^+(r\,t)|^2 = E^+(r\,t)\,E^-(r\,t)$$

$$= [A e^{i(k\cdot r_1 - \omega t')} + B e^{i(k'\cdot r_1 - \omega t')} + A e^{i(k\cdot r_2 - \omega t')} + B e^{i(k'\cdot r_2 - \omega t')}]$$

$$\cdot [A^* e^{-i(k r_1 - \omega t')} + B^* e^{-i(k'\cdot r_1 - \omega t')} + A^* e^{-i(k r_2 - \omega t')} + B^* e^{-i(k'\cdot r_2 - \omega t')}]$$

Take avg. since in 1 sec. more than $10^8$ waves pass with diff. phases. Our observ$^n$ time is not as small as $10^{-8}$ sec.

$$\langle |E^+(r\,t)|^2 \rangle = \langle [A A^* + B B^* + A A^* + B B^*$$
$$+ A B^* e^{i(k r_1 - \omega t')} e^{-i(k' r_1 - \omega t')}$$
$$+ A A^* e^{i(k r_1 - \omega t')} e^{-i(k r_2 - \omega t')}$$
$$+ A B^* e^{i(k r_1 - \omega t')} e^{-i(k' r_2 - \omega t')}$$
$$+ B A^* e^{i(k' r_1 - \omega t')} e^{-i(k r_1 - \omega t')}$$
$$+ B A^* e^{i(k' r_1 - \omega t')} e^{-i(k r_2 - \omega t')}$$
$$+ B B^* e^{i(k r_1 - \omega t')} e^{-i(k' r_2 - \omega t')}$$
$$+ A A^* e^{i(k r_2 - \omega t')} e^{-i(k r_1 - \omega t')}$$
$$+ A B^* e^{i(k r_2 - \omega t')} e^{-i(k'\cdot r_1 - \omega t')}$$
$$+ A B^* e^{i(k r_2 - \omega t')} e^{-i(k' r_2 - \omega t')}$$
$$+ B A^* e^{i(k' r_2 - \omega t')} e^{-i(k r_1 - \omega t')}$$
$$+ B B^* e^{i(k' r_2 - \omega t')} e^{-i(k'\cdot r_1 - \omega t')}$$
$$+ B A^* e^{i(k' r_2 - \omega t')} e^{-i(k r_2 - \omega t')}] \rangle$$

$\langle A \rangle = \langle B \rangle = 0$

$\langle AB^* \rangle$

$\langle |A|^2 A^* B \rangle = 0$

$$= 2\langle |A|^2 \rangle + 2\langle |B|^2 \rangle$$
$$+ \langle |A|^2 e^{ik(r_1 - r_2)} \rangle + \langle |B|^2 e^{ik'(r_1 - r_2)} \rangle$$
$$+ \langle |A|^2 e^{ik(r_2 - r_1)} \rangle + \langle |B|^2 e^{ik'(r_2 - r_1)} \rangle$$
$$+ \langle A B^* e^{i(k-k') r_1} + A B^* e^{i(k r_1 - k' r_2)}$$
$$+ B A^* e^{i(k'-k) r_1} + B A^* e^{i(k' r_1 - k r_2)}$$
$$+ A B^* e^{i(k-k') r_2} + A B^* e^{i(k r_2 - k' r_1)}$$
$$+ B A^* e^{i(k'-k) r_2} + B A^* e^{i(k' r_2 - k r_1)} \rangle$$

$\langle |A|^{2n} \rangle \neq 0$

$\langle B \rangle \neq 0$

$n = 1, 2, \ldots$

$\langle AB^* \rangle$

$= \langle A \rangle \langle B^* \rangle$

$= 0$

$$= 2\langle |A|^2 \rangle + 2\langle |B|^2 \rangle$$
$$+ \langle |A|^2 \{ e^{ik(r_1 - r_2)} + e^{-ik(r_1 - r_2)} \} \rangle$$
$$+ \langle |B|^2 \{ e^{ik'(r_1 - r_2)} + e^{-ik'(r_1 - r_2)} \} \rangle$$

$\to \cos(\ )$

$$= 2\,\mathrm{Re} \{ \langle |A|^2 + |B|^2$$
$$+ \langle |A|^2 \rangle e^{-ik\cdot(r_1 - r_2)}$$
$$+ \langle |B|^2 \rangle e^{-ik'\cdot(r_1 - r_2)} \} \qquad (2.11)$$

