Magnetic dipole moment of Deuteron

Magnetic Moment of the Deuteron.

Assumptions:

① n and p spins are coupled to make total spin $\vec{S}$,

② n and p angular momenta couple to make $\vec{L}$ as total angular momentum.

③ $\vec{L}$ & $\vec{S}$ couple to give the total angular momentum $\vec{J}$

i.e $\vec{J} = \vec{L} + \vec{S}$

We know that $\vec{\mu}$ (magnetic moment) is related to $\vec{J}$ by the following relation

$$\vec{\mu} = \left[(g_L \vec{L} + g_S \vec{S}) \cdot \vec{J} \right] \vec{J} \mu_N$$

where μ_N is nuclear magneton.

By definition

$$\vec{S} = \vec{S}_p + \vec{S}_n$$

$$\vec{\mu}_s = \frac{\left[(g_p \vec{S}_p + g_n \vec{S}_n) \cdot \vec{S} \right]}{S(S+1)} \vec{S} \mu_N \cong g_s \vec{S} \mu_N$$

The orbital angular moment-a of proton in COM system is $\frac{\vec{L}}{2}$, where $\vec{L}$ is the relative angular momentum of the proton w.r.t neutron.

Calculation of Magnetic Moment in ^3S state

$(L = 0, \quad S = 1, \quad J = 1)$

Since we know

$$\frac{\vec{\mu}}{\mu_N} = \frac{\left[\frac{1}{2}(\vec{L}\cdot\vec{J}) + \frac{1}{2}(g_p + g_n)(\vec{S}\cdot\vec{J})\right]}{J(J+1)}\vec{J}$$

Substituting $L=0$, $S=1$, and $J=1$
we get

$$\vec{L}\cdot J = 0$$

$$\vec{S}\cdot\vec{J} = \frac{1}{2}\left[J(J+1) + S(S+1) - L(L+1)\right]$$

$$= \frac{1}{2}\left[2+2-0\right]$$

$$\vec{S}\cdot\vec{J} = 2$$

Hence

$$\frac{\vec{\mu}}{\mu_N} = \frac{\frac{1}{2}(g_p+g_n)\times 2}{1\times 2}\vec{J} = \frac{1}{2}(g_p+g_n)\vec{J}$$

As $\mu_p = g_p s_p \mu_N = \frac{1}{2}(g_p)\mu_N$

$\mu_n = g_n s_n \mu_N = \frac{1}{2}(g_n)\mu_N$

$$g_s = \frac{g_p \vec{s}_p \cdot \vec{S} + g_n \vec{s}_n \cdot \vec{S}}{S(S+1)}$$

We also know that

$$\vec{S} \cdot \vec{s}_p = \frac{S(S+1) + s_p(s_p+1) - s_n(s_n+1)}{2} = L$$

As $\vec{s}_n = \vec{S} - \vec{s}_p$

$$\vec{s}_n \cdot \vec{s}_n = (\vec{S} - \vec{s}_p) \cdot (\vec{S} - \vec{s}_p)$$

$$s_n(s_n+1) = S(S+1) + s_p(s_p+1) - 2\, \vec{S} \cdot \vec{s}_p$$

$$\vec{S} \cdot \vec{s}_p = \frac{1}{2}\left[S(S+1) + s_p(s_p+1) - s_n(s_n+1)\right]$$

Similarly

$$\vec{S} \cdot \vec{s}_n = \frac{1}{2}\left[S(S+1) + s_n(s_n+1) - s_p(s_p+1)\right]$$

$$= 1$$

$$\vec{S}\cdot\vec{S}_n = \vec{S}\cdot S_p = 1$$

Hence

$$g_s = \left(\frac{g_p + g_n}{2}\right)$$

$$\frac{\vec{\mu}}{\mu_N} = \left[\frac{(g_L \vec{L}\cdot\vec{J} + g_s \vec{S}\cdot\vec{J})}{J(J+1)}\right]\vec{J}$$

Since the neutron being uncharged, makes no contribution to the orbital magnetic moment, the proton produces the entire orbital angular momenta

So

$$\mu = \frac{1}{2}\left[2\left(\mu_p + \mu_n\right)\right] \mu_N$$

$$= \left(\mu_p + \mu_n\right) \mu_N$$

$\mu_p = 2.7928.$

$\mu_n = -1.9130$

$$\boxed{\mu = 0.8798\ \mu_N}$$

Magnetic Moment in D state

As we already know

$$\frac{\vec{\mu}}{\mu_N} = \frac{\left[\frac{1}{2}(\vec{L}\cdot\vec{J}) + \frac{1}{2}(g_p + g_n)\,\vec{S}\cdot\vec{J}\right]\vec{J}}{J(J+1)}$$

for D state $L=2$, $S=1$, $J=1$

$$\vec{L}\cdot\vec{J} = \frac{1}{2}\left[J(J+1) + L(L+1) - S(S+1)\right]$$

$$= \frac{1}{2}\left[1\times 2 + 2\times 3 - 1\times 2\right]$$

$$\boxed{\vec{L}\cdot\vec{J} = 3}$$

$$\vec{S}\cdot\vec{J} = \frac{1}{2}\left[J(J+1) + S(S+1) - L(L+1)\right]$$

$$= \frac{1}{2}\left[1\times 2 + 1\times 2 - 2\times 3\right]$$

$$\boxed{\vec{S}\cdot\vec{J} = -1}$$

Hence

$$\frac{\vec{\mu}}{\mu_N} = \frac{\left[\frac{1}{2} \times 3 + \frac{1}{2}(g_p + g_n) \times (-1)\right]}{1 \times 2} \vec{J}$$

$$\vec{\mu} = \frac{1}{2}\left[\frac{3}{2} - \frac{1}{2}(g_p + g_n)\right] \vec{J} \mu_N$$

substituting the values of g_p & g_n in the above expression

$$\mu = \left(\frac{3}{4} - \frac{(2.7928 - 1.9130)}{2}\right) \mu_N$$

$$\mu = 0.3101 \ \mu_N$$

Magnetic Moment of the Deuteron

$\mu_{L=0} = 0.8798 \, \mu_N$ for S state

$\mu_{L=2} = 0.3101 \, \mu_N$ for D state

$\mu_{observed} = 0.8574 \, \mu_N$

$$\psi = a\psi(L=0) + b\psi(L=2)$$

$$\mu_{obs} = \{|a|^2 \times 0.8798 + |b|^2 \, 0.3101\} \, \mu_N = 0.8574 \, \mu_N$$

but $|a|^2 + |b|^2 = 1$

$|a|^2 \simeq 0.96, \quad |b|^2 \simeq 0.04$

The difference between μ_{abs} and $\mu_n + \mu_p$ can not be explained by experimental errors. It can be accounted for by the fact that neutron and proton are not in pure $^3S_1$ state, but in a mixture of $^3S_1$ and $^3D_1$ state. The admixture of 4% of $^3D_1$ state with 96% of $^3S_1$ explains the experimentally observed magnetic moment value.

The assumption of pure $l=0$ state, which was made in calculation for nuclear potential well depth, is pretty good, but not quite exact.

According to the quantum mechanical theory, the angular momentum L is a constant of motion for a system acted upon by a central force.

The simultaneous existence of two different L values ($L=0$ and $L=2$) reflect the non central character of Nuclear force.

Reference Books

- An Introduction to Physics of Nuclei and Particles by R.A. Dunlap
- Nuclear Physics by K.S. Krane

THANK YOU