Now correlation function

$$G^{(1)}(r t, r' t') = \langle E^-(rt)\, E^+(r't')\rangle$$

$$\therefore \quad G^{(1)}(r_1 t', r_2 t') = \langle E^-(r_1 t')\, E^+(r_2 t')\rangle$$

$$= \langle \{A^* e^{-i(kr_1-\omega t')} + B^* e^{-i(k'.r_1-\omega t')}\} \cdot \{A e^{i(kr_2-\omega t')} + B e^{i(k'.r_2-\omega t')}\}\rangle$$

$$= \langle |A|^2 e^{ik(r_2-r_1)} + |B|^2 e^{ik'(r_2-r_1)}\rangle \quad (2.12)$$

G is time indep ⇒ stationary character of field. $\quad \because \langle AB^*\rangle = 0$

from (2.11) $\quad \therefore \langle |E^+(rt)|^2\rangle = 2\,Re\{\langle |A|^2 + |B|^2\rangle + G^{(1)}(r_1 t', r_2 t')\}$ cf. (2.13)

The corrⁿ fn $G^{(1)}$ contains two spatially oscᵍ terms. The displacement $r_1 - r_2$ will decide that whether these terms reinforce or cancel each other. $G^{(1)}$ is the interference term.

If $\langle |A|^2\rangle = \langle |B|^2\rangle$ then

$$\langle |E^+(rt)|^2\rangle = 4\langle |A|^2\rangle + 2\,Re\{\langle |A|^2\rangle (e^{ik(r_2-r_1)} + e^{ik'(r_2-r_1)})\}$$

$$= 4\langle |A|^2\rangle + 2\langle |A|^2\rangle \{\cos k(r_2-r_1) + \cos k'(r_2-r_1)\}$$

$$= \quad \{2\cos\frac{(k+k').(r_1-r_2)}{2} \cdot \cos\frac{(k-k')(r_1-r_2)}{2}\}$$

$$= 4\langle |A|^2\rangle \{1 + \cos[\tfrac{1}{2} \quad \cdot \cos \quad \} \quad (2.14)$$

If $r_1 - r_2$ is adjusted so that $\cos\frac{1}{2}(k-k')(r_1-r_2) = 0$ then fringes will disappear.

- Appearance of fringes imply existence of two sources
- Dis " " " gives $r_1 - r_2$ and thereby the angular separation
- Large interferometers required (difficult) • Random variations of r.i. along the op. path can wash out interference pattern.

• Similar Instruments: for radioastronomy to determine size of celestial radio sources. Two separate antenna supply signal to common detector. If separation of antenna increased, random ph. diff occurs (difficulty)

# Radio interferometer of Hanbury Brown & Twiss

Fig.

- signals detected at antennas individually.
- high frequency components of the incoming radiation is filtered out.
- Only the measure of the 'fluctuations of the intensities' arriving at the receivers' is transmitted to central observ'n point.
- Detector signals are of relatively low frequency
  ∴ they are easy to transmit faithfully, over distances large compared to the limiting dimensions of Michelson interferometer
- deals with 'average of the product of two random intensities' rather than with a single intensity.'

Each detector responds to $|E^+(s,t)|^2$.

At $P_1$ $\quad E^+(s,t) \equiv E^+(r_1,t) = Ae^{i(k\cdot r_1-\omega t)} + Be^{i(k'\cdot r_1-\omega t)}$

At $P_2$ $\quad E^+(s,t) \equiv E^+(r_2,t) = Ae^{i(k\cdot r_2-\omega t)} + Be^{i(k'\cdot r_2-\omega t)}$

$$|E^+(r_1,t)|^2 = E^+(r_1,t)\,E^-(r_1,t)$$
$$=$$
$$= |A|^2 + |B|^2 + AB^* e^{i(k-k')\cdot r_1} + A^*B\, e^{-i(k-k')\cdot r_1}$$

$\because \langle AB^*\rangle = 0$
∴ no interference term

∴ Brown and Twiss multiplied the detected signal and only then took the statistical average.

$$\langle |E^+(r_1,t)|^2 |E^+(r_2,t)|^2\rangle = \langle\{|A|^2+|B|^2+AB^*e^{i(k-k')r_1}+A^*Be^{-i(k-k')r_1}\}\cdot\{|A|^2+|B|^2+AB^*e^{i(k-k')r_2}+\}\rangle$$

$\langle |A|^2A^*B\rangle = 0$ etc.

$$= \langle (|A|^2+|B|^2)^2 + |A|^2|B|^2 e^{i(k-k')\cdot(r_1-r_2)} + |A|^2|B|^2 e^{-i(k-k')(r_1-r_2)}\rangle$$

$$= \langle(|A|^2+|B|^2)^2\rangle + 2\langle|A|^2|B|^2\rangle \cos[(k-k')\cdot(r_1-r_2)]$$

quartic interference effect.

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Pedrotti



# Optical Data Processing

## Fraunhofer diffraction and the Fourier transform

Fraunhofer diffraction Pattern (within certain approx.) $\equiv$ F.T. of the E-field amplitude distribution in the object plane

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} g(k)\, e^{-ikx}\, dk \qquad \text{— (1)}$$

$$g(k) = \int_{-\infty}^{+\infty} f(x)\, e^{ikx}\, dx \qquad \text{— (2)}$$

Eq. (1) $\Rightarrow$ An arbitrary, non-periodic fn. $f(x)$ can be obtained by summing a continuous distribution of plane waves with amplitude distribution $g(k)$

- $f(x)$ and $g(k)$ are F.T. pair.
- In 2d

$$f(x,y) = \frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty} g(k_x, k_y)\, e^{-i(xk_x + yk_y)}\, dk_x\, dk_y \qquad \text{— (3)}$$

$$g(k_x, k_y) = \iint_{-\infty}^{+\infty} f(x,y)\, e^{i(xk_x + yk_y)}\, dx\, dy. \qquad \text{— (4)}$$

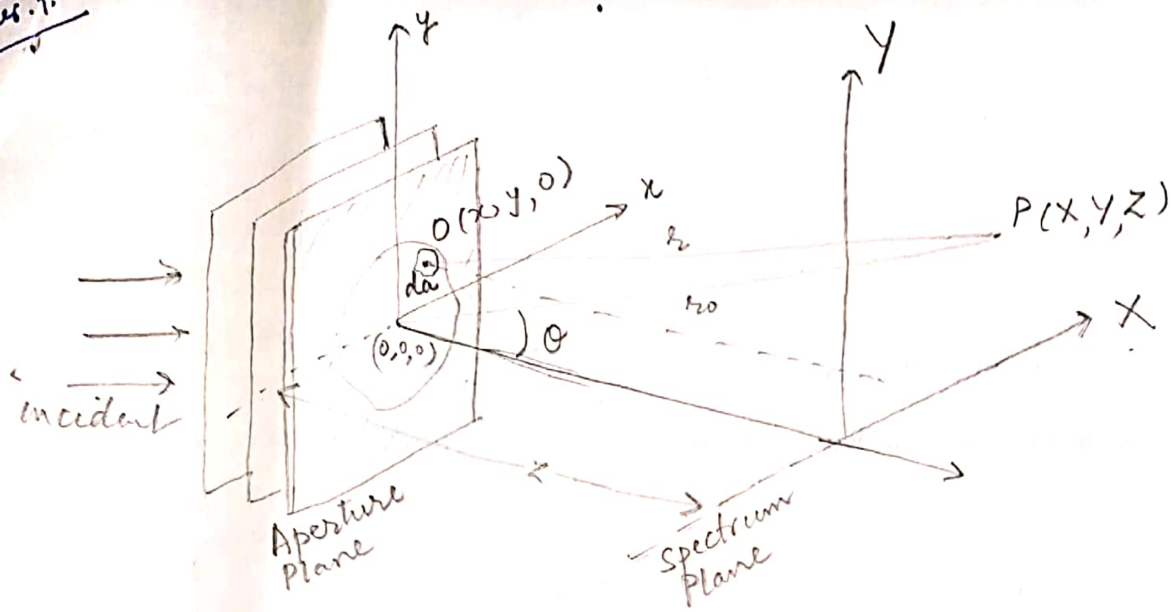
Thus,

Non-periodic fn. $f(x,y)$ = sum of a distribution of plane waves, each with amplitude $g(k_x, k_y)$ and constant phase i.e.

$$xk_x + yk_y = \text{constant} \qquad \text{— (5)}$$

$k_x$, $k_y$ $\cancel{\left(= \frac{2\pi}{\lambda}\right)}$ are spatial frequencies $\left(\frac{2\pi}{\lambda}\right)$ components

- [Eq. (5) is a straight line in x-y plane. As $k_x$, $k_y$ vary the slopes of these lines vary.
- Individual plane waves (of continuous distribution $g(k_x k_y) e^{-(\,)}$) intersect x-y plane along the straight lines given by Eq. (5).
- Thus, synthesis of $f(x,y)$ involves plane waves that vary in direction..




Arbitrary Aperture in $xy$ plane.

Plane monochro. waves diffract from aper. plane

Diff. observed in Spectrum Plane ($XY$ plane) at a distance $Z$

Contribution at P due to light amplitude from an area element $da$ surrounding pt. O $= dE_p$

$E_s \rightarrow$ source strength

$$dE_p = \left(\frac{E_s\, da}{r}\right) e^{i(\omega t - kr)} \quad \text{—— (6)}$$

Eq. (6) represents spherical wave. Its amplitude decreases with distance $r$.

$E_s \rightarrow$ source strength $\equiv$ amp./unit area of aperture near O.

For $r = 1$, amp. $= E_s\, da$

$E_s\, da \Rightarrow$ amp. due to area element $da$ at unit distance from pt. O

✓ If aperture is <u>not uniformly illuminated</u> or not uniformly transparent, then

$$E_s = E_s(x, y) \quad \text{—— (7)},$$ the aperture fn.

Now

$$r^2 = (X - x)^2 + (Y - y)^2 + (Z - 0)^2$$

$$r_0^2 = X^2 + Y^2 + Z^2$$

$$\therefore\ r^2 = r_0^2 + x^2 + y^2 - 2xX - 2yY \quad \text{—— (8)}$$

$x, y << r_0 \quad \therefore\ x^2, y^2$ negligible

$$\therefore r^2 = r_0^2 - 2(xX + yY) \Rightarrow r = r_0\left[1 - \frac{2(xX+yY)}{r_0^2}\right]^{1/2}$$

$$r = r_0\left[1 - \frac{xX+yY}{r_0^2}\right] \quad —(9)$$

In Eq. (8), distance $r$ appears in • amp as well as • phase

In amp. $r \cong Z$,

In phase $r$ = Eq. (9). Then
(∵ phase sensitive to displacement)

$$dE_p = \left[\frac{E_s(x,y)dx\, dy}{Z}\right] e^{i\omega t} \cdot e^{-ik\{r_0(1 - \frac{xX+yY}{r_0^2})\}}, \quad \boxed{da = dx . dy}$$

$$= \left[\frac{E_s\ dx\, dy}{Z}\right] e^{i\omega t}\, e^{-ik\{r_0 - \frac{xX+yY}{r_0}\}}$$

Integrate over the area of the aperture

$$E_p = \frac{e^{i(\omega t - kr_0)}}{Z} \underbrace{\iint E_s(x,y)\, e^{ik\frac{(xX+yY)}{r_0}} dx\, dy}_{\text{relative amplitude distribution of the electric field in the spectrum plane}} \quad —(11)$$

$$E_p \cancel{(k_x, k_y)} = \iint E_s(x,y)\, e^{\frac{ik(xX+yY)}{r_0}} dx\, dy \quad —(12)$$

$$E_p(k_x, k_y) = \iint E_s(x,y)\, e^{i(xk_x + yk_y)} dx\, dy \quad —(13)$$

Here $k_x = \frac{kX}{r_0}$, $k_y = \frac{kY}{r_0}$ are angular spatial freq. corresponding to each pt. $X, Y$ in the spectrum plane.

Compare (13) with (4)

- $E_p$ and $E_s$ are related through F.T.
- Inverse F.T. is

$$E_s(x,y) = \frac{1}{(2\pi)^2} \iint E_p(k_x, k_y)\, e^{-i(xk_x + yk_y)} dk_x\, dk_y \quad —(14)$$

Thus, the Fraunhofer diffract<sup></sup>n pattern described by $E_p(k_x, k_y)$ is the 2d-F.T. of the ~~spectrum~~ aperture fn. described by $E_s(x,y)$.

Thus, the continuous distribution of constituent multidirectional plane waves is responsible for 'bending' of the light into the various regions of the 2d diffraction pattern.